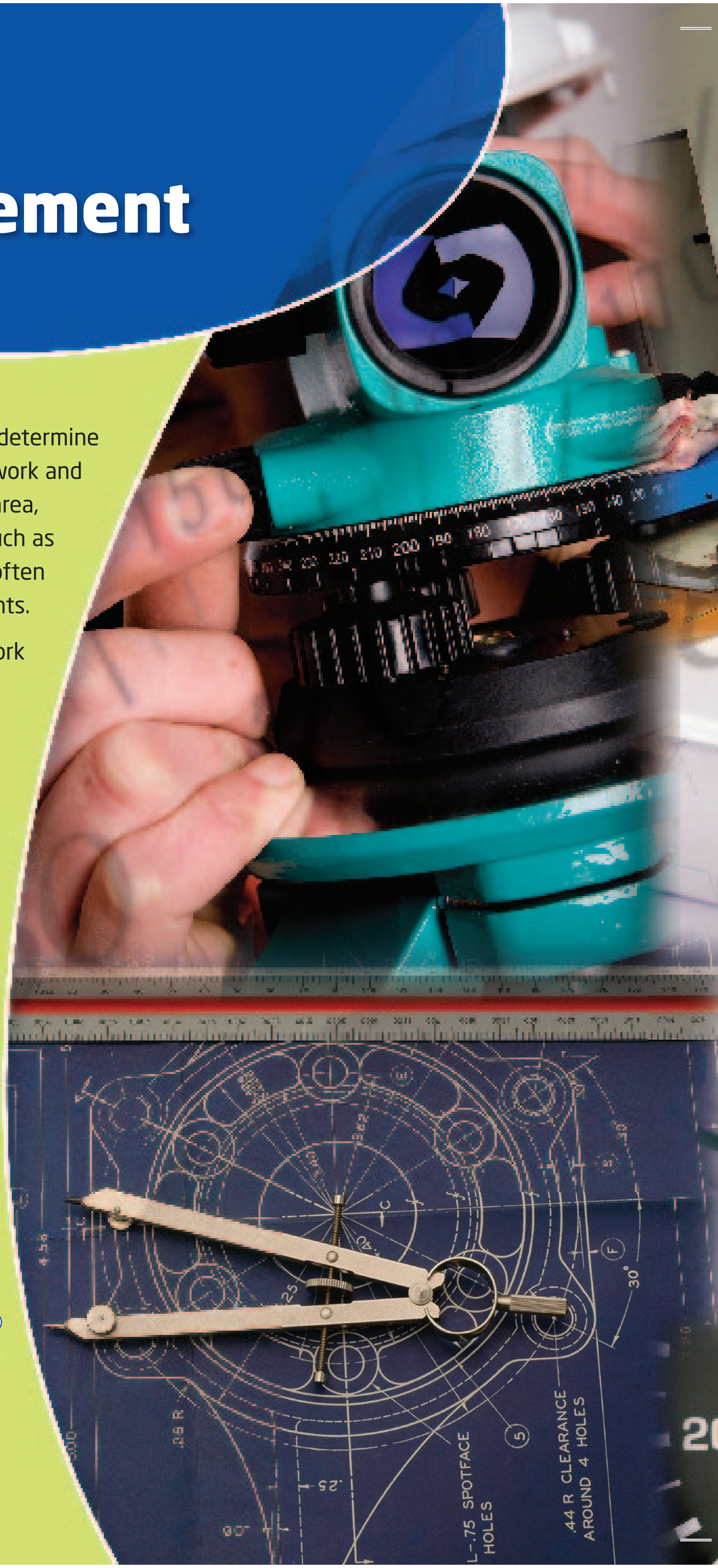
UNIT

1 Measurement

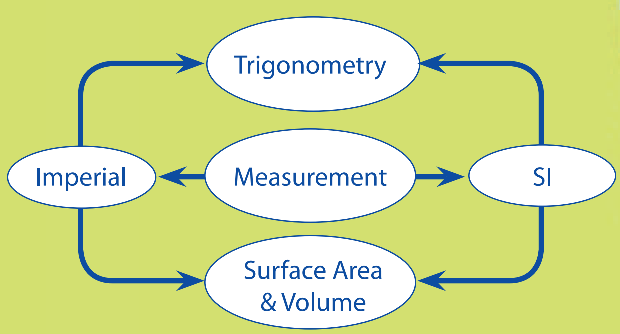
Measurement explores different ways you can determine lengths, areas, and volumes. In many fields of work and in your daily life, you may need to use length, area, and volume measurements. Workers in fields such as architecture, road construction, and surveying often use trigonometry to help calculate measurements.

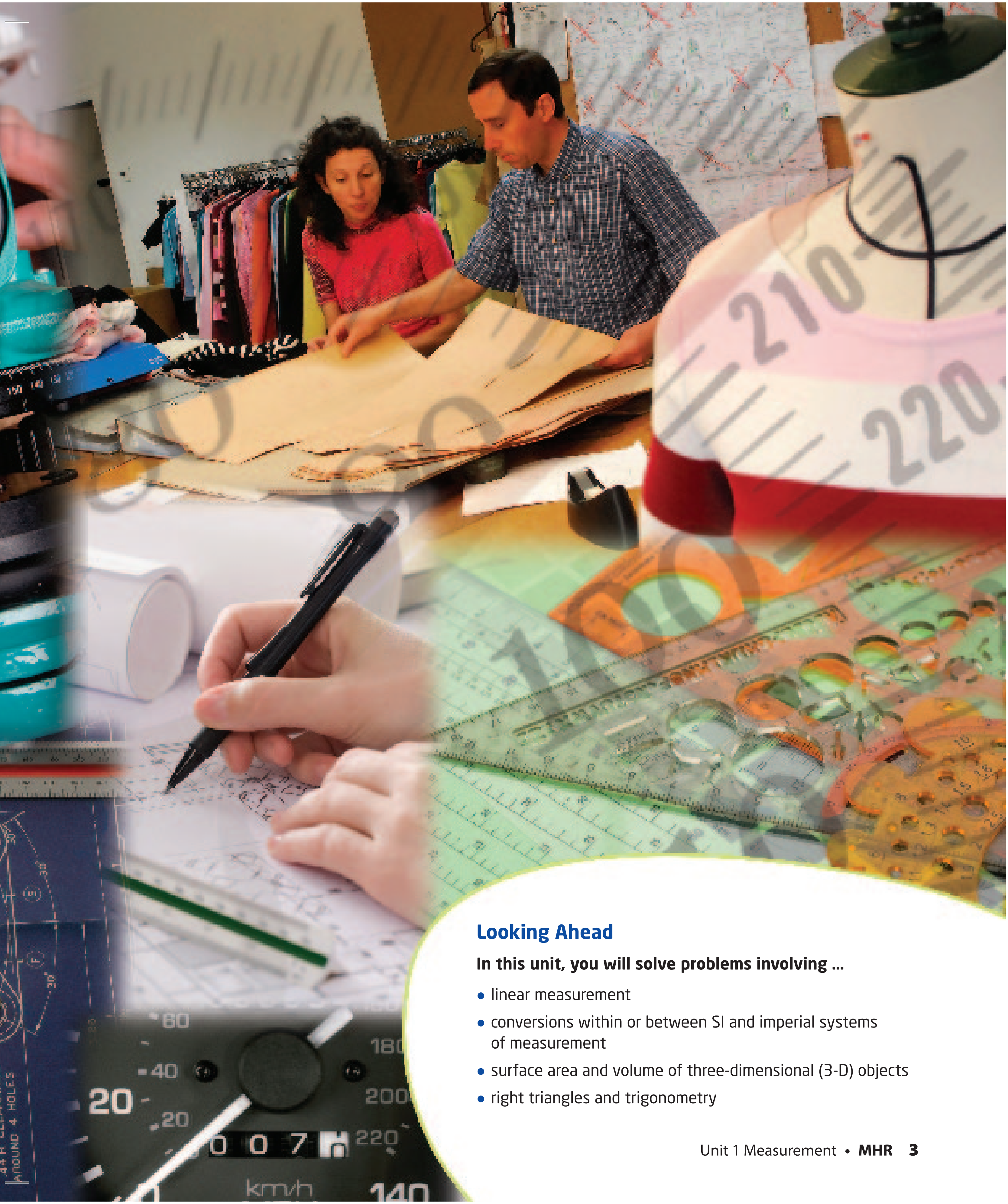
Depending upon your task, you may need to work in the imperial or SI system, or even convert between systems. In this unit, you will take some measurements directly and learn how to calculate surface area and volume of three-dimensional objects. You will learn how to use trigonometry to calculate distances that are difficult to measure.



Your Measurement Organizer

You can use this measurement organizer to see how the concepts in this unit are connected. You will see this organizer on the first page of each chapter. The concepts covered in that chapter are highlighted.





Looking Ahead

In this unit, you will solve problems involving ...

- linear measurement
- conversions within or between SI and imperial systems of measurement
- surface area and volume of three-dimensional (3-D) objects
- right triangles and trigonometry

Unit 1 Project

Changes in Music Distribution

The music industry has been big business for decades. Technological advances have affected how music is recorded, distributed, and listened to. Over the years, different devices have been used to store and transfer music. Many of these devices could hold only a few songs. Today, the increasing demand for greater storage and distribution capabilities has created a need for improved communication networks.

In the Unit 1 project, you will prepare a presentation that includes research on the history of music recording, a comparison of storage devices, a description of the impact of technology on music distribution, and a prediction of the next technological advance.

Throughout Chapters 1, 2, and 3, **Unit Project** questions will help you gather some of the information for your project. You may need to conduct additional research to complete your project.

While completing your project, you will ...

- research the history and storage of music over time (throughout Unit 1)
- estimate and calculate linear measurements of music storage devices (Chapter 1)
- apply your understanding of unit conversions (SI and imperial) to compare music storage devices (Chapters 1 and 2)
- use surface area and volume to compare music storage devices (Chapter 2)
- use the Pythagorean theorem and trigonometry to explore how wireless systems have affected music distribution (Chapter 3)
- predict the next technological advance in music distribution (end of Unit 1)

CHAPTER

1 Measurement Systems

Measurement is the process of finding the size of an object. It relates the unknowns in our world to accepted or standard units. Linear measurement is critical to many jobs. How is measurement being used in each picture? Challenge yourself to think of a career that does not have a connection to measurement.

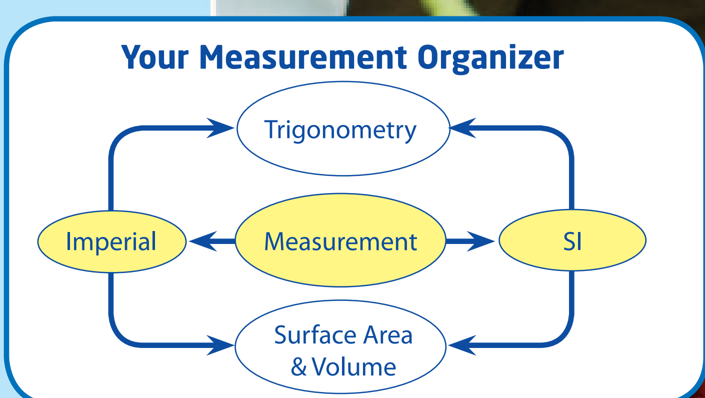
Big Ideas

When you have completed this chapter, you will be able to …

- estimate a linear measurement using a referent and justify your choice of units
- solve problems involving linear measurement using measuring instruments
- provide referents for linear measurements
- convert measurements within or between the SI and imperial systems

Key Terms

SI
referent
imperial system

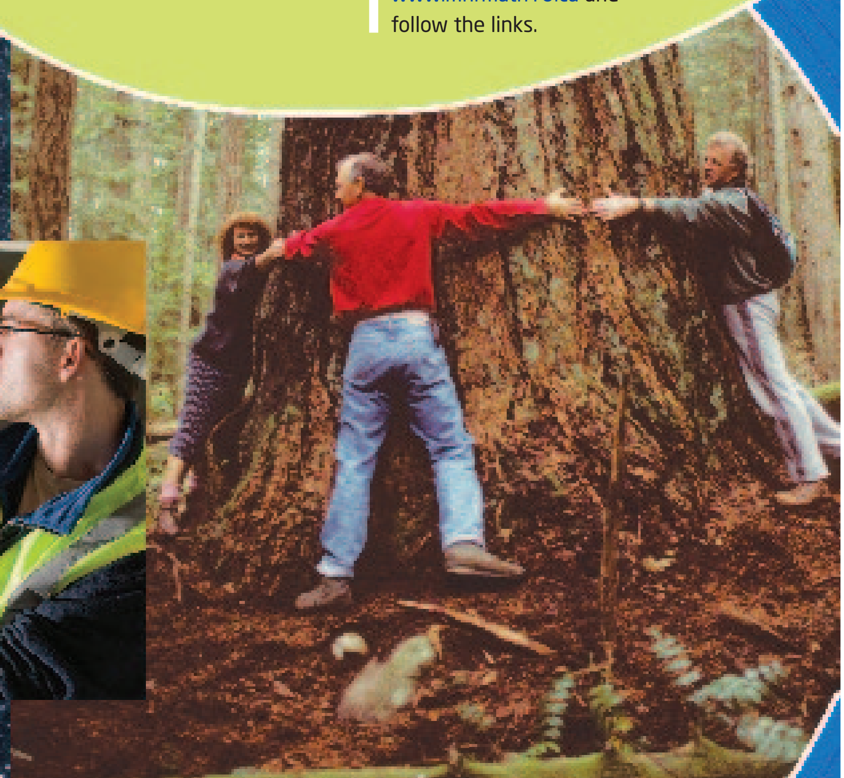


Photogrammetrist

Photogrammetrists analyse, measure, and interpret aerial photographs. Using these skills, they get information about 3-D objects. The information is used in making maps, in ecological studies, and in forestry. A photogrammetrist may be hired to track an oil spill or engineer a roadway.

Web Link

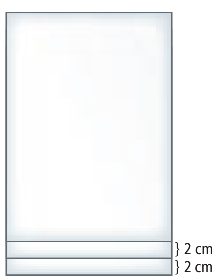
To learn more about photogrammetrists, go to www.mhrmath10.ca and follow the links.



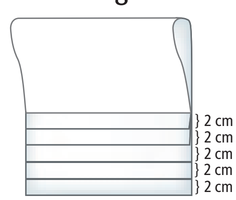
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 1.

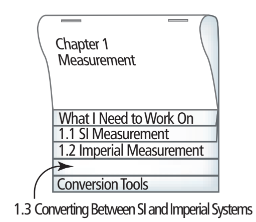
1. Create a booklet by staggering three sheets of $8\frac{1}{2}$ in. by 11 in. paper to create tabs that are approximately 2 cm wide.



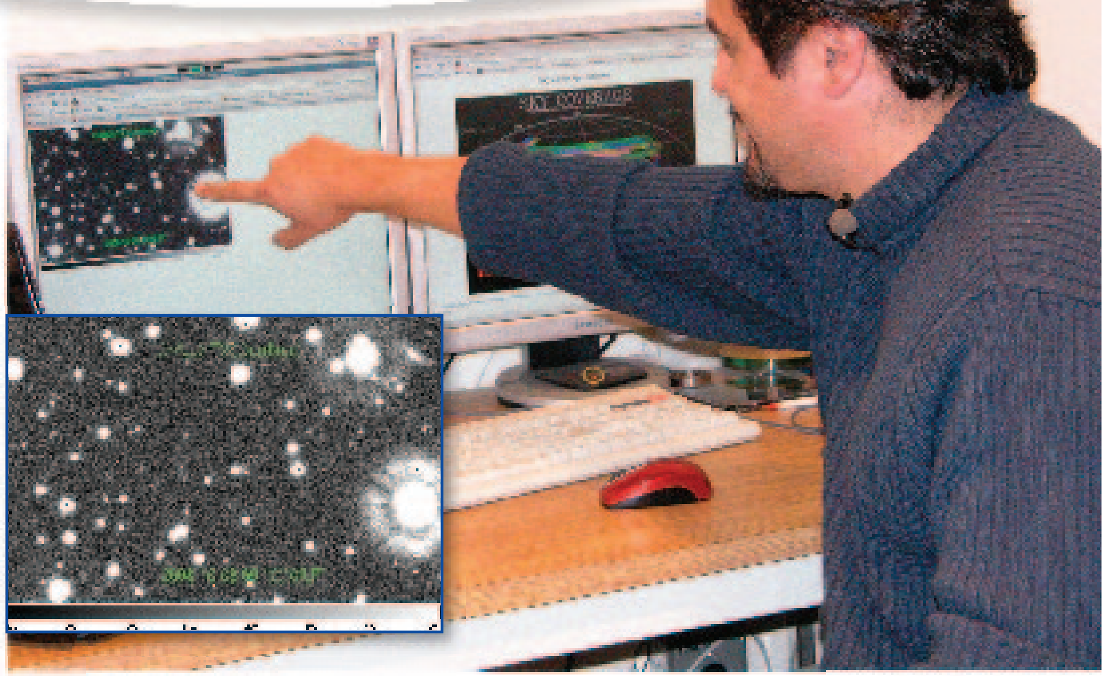
2. Fold the top of the booklet toward you and align these tabs with the existing ones. All of the tabs should appear the same height.



3. Staple the top to hold the pages in place. Write the title of each section on the tabs.



1.1 SI Measurement



Focus on ...

- justifying the units used for a measurement
- solving problems that involve linear measurement
- explaining the process used to estimate a linear measurement
- selecting appropriate referents

Rob Cardinal, who is of the Siksika First Nation, is a researcher at the University of Calgary's Rothney Astrophysical Observatory. On October 1, 2008, he took some images that he thought were of an asteroid. It turned out to be an undiscovered comet and was named Comet Cardinal. Whether you are exploring the universe, hiking in the Rockies, or travelling in the Prairies you will be using linear measurement with SI units. SI stands for Système International d'Unités. What SI units can you name? What type of measurement is associated with each unit?

Materials

- three items that are non-standard measuring units (e.g., coin, paperclip)
- SI measuring tape
- grid paper

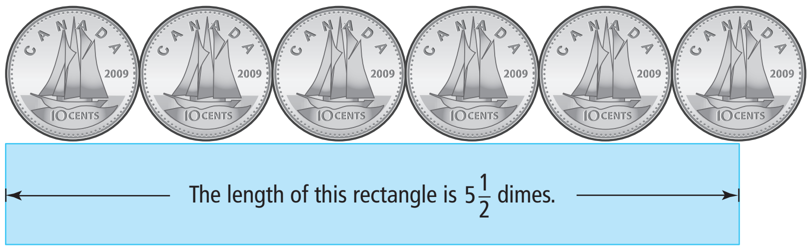
Investigate Dimensions of a Rectangle

What effect does the unit of measurement have on the length-to-width ratio of a rectangle?

1. Work with a partner. Draw a rectangle on half of a sheet of paper.

2. Choose three non-standard units to measure the dimensions of your rectangle. Estimate how many of each non-standard unit it takes to measure the length and width of your rectangle.

3. Measure the dimensions using each non-standard unit. Record all data in a chart or table. You may need to record measurements using fractions or decimals.



4. Estimate the length and width of your rectangle in millimetres and centimetres. Then, confirm your estimate by measuring using each of these standard units. Record your estimates and measurements.

5. For each unit of measurement, plot your measurements as (length, width) coordinate pairs on a grid.

6. **Reflect and Respond**
   - **a)** Describe any patterns you see in the graph.
   - **b)** Does the ratio of length to width for your rectangle vary when you change the units of measurement? Explain.

7. Discuss the advantages and disadvantages of using standard units for measuring distance.

Link the Ideas

Canada's official measurement system is **SI (Système International d'Unités)**. Some SI units for linear measurement are listed in the table.

Unit	Abbreviation	Multiplying Factor
kilometre	km	1000
hectometre	hm	100
decametre	dam	10
metre	m	1
decimetre	dm	0.1
centimetre	cm	0.01
millimetre	mm	0.001

Various measuring instruments allow accurate measurement of distances in standard units. You can also develop personal **referents** to use when estimating measurements.

SI (Système International d'Unités)
- a system of measurement in which all units are based on multiples of ten
- the metre is the basic unit of length

referent
- an item that an individual uses as a measurement unit for estimating
- for example, the height of a doorknob above the floor is about 1 m, or the thickness of a dime is about 1 mm

Measuring Instruments

SI rulers, metre sticks, and measuring tapes give measurements to the nearest millimetre, or 0.1 cm. A caliper can accurately measure to the nearest tenth of a millimetre, or 0.01 cm, depending on the scales.

Follow these steps to read a caliper.

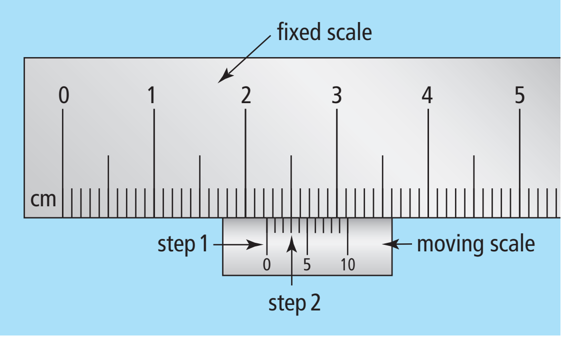
1. Read the value on the fixed scale that is located exactly at or just to the left of the zero on the moving scale.

   For the caliper shown, the reading is 2.2.

2. Identify the next line on the moving scale that aligns with a line on the fixed scale. Read the value on the fixed scale.

   For this caliper, it is the line on the moving scale that represents 0.03.

3. The final reading is 2.23 cm. (2.2 + 0.03 = 2.23)



Web Link

To watch a video showing how to read an SI caliper, go to www.mhrmath10.ca and follow the links.

Referents

A non-standard measuring unit can be used as a personal referent. Referents help individuals estimate in standard units, such as SI units. For example, suppose you use the width of your fingernail to approximate 1 cm. Then, when you measure something that appears to be as wide as 4 of your fingernails, you can estimate that it is 4 cm wide.

Several referents are possible for each of the main linear SI units, the millimetre, centimetre, metre, and kilometre. Finding a referent for a kilometre is more difficult, because it is a large unit. You might choose a referent for a kilometre to be 12 or 13 city blocks.

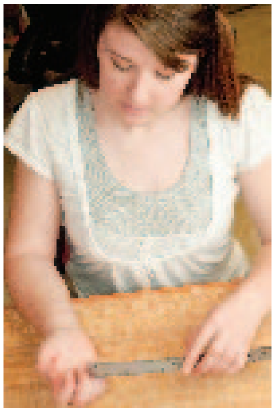
Example 1 Estimate and Measure Using SI Units

Estimate each distance using an appropriate referent. Then, measure each distance.

a) the thickness of a CD case
b) the height of the seat of a chair
c) the width of this page

Solution

a) Use the width of one fingernail as a referent for 1 cm.



The thickness of a CD case is approximately as wide as half of one fingernail. Estimate the thickness of a CD case as 0.5 cm. Measure, using an SI ruler or caliper. The thickness of a CD case is 4.5 mm or 0.45 cm.

b) The height of the seat of a chair is approximately half of waist height. Use waist height as a referent for 1 m. Estimate the seat of the chair as 0.5 m high, or 50 cm. Measure, using a measuring tape. The height of the seat of a chair is 46 cm.

c) Use the width of one fingernail as a referent for 1 cm. Count the number of fingernails that fit across half of this page and double the number. An estimate of the width of this page is 22 cm. Measure the width using an SI ruler. The width of this page is 21.5 cm.

Your Turn

Estimate the height of the chalk or marker tray on a blackboard or whiteboard using an appropriate referent. Then, measure this height.

Example 2 Convert Between SI Units for Length

A newspaper reported the following measurements in different stories.

The distance from Earth to the moon is 38 440 300 000 cm.

A worm measures 0.0019 m.

a) For each measurement, state a more appropriate SI unit. Justify your choice.
b) Convert the given measurement to the more appropriate unit.

Solution

a) The distance from Earth to the moon is very large. It could be measured in kilometres rather than centimetres. The length of a worm is very small. It could be measured in millimetres rather than metres.

b) Convert 38 440 300 000 cm to kilometres.

Use *unit analysis* to calculate the number of centimetres in 1 km.
1 km = 1000 m and 1 m = 100 cm
So, 1 km = (1000)(100) cm
1 km = 100 000 cm

$$38\ 440\ 300\ 000 \text{ cm}\left(\frac{1 \text{ km}}{100\ 000 \text{ cm}}\right) = 384\ 403 \text{ km}$$

The measurement 38 440 300 000 cm can be converted to 384 403 km.

Convert 0.0019 m to millimetres.

Let x represent the number of millimetres.
Use proportional reasoning.

$$1000 \text{ mm} = 1 \text{ m}$$
$$\frac{1000 \text{ mm}}{1 \text{ m}} = \frac{x \text{ mm}}{0.0019 \text{ m}}$$
$$1000(0.0019) = x$$
$$1.9 = x$$

The measurement 0.0019 m can be converted to 1.9 mm.

How do you decide which values to use for the numerator and denominator?

Your Turn

Convert each measurement to a more appropriate SI unit. Justify your choice of unit.
a) A tube of toothpaste is 205 mm long.
b) The circumference of a highlighter measures 0.06 m.
c) You travel 590 000 m from Regina to Winnipeg.
d) The top of a door is 2110 mm high.

Did You Know?

In a fraction, common factors in the numerator and denominator divide to make 1. This concept is used to simplify fractions to lowest terms.

Example:

$$\frac{18}{45} = \frac{(\cancel{9}^1 \times 2)}{(\cancel{9}_1 \times 5)} = \frac{2}{5}$$

In *unit analysis*, the same concept can be applied to units of measurement.

Example:
To convert from metres to millimetres,

$$(25 \text{ m})\left(\frac{1000 \text{ mm}}{1 \text{ m}}\right)$$
$$= \left(\frac{25\ \cancel{\text{m}}}{1}\right)\left(\frac{1000 \text{ mm}}{1\ \cancel{\text{m}}}\right)$$
$$= 25\ 000 \text{ mm}$$

Example 3 Solve a Problem Involving Linear Measurement

Kyla buys an oversized wooden barrel. She cuts it in half to make a planter. She wants to place a metal band around the planter, 4 cm from the top, to hold the planter together.

a) If the radius 4 cm from the top of the planter is 0.6 m, what length of band will she need? Express your answer to the nearest centimetre.

b) If the bottom band of her planter is 1 m shorter than the top band, what is the radius of the planter at the bottom band? Express your answer to the nearest centimetre.

c) What is the difference between the radius of the planter at the top band and the radius at the bottom band?

d) Show how much the radius of any barrel increases if 1 m is added to the length of a band. State your answer as an exact value. Then, express your answer to the nearest centimetre.

Solution

a) The length of the band is equal to the circumference of the planter at 4 cm from the top.

$C = 2\pi r$

$C = 2\pi(0.6)$

$C = 3.7699\ldots$

How do you convert 3.77 m to centimetres?

The length of the band 4 cm from the top is 3.77 m or 377 cm.

b) The circumference of the planter at the bottom band is equal to the length of the bottom band. The bottom band is 1.0 m shorter than the top band.

$3.77 - 1.0 = 2.77$

The length of the bottom band is 2.77 m.

Calculate the radius of the planter at the bottom band.

$$\begin{aligned} C &= 2\pi r \\ 2.77 &= 2\pi r \\ \frac{2.77}{2\pi} &= r \\ 0.4408\ldots &= r \end{aligned}$$

The radius of the planter at the bottom band is 0.44 m or 44 cm.

c) Calculate the difference between the two radii.

$0.6 - 0.44 = 0.16$

The difference between the radius of the planter at the top band and the radius at the bottom band is 0.16 m or 16 cm.

Did You Know?

Wooden barrels bulge in the middle. The bulge or bilge, as it is known, is designed to make it easier to roll and to change direction while rolling a barrel.

d) The formula $C = 2\pi r$ represents the circumference, C, for any barrel with radius r. A barrel whose circumference increases by 1 m and radius increases by x metres can be represented by the following formula.

$$C + 1 = 2\pi(r + x)$$
$$C + 1 = 2\pi r + 2\pi x$$
$$C + 1 = C + 2\pi x$$

Substitute C for $2\pi r$.

$$1 = 2\pi x$$
$$\frac{1}{2\pi} = x$$

The radius of the barrel increases by the exact value $\frac{1}{2\pi}$.

$$\frac{1}{2\pi} = 0.1591549431$$

The radius of the barrel increases by approximately 0.16 m or 16 cm for every 1 m increase in circumference.

How does this answer relate to your answer to part c)?

Why do you think this increase will hold true for any size barrel?

Did You Know?

Red River carts were used by Métis to transport goods and belongings. The carts were pulled by horses or oxen. Sometimes several carts were attached in a line to haul a greater amount of freight. Red River carts were built entirely of wood, sinew, and rope. This allowed the carts to float easily with the wheels removed. Prairie dust prevented the axles from being effectively greased. As a result, the carts made a loud squeaking noise.

Your Turn

Suppose the inner rim of a Red River cart wheel has a circumference of 7.1 m. Each spoke is 1 m long. What is the diameter of the centre circular hub of the wheel?

Key Ideas

- Each unit in the SI measurement system is a multiple of 10. All linear measurements are derived from the metre. The most common units are the kilometre (km), metre (m), centimetre (cm), and millimetre (mm).
- The kilometre is a large unit (1 km = 1000 m) and is suitable for measuring large distances.

- The millimetre is a small unit $\left(1 \text{ mm} = \frac{1}{1000} \text{ m}\right)$ and is suitable for measuring small distances.
- A referent is a personal measurement unit that you can use to estimate measurements in standard units, such as SI units.

  Estimate the length of an eyeglass case.

  Use a personal referent, such as "the width of your palm is 7.5 cm."

  The length of an eyeglass case measures 2 times as wide as your palm.
  $7.5(2) = 15$

  Estimate the length of the eyeglass case to be 15 cm.

Check Your Understanding

Practise

1. a) Estimate the perimeter of each figure in an appropriate SI unit.

i)

ii)

b) Measure the perimeter of each figure. If all the angles in each figure are right angles, is it necessary to measure all sides of the figures? Explain.

2. a) On a plain piece of paper, draw a letter S whose curve length you estimate to be each distance.

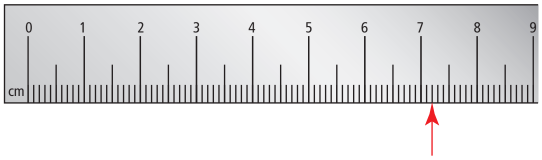
   i) 25 mm   ii) 20 cm

   b) Explain how you could measure the distance of each curved letter you drew.

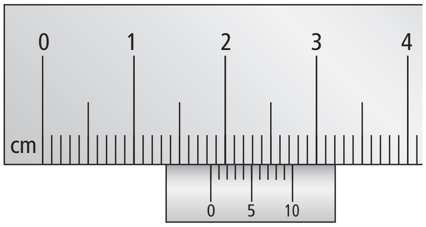
   c) Measure each S and compare your measurements with the required distances. If you are out by more than 5 mm for part i) or 2 cm for part ii), try drawing the letter again.

3. What reading is shown on each measuring instrument? Give each reading in both millimetres and centimetres.

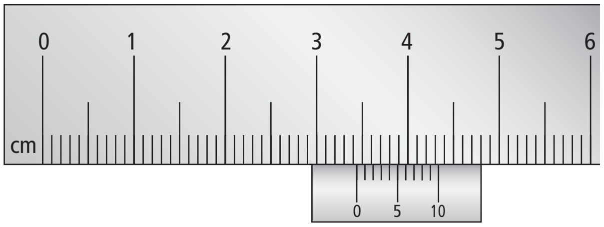
   a) SI ruler



   b) SI caliper



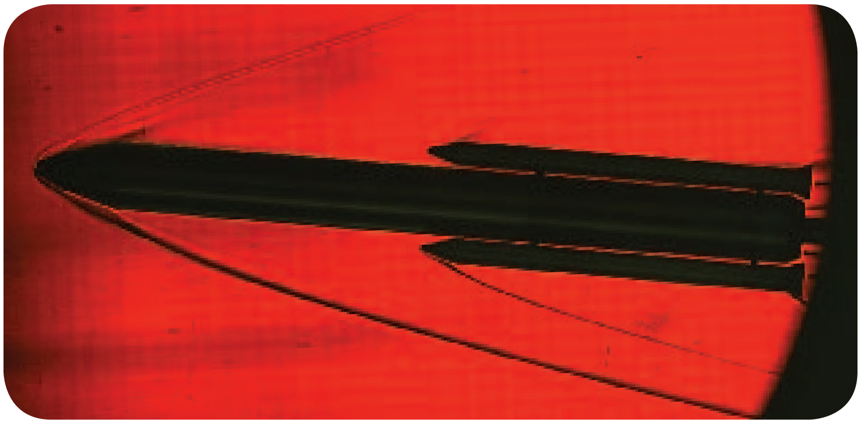
   c) SI caliper



Did You Know?

The Ares V is the cargo launch component for Project Constellation. NASA plans manned space travel in this vehicle, in 2019. The Ares V can carry 188 000 kg into low Earth orbit and 71 000 kg to the moon.

4. The photograph shows a wind-tunnel test of the airflow over a model of NASA's heavy launch vehicle, called Ares V.



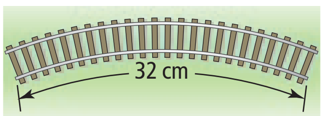
   a) Assume the photo and model show a reduction of approximately 1 : 1000. What is the actual length of Ares V?

   b) What is the diameter of one of the actual solid-rocket boosters?

5. Consider each measurement. State whether it is reported in the most appropriate unit. If it is not, explain why and convert to a more appropriate unit.
   a) The highest mountain in Canada, Mount Logan, has a height of 595 900 cm.
   b) The diameter of a water bottle is 0.064 m.
   c) The world's tallest bear was 4200 mm.
   d) A whooping crane's wingspan is 0.001 95 km.

Apply

6. A circular model railway track is made by connecting 12 pieces like the one shown.



   a) When the 12 track pieces are assembled, what distance does a toy train travel along the inside of the track?
   b) Calculate the radius of the inside edge of the track, to the nearest millimetre.
   c) Estimate the radius of the outside edge of the track.

7. Give some examples of measuring instruments that are used to measure distance in the home, community, or workplace. Show or explain how one of these instruments works.

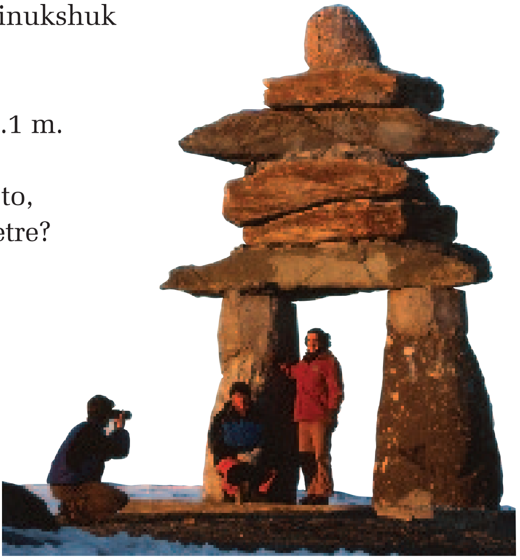
8. Jacques wants to build a trundle wheel. He wants the wheel to go around once for every metre the trundle wheel is pushed. What will be the radius of his trundle wheel?

9. A magazine editor needs to assess whether the photograph can be reduced proportionately to fill the rectangle below. Can it be? Explain, using measurements and ratios.

Did You Know?

The inukshuk is a symbol of Inuit culture. It was traditionally used to mark a place of respect, to help hunt caribou, or as a landmark. Inukshuks are made by piling rocks on top of each other. They may be created in many forms; however, few have the form of a person.

10. The photograph shows an inukshuk overlooking Rankin Inlet in Nunavut. Suppose the height of the inukshuk is 4.1 m. What is the height of the person standing in the photo, to the nearest tenth of a metre?

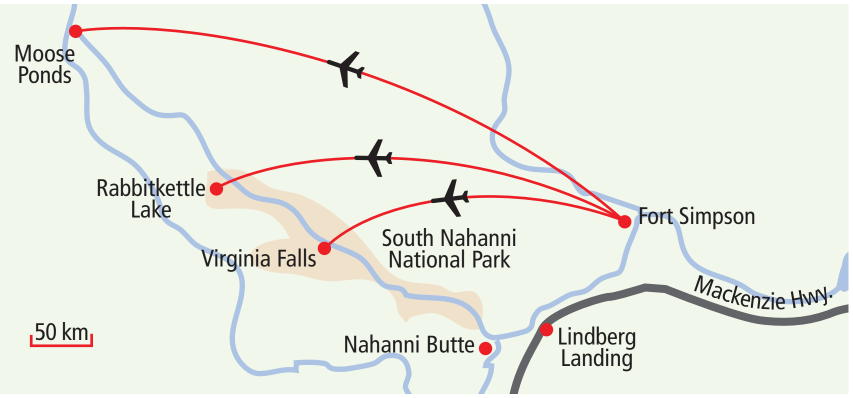


11. a) Measure the diameter of a Canadian dime and quarter.

b) Calculate the ratio of the diameter of a dime to the diameter of a quarter.

c) If this ratio applies for a quarter and a loonie, what would the diameter of the loonie be? Does the ratio apply? Justify your reasoning.

12. Use the map of part of the Northwest Territories to help answer the following questions.



a) Express the scale of the map as a ratio in lowest terms.

b) Estimate the distance from Fort Simpson to Moose Ponds. Measure and compare the distance with your estimate.

c) Compare the distances from Virginia Falls and Rabbitkettle Lake to Fort Simpson. How much greater is the distance from Rabbitkettle Lake?

13. A geostationary satellite is in orbit 35 800 km above Earth's equator. The average radius of Earth at the equator is 6380 km.

a) Draw and label a diagram of Earth and the path of the satellite.

b) What distance does an observer on the equator travel in one day due to Earth's rotation? Express your answer to the nearest tenth of a kilometre

c) How far must the geostationary satellite travel in one day to appear stationary above Earth? Express your answer to the nearest tenth of a kilometre.

d) How much faster is the satellite travelling than the observer on Earth? Hint: Use the following formula.

$$\text{Velocity (in kilometres per hour)} = \frac{\text{distance (in kilometres)}}{\text{time (in hours)}}$$

Did You Know?

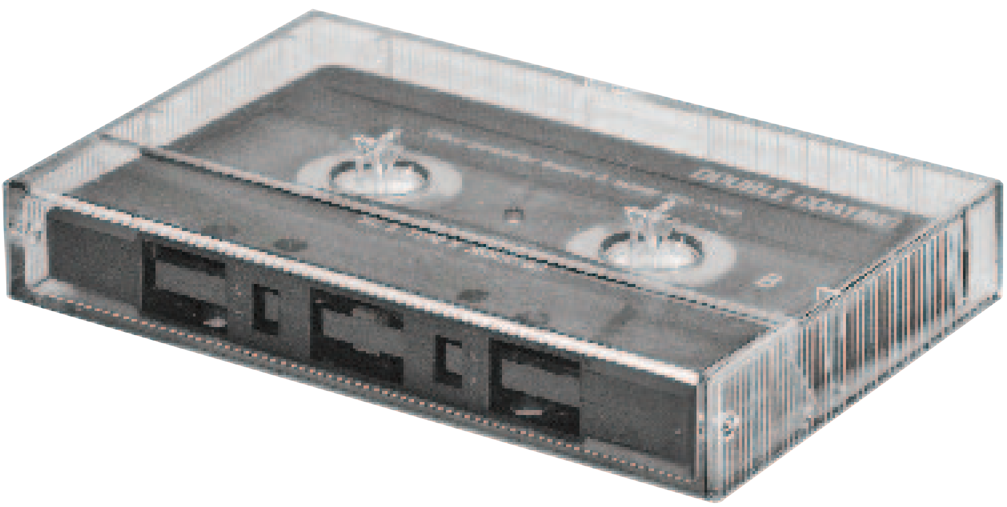
A geostationary satellite is a satellite that appears in a fixed position to an observer on Earth. The satellite revolves around Earth at the same distance above the equator. These satellites are used for communications such as direct TV distribution.

14. **Unit Project** The music industry involves the production, distribution, and sale of music in a variety of forms. Since the 1960s, music distribution has evolved from vinyl records to cassette tapes, to CDs, and to MP3s. Each change emphasized that smaller is better.

a) For the actual-size cassette shown, use a suitable referent to estimate the dimensions of the cassette case. Explain why you used that referent.

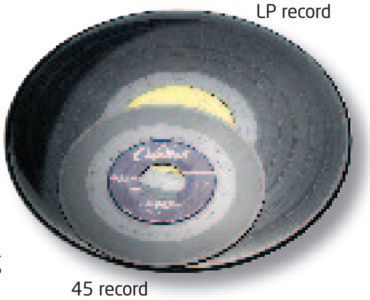
WWW Web Link

To learn more about the history of music distribution, go to www.mhrmath10.ca and follow the links.



b) Measure and calculate the perimeter of each different face of the cassette case, in millimetres. How many perimeters do you need to calculate?

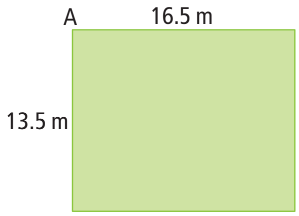
c) Vinyl records are available in three sizes—45 rpm (revolutions per minute), 78 rpm, and $33\frac{1}{3}$ rpm or LP size. A 45 record has an actual diameter of 17.5 cm. Estimate the diameter of the LP in the photograph. Then, by measuring and determining a scale, calculate the actual diameter of the LP, in millimetres.



Extend

15. The scale of an aerial photograph can be approximated as the ratio of the camera's focal length to the airplane's altitude. For this aerial photograph taken near Fort McMurray, AB, the airplane's altitude is 305 m. The camera's focal length is 45 mm. What is the greatest distance across the crater shown in the photo?

16. Your lawn has the dimensions shown. You cut the grass using a mower that cuts with a width of 52 cm.



a) To mow the entire lawn, what is the minimum distance you must walk? You start at A and return to A when finished.

b) Does your route, for example, along the perimeter versus in rows, affect the distance you walk? Explain.

Create Connections

17. Imagine a band or ring placed tightly around Earth at the equator. You cut the band and lengthen it by 1 m. If you could block the new band so it is equally spaced above the equator, what distance would it be away from Earth? Assume Earth is circular at the equator and the radius of Earth is about 6400 km.

18. Work with a partner. Sometimes a sprained ankle needs to be wrapped with a compression bandage.

a) Estimate the length of elastic bandage needed to wrap your ankle then foot repeatedly for four cycles.

b) Wrap your ankle and foot using the figure eight wrapping technique with a strip of cloth. Measure the length required. Compare your estimate with the actual measurement.

c) Darwin estimates the length of bandage by determining the approximate circumferences of his ankle and foot and multiplying the sum by four. He notices that his estimate is not close to his actual measurement of the bandage. Explain why.

19. Sandra lives in Salmon Arm, BC. She wants blue orchard mason bees to live in her backyard. She purchases a bee nesting box and intends to place it in one of her flower beds. The front of the box measures 15 cm by 15 cm on the inside. The outer diameter of each nesting tube is 8 mm.

a) Estimate the number of tubes that can fit inside the box.

b) Draw and label a diagram showing the dimensions of the nesting box.

c) Calculate the maximum number of tubes that can fit inside the box. Then, describe one way to check your work.

Did You Know?

Blue orchard mason bees tend to be quite gentle around people and pets. They are native to North America, and recognized as effective pollinators. Each female builds a nest by herself. She forages nectar and pollen and lays eggs.

20. A factory makes frying pans. The inside surface of each pan is coated with a non-stick coating. You are hired to verify the formula that the factory uses to calculate the area of the inside surface. The company formula is $S = \pi d\left(\frac{d}{4} + h\right)$, where S represents the inside surface area, in square centimetres; d represents the diameter of the pan across the top, in centimetres; and h represents the height up the side of the pan, in centimetres.

a) Measure a frying pan. Calculate the inside surface area using a technique of your choice. Give your answer in terms of π. Then, calculate the surface area using the company formula. Compare the values. Is the factory formula correct? Justify your response.

b) Li says that most frying pans will have an inside surface area slightly greater than the one calculated using the formula. Do you agree with Li? Explain why. Use an example and a diagram to support your explanation.

21. **MINI LAB** Work in a small group to establish a personal referent for a kilometre. Mark off a square that has sides of length 25 m. For each group member, measure the time it takes to walk around the square once.

Step 1 Estimate how long it might take to walk 1 km.

Step 2 Measure actual times by having each group member walk at a normal pace 10 times around the square. Why might your estimate not be close to the actual time?

Step 3 Walk along a street or road for your length of time from step 2. Measure the distance using an outdoor measuring device.

Step 4 List some places that are about 1 km from your school or home.

Materials

- SI measuring tape
- watch
- outdoor measuring device, such as an odometer, pedometer, trundle wheel, or measuring tape

1.2 Imperial Measurement

Focus on …

- providing referents for linear measurements
- describing a strategy for taking a linear measurement
- solving problems that involve linear measurement using instruments
- estimating linear measurements

Most of the world uses the SI measurement system. The United States uses the **imperial system** for linear measurement. This system is based on the older English units of measurement derived from nature and everyday activities.

Canada began a transition from the imperial system to SI in 1970, but imperial measurement is still used. Why do you think this is true? Where have you seen imperial units used to measure distance? What other imperial units can you identify?

The Royal Observatory in Greenwich, United Kingdom displays metal representations for the smaller imperial distance measurements. These include the inch, foot, and yard.

imperial system

- a system of measurement based on British units

Materials

- imperial linear measuring instrument

Investigate Referents for Imperial Measurement

One of the smallest imperial units for measuring distance is the inch. The next unit larger than the inch is the foot. The next unit larger than the foot is the yard. Work with a partner. Share your answers with your classmates.

1. Identify the length of one inch, one foot, and one yard on your measuring instrument.

a) How many inches are in one foot?

b) How many feet are in one yard?

c) How many inches are in one yard?

2. List objects at school or at home that you could use as a referent for one inch, one foot, and one yard.

3. What could you use as a referent for one mile?

4. **Reflect and Respond**
   a) Choose an object in your classroom. Describe how to use a referent to measure the dimensions of the object.
   b) Explain why you chose your referent.

Link the Ideas

The following units are the basic imperial units used for measuring distances. They are in order from smallest to largest. The abbreviations and symbol are in brackets.

inch (in. or ″)	
foot (ft or ′)	1 ft = 12 in.
yard (yd)	1 yd = 3 ft or 36 in.
mile (mi)	1 mi = 1760 yd or 5280 ft

How many inches are in $3\frac{1}{2}$ yd?

$$\begin{aligned} 1 \text{ yd} &= 36 \text{ in.} \\ 3\tfrac{1}{2} \text{ yd} &= 3\tfrac{1}{2} \text{ yd}\left(\frac{36 \text{ in.}}{1 \text{ yd}}\right) \\ 3\tfrac{1}{2} \text{ yd} &= 126 \text{ in.} \end{aligned}$$

$\left(3\frac{1}{2}\right)(36) \approx 4(30)$ or $3(40)$.
So, an estimate is 120.

There are 126 in. in $3\frac{1}{2}$ yd.

Approximately how many miles are in 12 640 ft?

$$\begin{aligned} 1 \text{ mi} &= 5280 \text{ ft} \\ \frac{1 \text{ mi}}{5280 \text{ ft}} &= \frac{x \text{ mi}}{12\,640 \text{ ft}} \\ \frac{1(12\,640)}{5280} &= x \\ 2.3939\ldots &= x \end{aligned}$$

There are approximately 2 mi in 12 640 ft.

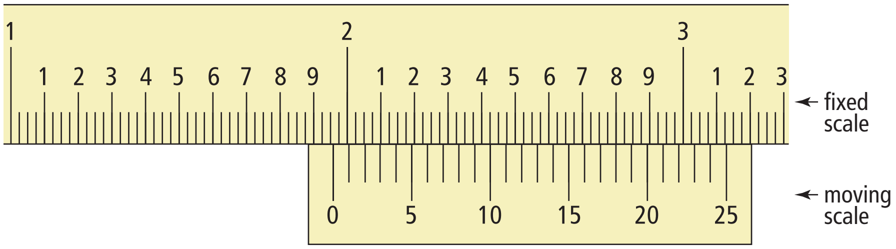
Measuring Instruments

Different measuring devices are used depending on the precision required. An imperial ruler or measuring tape can measure distances to the nearest $\frac{1}{16}$ in. A caliper can measure to the nearest $\frac{1}{1000}$ in.

WWW Web Link

To watch a video showing how to read an imperial caliper, go to www.mhrmath10.ca and follow the links.

Follow these steps to read an imperial caliper.



1. Read the whole number and tenth values on the fixed scale.

This reading is 1.9 in.

2. Determine where zero on the moving scale lies relative to, in this case, the 9 on the fixed scale.

It is 2 small divisions beyond the 9.

$\frac{2}{4}$ of $\frac{1}{10} = \frac{1}{20}$ or 0.05.

This reading is 0.05 in.

3. Identify the next line on the moving scale that aligns with a line on the fixed scale. In this example, it is 10.

This reading is 0.010 in.

4. Add the measurement readings from steps 1 to 3.

The final reading is 1.960 in. $(1.9 + 0.05 + 0.010 = 1.960)$

Example 1 Determine Imperial Distances

The photograph shows a polar bear near Churchill, MB. The scale of the photograph is 1:24.

a) Calculate the height of the bear's back, to the nearest inch.
b) What is the length of the bear? State your answer in feet and whole inches.

Solution

a) Use an instrument to measure the distance from the highest point on the bear's back to the ground.
height of bear in photo = 2 in.

Let x represent the height of the actual bear.

$$\text{Scale} = \frac{\text{distance on photograph}}{\text{actual distance on ground}}$$

$$\frac{1}{24} = \frac{2}{x}$$

What measurement unit will the actual height be in? How do you know?

$$\frac{1}{24}(24x) = \frac{2}{x}(24x)$$

$$x = 48$$

The height of the bear is 48 in.

b) Measure the distance from the bear's nose to the rear leg.
length of bear in photo = $3\frac{3}{8}$ in.

Let z represent the actual length of the bear.

$$\frac{1}{24} = \frac{3\frac{3}{8}}{z}$$

$$\frac{1}{24} = \frac{\frac{27}{8}}{z}$$

$$z = 24\left(\frac{27}{8}\right)$$

$$z = 81$$

The length of the bear is 81 in.

Convert 81 in. to feet and inches. To do this calculate the number of whole feet and then find the number of inches remaining.

$$\frac{81 \text{ in.}}{12} = 6 \text{ ft } 9 \text{ in.}$$

Use 1 ft = 12 in. to find the number of whole feet.

The number of inches in 6 ft is 6(12 in.) = 72 in.

Remainder = 81 − 72
= 9

The length of the bear is 6 ft 9 in.

Your Turn

The photograph of a muskox uses a scale of 1 : 30. Calculate the height of the muskox and the distance between the tips of its horns. State each answer in feet and inches.

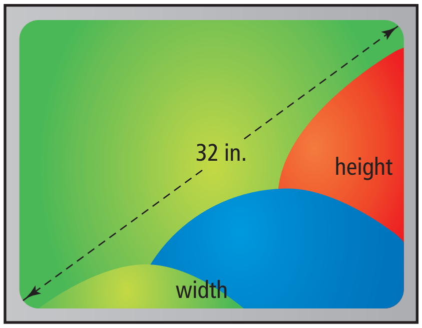
Example 2 Apply Linear Measurement

The Carsons want to buy a 32″ television. The size of a television is measured across the screen diagonally. They are choosing between a standard 4:3 television set and a widescreen 16:9 HDTV. To help them decide, calculate the screen dimensions and the viewing area for each television. Which television has the greater viewing area?

Solution

Standard 4:3 Television Screen

Using the ratio of width to height (4:3), draw a scale diagram to visualize the television screen.



Measure the diagonal, the width, and the height of the television screen in the diagram.

diagonal = 2.5 in.
width = 2 in.
height = 1.5 in.

To calculate the width, w, and height, h, of the actual TV screen, calculate the scale factor using the measurement of the diagonal. Let s represent the scale factor.

$$32s = 2.5$$
$$s = \frac{2.5}{32}$$
$$s = 0.078\ 125$$

Scale factor(width of actual TV) = width of screen in diagram

$$0.078\ 125w = 2$$
$$w = \frac{2}{0.078\ 125}$$
$$w = 25.6$$

Scale factor(height of actual TV) = height of screen in diagram

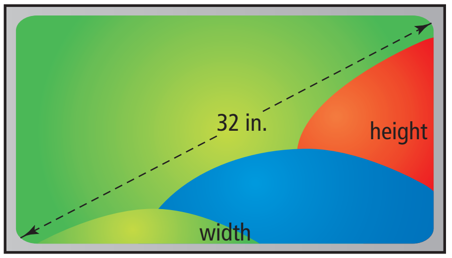
$$0.078\ 125h = 1.5$$
$$h = \frac{1.5}{0.078\ 125}$$
$$h = 19.2$$

Area of screen = width × height

$$A = 25.6(19.2)$$
$$A = 491.52$$

The viewing area of the standard television is 491.52 in.^{2}.

Widescreen 16 : 9 HDTV



The ratio of width to height in the diagram is 16 : 9.

The actual TV is an enlargement of the diagram.

To determine the dimensions of the actual TV screen, you could use an enlargement factor of x.

Then, the actual width, w, is represented by $16x$ and the actual height, h, is represented by $9x$.

The actual diagonal is 32 in.

You can use the Pythagorean relationship to determine the enlargement factor.

$$\begin{aligned}(16x)^2 + (9x)^2 &= 32^2\\ 256x^2 + 81x^2 &= 1024\\ 337x^2 &= 1024\\ x^2 &= \frac{1024}{337}\\ x^2 &= 3.0385\ldots\\ x &= 1.7431\ldots\end{aligned}$$

Calculate the actual width.

$w = 16x$
$w = 16(1.7431\ldots)$
$w = 27.8904\ldots$

Calculate the actual height.

$h = 9x$
$h = 9(1.7431\ldots)$
$h = 15.6883\ldots$

$$\begin{aligned}\text{Area of screen} &= \text{width} \times \text{height}\\ A &= (27.8904\ldots)(15.6883\ldots)\\ A &= 437.5548\ldots\end{aligned}$$

The viewing area of the standard TV is about 491 in.^{2} and the viewing area of the HDTV is about 438 in.^{2}.

The standard TV has the greater viewing area.

Your Turn

What is the difference in the viewing area for a 46″ standard television (4 : 3) and a 46″ widescreen television (16 : 9)?

Did You Know?

Scientists have developed an ultra-thin flexible screen that folds to fit in a pocket. These screens could be used for computers, telephones, and advertising.

Example 3 Solve a Problem Using Imperial Measurements

Alashun wants to make a drum, or qilaut, that resembles the one used by a drum dancer in Iqaluit, NU. He has a circular frame, over which to stretch caribou skin. Then, he will lash it into place along the frame with sinew. Alashun uses $3\frac{1}{2}$ in. of sinew for each inch of the frame.

a) Estimate the diameter of the drum frame in imperial units. The scale of the photo is 1:15.

b) Approximately what length of sinew does Alashun need to make the drum? State your answer in yards and inches.

Inuit drum dancers performing at the inaugural event in Iqaluit, Nunavut. Nunavut became Canada's newest territory on April 1, 1999.

Solution

a) The diameter of the drum in the photo appears to be just over 1″.
$1''(15) \approx 15''$
So, the diameter of the drum is approximately 15″.

How would you estimate the diameter of the drum in the photo?

b) Estimate the circumference of the drum frame.

$C = \pi d$
$C = \pi(15)$
$C = 15\pi$

The circumference of the drum frame is approximately 15π inches.

Alashun uses $3\frac{1}{2}$ in. of sinew for each inch of the frame.

Let l represent the length of sinew needed.

$l \approx \left(3\frac{1}{2}\right)(15\pi)$
$l \approx 164.9336...$

Round this distance to whole inches for converting.

Convert 165 in. to yards and inches.

$\frac{165\text{ in.}}{36} = 4\text{ yd }21\text{ in.}$

What steps do you follow to convert between imperial units?

$\frac{160}{40} = 4$
So, 165 in. ≈ 4 yd.

Alashun needs approximately 4 yd 21 in. of sinew.

Your Turn

A round Inuit drum needs to have its skin restretched and then lashed into place with sinew. For each inch of the frame, $3\frac{1}{2}$ in. of sinew are needed. The diameter of the frame is $1\frac{1}{4}$ ft. What length of sinew is needed? Express your answer to the nearest quarter of a foot.

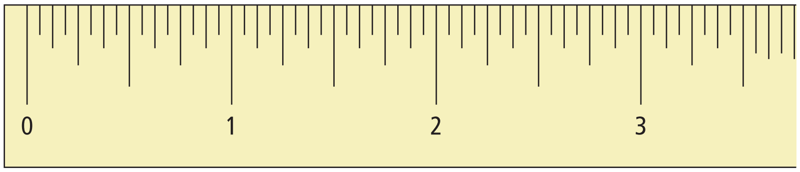
Key Ideas

- The imperial system of measurement is widely used in the United States for measuring distances.
- Even though SI is Canada's official measurement system, some Canadian industries still use imperial units.
- In the imperial system, common units for linear measurement are the inch (in.), foot (ft), yard (yd), and mile (mi). The imperial units for length are related according to the following conversions:
  1 mi = 1760 yd 1 yd = 3 ft 1 ft = 12 in.

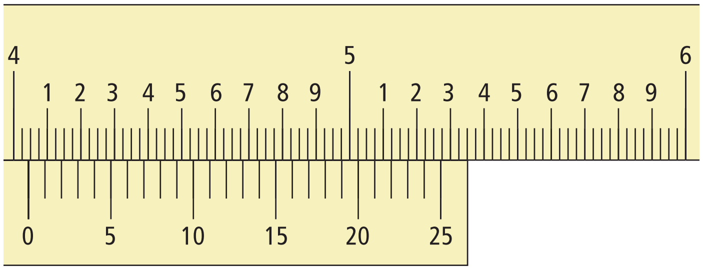
Check Your Understanding

Practise

1. a) What does the smallest subdivision on this imperial ruler represent?



b) Look at the caliper that measures in inches. What is the value of each of the smallest subdivisions on the fixed scale? State your answer as both a fraction and a decimal.



c) What is the value of each of the smallest subdivisions on the moving scale of the caliper in part b)? State your answer in fraction and decimal form.

2. Convert each measurement to the unit indicated.

   a) The world's longest earthworm measured 1 ft $1\frac{1}{2}$ in. (nearest half of an inch)

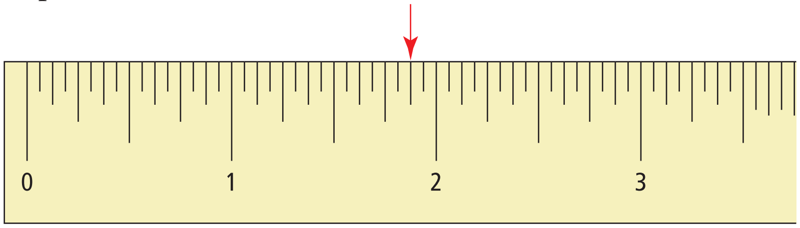
   b) The world's shortest man is 2′ 3″. (nearest quarter of a yard)

   c) A rocket separates from its space capsule at 400 000 ft. (nearest mile)

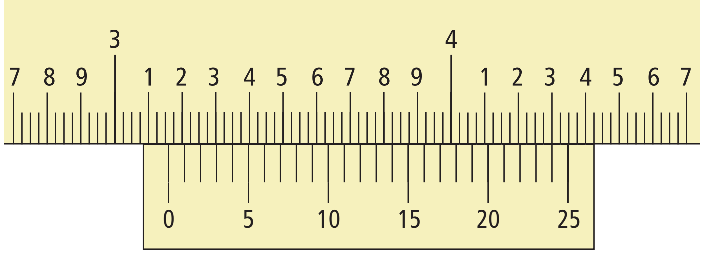
   d) The altitude of a balloon is 3 mi. (nearest foot)

3. What reading is shown on each measuring scale? For each measurement, name one item that might have this dimension.

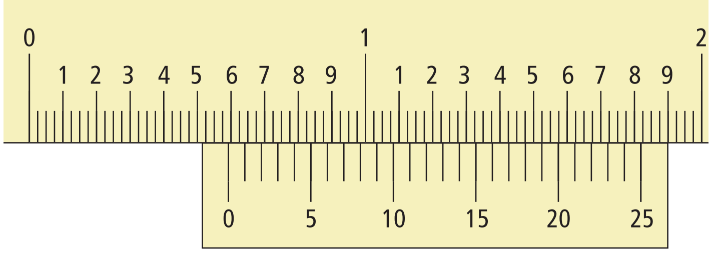
   a) imperial ruler



   b) imperial caliper



   c) imperial caliper



4. Name a measuring device that would be appropriate to measure each distance. Explain your choices. Then, measure each distance, to the nearest sixteenth of an inch.

   a) the diameter of a pen

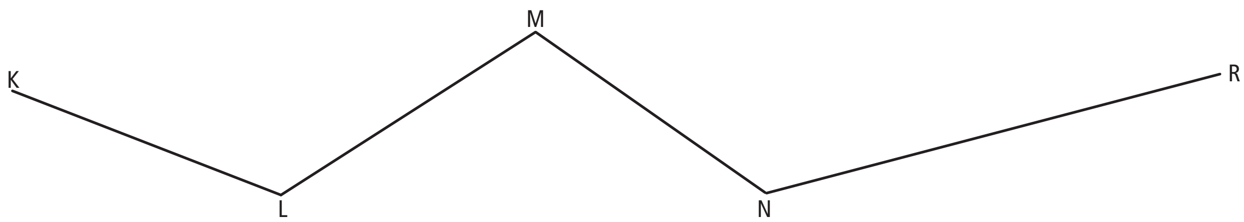
   b) the circumference of a pen

   c) the length of a pen

5. Use your referent for an inch to estimate the total length of each figure. Then, measure each distance. Express answers to the nearest quarter of an inch.

a)

b)



6. Explain how you can use a personal referent to help you estimate. Then estimate and measure each distance. State each measurement in feet and inches.

a) the width of your classroom

b) the perimeter of your desk or table top

7. Billy Loutit was a Métis mail carrier for the Hudson's Bay Company. He ran 100 mi, from Athabasca, AB, to Edmonton, in 16 h through flooded terrain.

a) What was Billy's average speed in miles per hour?

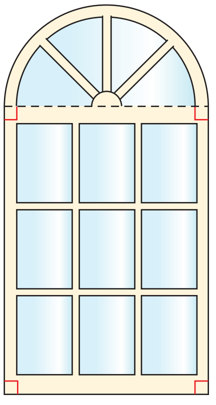
b) How long did it take Billy to run a mile?

Did You Know?

In 1904, William (Billy) Loutit was sent from Athabasca, AB, to Edmonton seeking emergency help against the flood that threatened to destroy Athabasca. By completing the 100-mi journey on foot in 16 h, he became a hero.

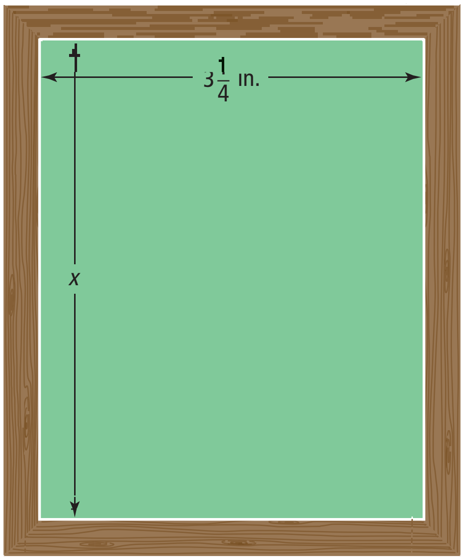
Apply

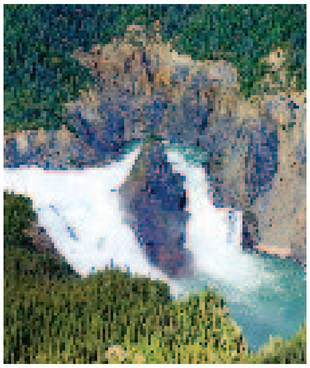
8. An interior designer wants to present a client with some options for wood trim to frame the Norman window shown in the diagram. The scale of the drawing is 1:32. What is the distance around the outside of the window? Express your answer to the nearest half inch. Assume the curve is a semicircle.



9. Leslie has a new manual wheelchair. It has 3 in. diameter micro-caster wheels and 24 in. diameter drive wheels.
   a) Leslie wants to know how many times the caster wheels rotate for each rotation of the drive wheels. Explain the calculations you would perform to obtain the answer. Then, give the answer as a ratio of drive wheel rotations to caster wheel rotations. Write the ratio in lowest terms.
   b) How many rotations of the drive wheels are needed to travel 250 yd?
   c) Suppose Leslie travels $1\frac{1}{2}$ mi. How many rotations will the drive wheels make?

10. Marcus works in a photography laboratory. He needs to enlarge a photograph of Virginia Falls in Nahanni National Park, NT, and make it fit into the frame shown.





   a) What is the scale factor for the enlargement?
   b) What is the length of the unknown side of the frame?

11. Gail and Bram are calculating the area of their washroom floor to order new tiles. Together, they measure the length and width of the floor to be $7\frac{1}{2}$ ft and 5 ft.
   a) Gail calculates the area to be 37 ft² 5 in.². Is Gail correct? Explain.
   b) The tiles they select are 6 in. by 12 in. How many tiles are needed to cover their washroom floor?

Did You Know?

The water at Virginia Falls in Nahanni National Park, NT, plunges 295 ft. In the centre of the falls stands Mason's Rock. It is named after Bill Mason, a well-known Canadian adventurer and canoeist.

12. **Unit Project** Today's music storage devices tend to be smaller than those of the past, but they can store many more songs. Find a cassette tape case, a CD, and an MP3 player. Use an imperial unit to measure each of the following dimensions. Justify your choice of unit.

a) the diameter of the CD

b) the dimensions of the cassette case

c) the perimeter of the largest face of the MP3 player

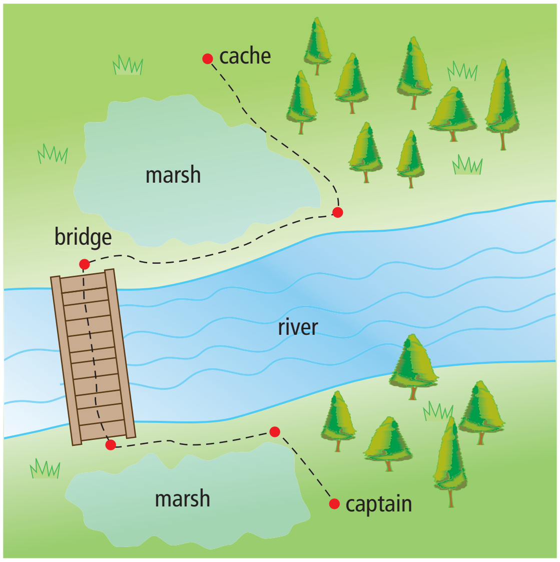
Web Link

To learn more about past and present music storage devices, go to www.mhrmath10.ca and follow the links.

13. A geocaching team captain estimates that a cache is located 500 yd northwest from his position. However, the team must cross the river using the bridge shown.

a) Estimate the distance that the team travels from the captain's position to the cache. Justify your answer.

b) Global Positioning System (GPS) readings provide straight line distances. Estimate the total of the GPS distances between the red dots on the map. Give your answer in yards and feet. How does this total distance compare with the actual distance walked? Explain why.



Extend

14. Sometimes it is difficult to measure the diameter of an object. It may be easier to measure its circumference. The distance around an above-ground circular pool is 60 ft.

a) What is the diameter of the pool? Express your answer to the nearest inch.

b) The owner wants to build a circular wall outside the existing one to help insulate the pool. The material she wants to use is only available in 62 ft, 65 ft, and 70 ft lengths. Determine the diameter of the new insulating wall using each of the available lengths. Express your answer to the nearest inch.

c) Which length would you recommend the owner choose? Explain your reasoning.

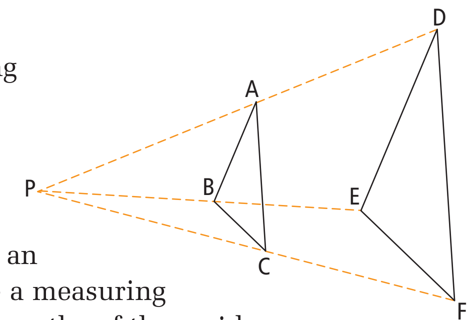
15. The astronomical unit (AU) is a unit of length based on the average distance from Earth to the sun. The AU is currently accepted as 92 955 887.6 mi.

Two comets appeared in rapid succession in 1996 and 1997.
- Comet Hyakutake came within 0.1018 AU of Earth.
- Comet Hale-Bopp came within 1.315 AU of Earth.

a) Within how many miles of Earth did each comet pass?

b) What is the difference in the distance from Earth between the paths of Comets Hale-Bopp and Hyakutake? Give your answer to the nearest mile.

16. You can enlarge a shape by using a point, P, and measuring distances from it to the vertices of the shape and to the enlargement.



a) What ratios of side lengths should be equal if $\triangle DEF$ is an enlargement of $\triangle ABC$? Use a measuring instrument to measure the lengths of these sides and compare the ratios.

b) Explain the mathematics behind this method of enlarging a figure.

c) Use this method to enlarge a figure by a factor of three.

Create Connections

17. Sam's room measures 11 ft by $13\frac{1}{2}$ ft. He wants to buy a new queen-size bed, if it will fit with his existing furniture. Sam has a desk, which measures 4 ft wide by 22 in. deep, and a night stand.

a) Find the dimensions of a double bed and a queen-size bed.

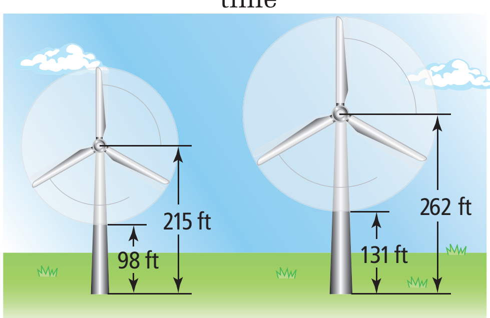
b) Design a layout for Sam's room using a scale diagram. Hint: You will need to estimate the dimensions of a night stand.

c) Which bed do you suggest Sam buy? Why?

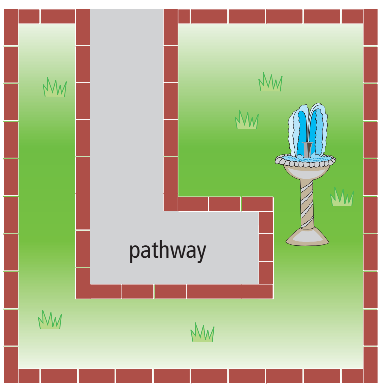
18. Manitoba Hydro announced plans to build a 300-MW wind farm at St. Joseph, MB. Measurements of some of the wind turbines being considered are shown below.

a) What is the difference in the length of the blades for the wind turbines shown?

b) Suppose each turbine makes 30 revolutions per minute. The tip speed of the blades can be up to 6 times the wind speed. Determine the tip speed of each blade. What is the maximum wind speed for each turbine? Hint: speed $= \frac{\text{distance}}{\text{time}}$.



19. A pathway leads to a fountain in a small park. The park and the pathway are outlined with a brick border.



a) Estimate the perimeter of the border in the diagram using imperial units. Explain how you estimated your answer.

b) Measure the perimeter. How close was your estimate to the actual measure?

c) Draw a new diagram for the same park but make the pathway to the fountain half the width of the one shown in the diagram. What is the perimeter of the border in your diagram?

d) Predict how the perimeter of the border changes as the width of the pathway changes. Check your prediction. Use words, diagrams, and imperial measurements to support your answer.

1.3 Converting Between SI and Imperial Systems

Focus on ...

- comparing SI and imperial units using referents
- solving problems that involve conversion of linear measurements between SI and imperial systems
- using mental mathematics to confirm the reasonableness of a solution to a conversion problem

Some people use music to express their thoughts through song lyrics. Many song lyrics tell a story.

- Name some lyrics or song titles that include imperial measurements, such as miles, feet, or miles per hour.
- What SI measurements are comparable to those measurements?
- Describe how the converted measurements could change the song.

Materials

- compact disc (CD)

Investigate Relationships Between SI and Imperial Measurements

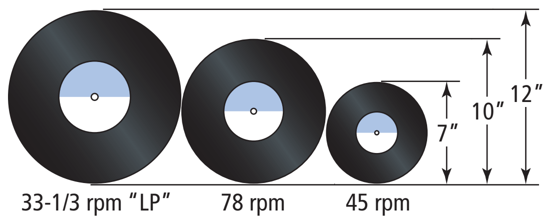
Unit Project

1. a) Use a referent to estimate the diameter of a CD in SI units. How did you make your estimate?

b) Use a referent to estimate the diameter of a CD in imperial units.

2. Measure the diameter of a CD, to the nearest millimetre.

3. Calculate the diameter of each vinyl record in SI units.



4. a) Calculate the circumference of a CD and each of the three vinyl records shown, in SI units.
   b) Compare the sizes of the four recording devices.

5. **Reflect and Respond**
   a) Discuss with a partner. Which recording device is able to store more music—a vinyl record or a CD?
   b) Develop a method you could use to compare the amount of music that each device stores to the size of the device. Test your method in both imperial and SI units.
   c) Describe how you think laser technology has affected the storage of music. Support your answer with information you have gathered in your research.

WWW Web Link

To learn more about laser technology, go to www.mhrmath10.ca and follow the links.

Link the Ideas

To convert from one measurement system to another, you need to understand the relationships between the units of length in each system. Conversions involve proportional reasoning and unit analysis.

Conversions between measurement systems may be approximate or exact. The imperial yard has been defined as 0.9144 m. This would be considered an exact conversion.

Since 1 yd = 0.9144 m and 1 yd = 3 ft, 3 ft = 0.9144 m.

$$1 \text{ ft} = 1 \text{ ft}\left(\frac{0.9144 \text{ m}}{3 \text{ ft}}\right)$$
$$1 \text{ ft} = 0.3048 \text{ m}$$

To convert from metres to yards, 1 m is often given as 1.094 yd. Verify whether this conversion is exact or approximate.
Let x represent the number of yards in 1 m.

$$0.9144 \text{ m} = 1 \text{ yd}$$
$$\frac{0.9144 \text{ m}}{1 \text{ yd}} = \frac{1 \text{ m}}{x \text{ yd}}$$
$$x = \frac{1}{0.9144}$$
$$x = 1.0936...$$

The conversion 1 m = 1.094 yd is approximate.

The following are some common conversions.

Exact Conversions

1 in. = 2.54 cm	1 ft = 30.48 cm	1 yd = 0.9144 m

Approximate Conversions

1 mm ≈ 0.0394 in.	1 cm ≈ 0.3937 in.	1 m ≈ 1.094 yd
1 m ≈ 3.281 ft	1 km ≈ 0.6214 mi	1 mi ≈ 1.609 km

Example 1 Convert Between SI and Imperial Units for Length

Researchers at the Harvard-Smithsonian Center for Astrophysics made an announcement in January, 2001. They stated that they had "frozen light" by using super-cooled vapour to slow the speed of light waves to zero. The speed of light in a vacuum is defined as 299 792 458 m/s.

a) Estimate the speed of light in miles per second.

b) Predict whether the actual speed of light is greater than your estimate. Justify your prediction.

c) Calculate the answer, to the nearest mile per second.

Solution

a) Since there are 1000 m in 1 km, 299 792 458 m/s can be converted to 299 792.458 km/s or approximately 299 792 km/s.

$$299\ 792\ \frac{\text{km}}{\text{s}}\left(\frac{1\text{ mi}}{1.6\text{ km}}\right) \approx 187\ 370\ \frac{\text{mi}}{\text{s}}$$

Recall that 1 mi ≈ 1.6 km.

The speed of light can be estimated as 187 370 mi/s.

b) The actual speed of light, in miles per second, is less than the estimate because the SI speed was rounded down before converting.

c) Use unit analysis.

$$1\text{ mi} = 1760\text{ yd}\left(\frac{0.9144\text{ m}}{1\text{ yd}}\right)$$

$$1\text{ mi} = 1609.344\text{ m}$$

$$299\ 792\ 458\ \frac{\text{m}}{\text{s}}\left(\frac{1\text{ mi}}{1609.344\text{ m}}\right) = 186\ 282.397\ \frac{\text{mi}}{\text{s}}$$

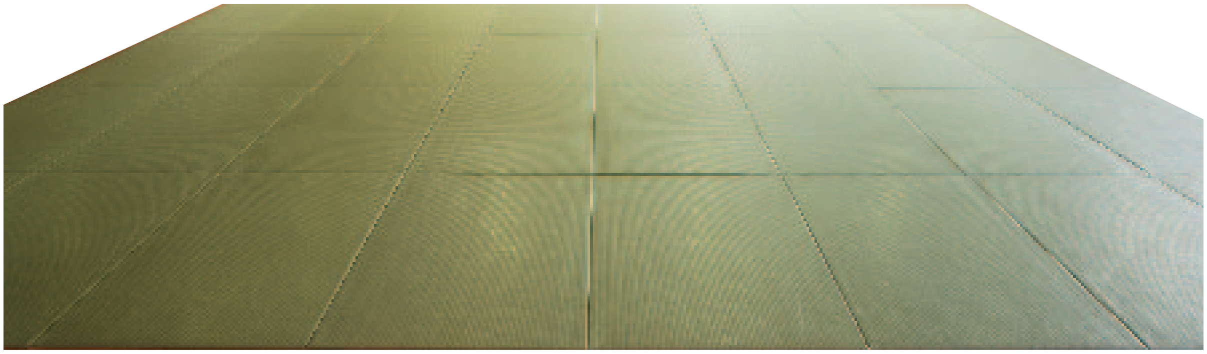
Why is the speed of light used here in metres per second?

The speed of light is approximately 186 282 mi/s.

Your Turn

Swimmer Brian Johns of Richmond, BC, represented Canada at the 2008 Olympics in Beijing. He finished 7th in a race that one news report referred to as 400 m long and another news report referred to as $\frac{1}{4}$ mi. Are the two measurements equivalent? If not, which distance do you think is more accurate? Justify your reasoning.

Example 2 Solve a Problem Involving Linear Measurements



Your class needs to lay mats on the gymnasium floor for a gymnastics meet. The gym measures 84′ by 50′. A scale drawing of one mat is shown. The scale is 1 : 30.5.

A classmate thinks that 131 mats are needed. Do you agree? Explain.

Solution

The mat in the diagram measures 8 cm by 4 cm.
Use the given scale.

$$\text{length of mat} = 8(30.5) \qquad\qquad \text{width of mat} = 4(30.5)$$
$$l = 244 \qquad\qquad w = 122$$

The length of the mat is 244 cm and the width is 122 cm.

Length of mat

$$244 \text{ cm}\left(\frac{1''}{2.54 \text{ cm}}\right) \approx 96''$$

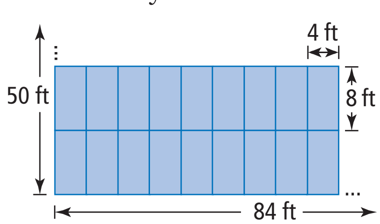
Width of mat

$$122 \text{ cm}\left(\frac{1''}{2.54 \text{ cm}}\right) \approx 48''$$

There is a 2 : 1 ratio of length to width in the measurements of the mat. The SI-to-imperial conversions confirm a 2 : 1 ratio of length to width.

The dimensions of the mat are 8 ft by 4 ft.
Sketch a layout of the mats in the gym.

How do you know the dimensions are 8 ft by 4 ft?



Suppose you do not visualize the mats and you only work with areas.

$$\frac{\text{area of gym}}{\text{area of mat}} = \frac{84(50)}{8(4)}$$

Therefore, about 131 mats would be needed.

The width of each mat is 4 ft.

$\frac{84}{4} = 21$

So, there are 21 columns of mats.

The length of each mat is 8 ft.

$\frac{50}{8} = 6\frac{1}{4}$

So, there are 6 rows of mats and some uncovered space.

To cover the gym floor, there are 21 columns and 6 rows of mats.

$21(6) = 126$

Therefore, 126 mats are needed to cover most of the gym floor.

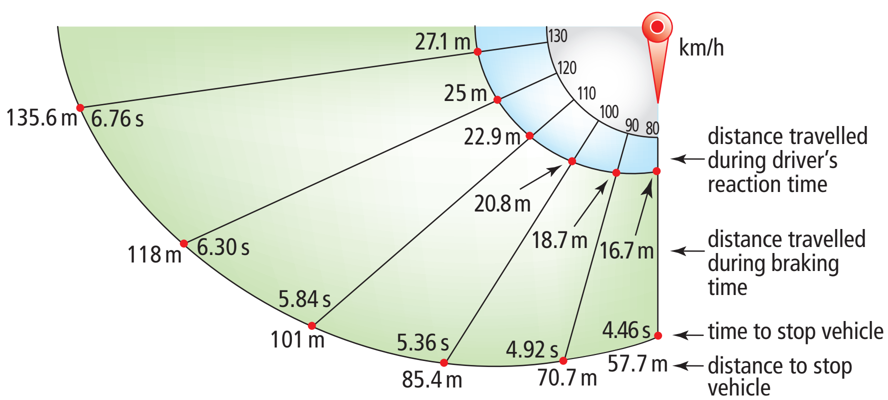
Your Turn

a) How many paving stones measuring $7\frac{1}{2}$ in. by $7\frac{1}{2}$ in. are needed to cover an area that is 1 yd by 1 yd?

b) To tile a floor that is 3 m by 4 m, how many tiles measuring 30 cm by 50 cm would you buy? Add 10% extra tiles for areas that require tiles to be cut.

Example 3 Determine Stopping Distances

The distance required to stop a moving vehicle is the sum of the distances travelled during the reaction time and the braking time. The diagram shows the theoretical stopping distance at various speeds.



a) What factors might affect the reaction time and braking distances?

b) If a vehicle is travelling at 100 km/h, approximately what distance is travelled while the brakes are being applied?

c) Convert 55 mph into kilometres per hour. What is the approximate stopping distance when a vehicle is travelling at this speed? Express your answer in feet.

Solution

a) Factors that affect reaction time may include being tired, distraction by passengers, time in the vehicle, loud music, or reduced visibility due to weather or dirty windows.

Factors that affect braking distance may include condition of brakes, condition and/or type of tires, speed, type of road surface, outside temperature, or road conditions (dry, icy, wet).

b) Find 100 km/h on the diagram.
The reaction-time distance at 100 km/h is 20.8 m.
The total stopping distance at 100 km/h is 85.4 m.

Total stopping distance = reaction-time distance + braking distance

$$85.4 = 20.8 + b$$
$$85.4 - 20.8 = b$$
$$64.6 = b$$

If travelling at 100 km/h, a vehicle travels approximately 65 m while the brakes are being applied.

Subtract the distances represented by the line segments on the diagram.
Stopping distance ≈ 85 − 20 or 65

ME

c) Convert 55 miles to kilometres.
Use proportional reasoning.

Recall that 1 mi ≈ 1.609 km.

Let x represent the number of kilometres.

$$\frac{1 \text{ mi}}{55 \text{ mi}} \approx \frac{1.609 \text{ km}}{x \text{ km}}$$
$$x \approx (55)1.609$$
$$x \approx 88.495$$

Therefore, 55 mph converts to approximately 88 km/h.

From the diagram, when a vehicle is travelling at 90 km/h, the stopping distance is 70.7 m.

The speed 88 km/h is just under 90 km/h, so convert 70 m to feet.
Use the conversion 0.9144 m = 1 yd or 3 ft.

$$70 \text{ m}\left(\frac{3 \text{ ft}}{0.9144 \text{ m}}\right) = 229.658\ldots \text{ ft}$$

When travelling at a speed of 55 mph, a vehicle takes approximately 230 feet to stop.

Your Turn

a) Use the diagram from Example 3 on page 40. Determine the difference between the reaction-time distances for speeds of 110 km/h and 120 km/h. Express your answer in feet.

b) Convert 90 km/h into miles per hour. What is the approximate stopping distance for a vehicle travelling at this speed? Express your answer in yards.

Key Ideas

- When solving problems involving measurement, it is crucial to work with the same units. You may need to convert units within one measurement system (for example, inches to feet) or between imperial and SI units.
- If an exact conversion between systems is required, use 1 yd = 0.9144 m to find a conversion between the required units.

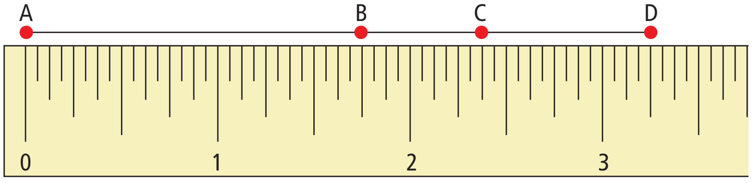
$$\begin{aligned} 1 \text{ yd} &= 0.9144 \text{ m} \\ 36 \text{ in.} &= 0.9144 \text{ m} && \text{Divide both sides by 36.} \\ 1 \text{ in.} &= 0.0254 \text{ m} \\ 1 \text{ in.} &= 2.54 \text{ cm} && \text{This is an exact conversion} \end{aligned}$$

- Sometimes you use approximate values, such as 1 in. ≈ 2.5 cm or 1.6 km ≈ 1 mi when estimating between measurement systems.

Check Your Understanding

Practise

1. Use the diagram of an imperial ruler to help answer the questions below.



a) What is the length of AC? Give your answer as a fraction.

b) Suppose you replace the imperial ruler with an SI ruler. What is the length of AD, in millimetres?

c) What is the difference in the lengths of segments AB and CD? Give your answer in SI and imperial units.

2. Convert each measurement to the unit specified.

a) The diameter of a human hair is 0.001 in. (hundredth of a millimetre)

b) On an NBA basketball court, the width of the key (painted area beneath the basket) is 4.9 m. (feet)

c) A snowmobile trail in Alberta is 26 mi 385 yd long. (hundredth of a kilometre)

d) An envelope has a height of $3\frac{7}{8}$ in. (hundredth of a centimetre)

3. People have always used parts of the body for measuring length. For example, you may have heard the measurements of a horse stated by the number of "hands." Work with a partner. State your answers in both SI and imperial units.

a) Estimate and then measure your hand span.

b) How many of your hand spans are needed to measure the length of your desk?

c) Estimate and then measure your pace. This is the distance between your heels when you take a step while walking. To be more accurate, you may wish to measure 10 paces and calculate the average.

d) How many of your paces would it take to walk a mile? a kilometre?

e) Which measurement system do you prefer to use when estimating? Why?

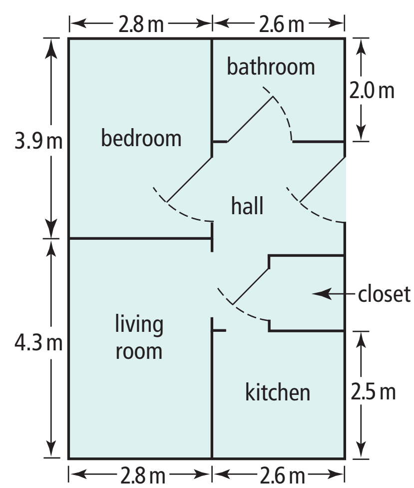
4. The floor plan for a one-bedroom apartment is shown.

a) If the closet is square, estimate the length of one of its sides.

b) Determine the scale of the floor plan.

c) Calculate the length of one side of the closet. Express your answer to the nearest tenth of a metre.

d) What are the dimensions of the bedroom, in imperial units?

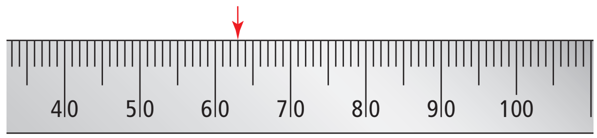


5. Read the following paragraph about the Columbia Icefield. Convert each SI measurement to an equivalent imperial measurement.

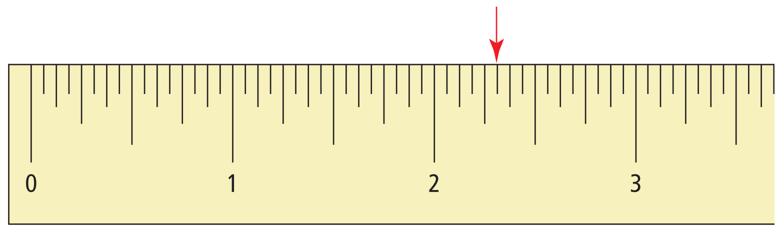
The Columbia Icefield straddles the boundary between Alberta and British Columbia. It is the largest glacier in North America, south of the Arctic Circle. It has a maximum depth of 365 m. Its highest points are Mount Columbia, at 3745 m, and Mount Athabasca, at 3491 m. The average snowfall across the icefield is 100 cm per year.

6. What is the reading represented on each measuring device? Estimate and then calculate each equivalent measurement in the other system (SI or imperial).

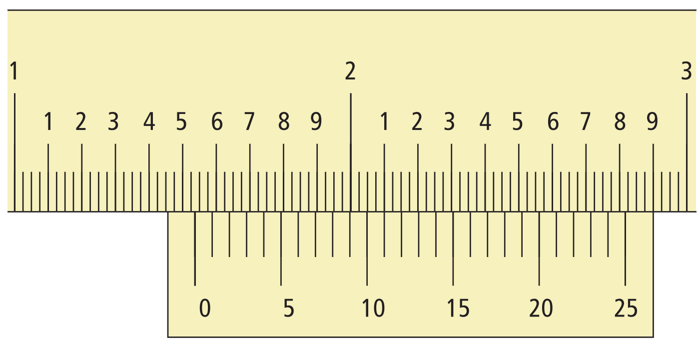
   a) SI ruler



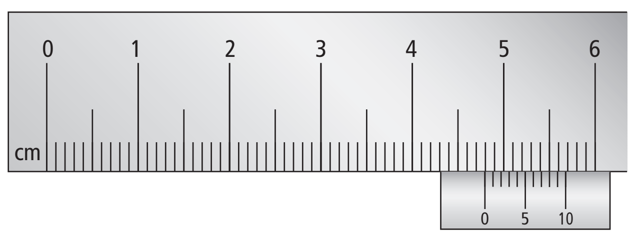
   b) imperial ruler



   c) imperial caliper



   d) SI caliper



Apply

7. A traditional Inuit dog sled uses teams of Qimmiq or sled dogs on separate lines. The lines are fastened directly to the komatik or sled. Each dog has a harness with an average length of $3\frac{1}{2}$ ft. Suppose a dog sled uses a team of 13 dogs.

   a) Calculate the approximate total length of rope needed to harness the team.

   b) Calculate the total length of the harness in SI units. Explain why you chose this particular unit.

8. Alex is on his way home to Moosomin, SK. He was visiting friends in Mohall, North Dakota. As soon as he gets on the highway, he sees the sign to the right.

As Alex passes the Canada-U.S. border, he sees this sign. What is the driving distance between Mohall and Moosomin, in SI units?

9. a) Discuss with a classmate your preferred method of converting the following units.
- metres to kilometres
- metres to centimetres
- yards to miles

b) Develop a strategy for converting from a smaller unit of length to a larger unit. Does your strategy apply in both SI and imperial systems of measurement? Explain.

c) How might your strategy change when you are converting from a larger unit of length to a smaller one?

10. Margaux and Penny each travel with their families to St. Pierre-Jolys, MB, for the annual Frog Follies. When the girls arrive, they compare how far they travelled from their homes.
- Margaux lives in Arborg, MB, and travelled 164 km.
- Penny lives in Grand Forks, North Dakota, and travelled 113 mi.

Penny is not familiar with SI distances. Explain how to determine the approximate conversion from kilometres to miles. Who travelled farther? Explain.

11. The deepest lake in the world is Lake Baikal, in Russia. It has a depth of 5369 ft. Canada's deepest lake is Great Slave Lake, named after the Slavey (Dene Tha) peoples who live there. It is located in the Northwest Territories and has a depth of 2015 ft. Quesnel Lake, south of Prince George, BC, is the deepest fjord lake in the world. It has a depth of 506 m.

Compare the depth of Quesnel Lake with the depths of Lake Baikal and Great Slave Lake. Give each answer as a decimal, to the nearest metre.

Did You Know?

Canada has two Niagara Falls. Most people know about Niagara Falls in Ontario. The other Niagara Falls is on Quesnel Lake in British Columbia. At the mouth of Niagara Creek, water falls over 200 ft into the east arm of Quesnel Lake.

Did You Know?

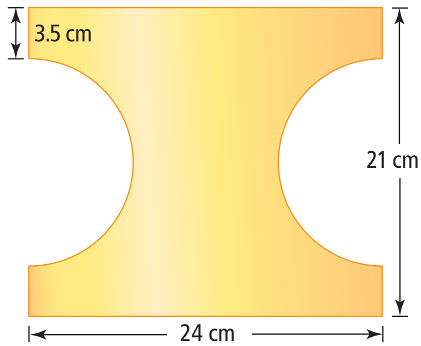
The pull of gravity is greater closer to the centre of Earth. For this reason, you would weigh about 2.2 lb more at the North Pole than on the equator.

12. Earth is not a perfect sphere. It flattens slightly at the poles. A person standing at the North Pole is about 13 mi closer to the centre of Earth than a person standing on the equator.

How far would you be from the centre of Earth when standing on the North Pole? Give your answer in miles. Assume the equatorial radius of Earth is 6380 km.

13. a) Calculate the perimeter of the figure shown. Express your answer to the nearest quarter of an inch.

b) Discuss your steps with a partner. Then, describe what you think is the easiest way to calculate the perimeter of the figure in inches.



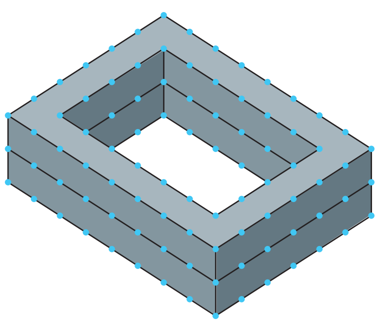
14. **Unit Project** One type of 80 GB MP3 player has dimensions of 4.14 cm (width) by 9.15 cm (height) by 0.85 cm (thickness). The storage capacity is about 20 000 songs. Each LP vinyl record holds an average of 12 songs and is approximately $\frac{1}{9}$ in. thick.

a) Calculate the number of LPs you would need to store as many songs as the MP3 player. Use mental mathematics to show that your answer is reasonable.

b) Suppose you stack the LPs. Calculate the height of the stack. Compare it with the height of the MP3 player. Give your answer as a ratio in lowest terms.

Extend

15. A farmer wants to build a wall using concrete blocks. She draws a diagram showing the wall and identifies the corners of the "cubes." Each concrete block is the same size as two cubes.



a) How many concrete blocks does the farmer need?

b) Each concrete block is 20 cm by 20 cm by 40 cm. What are the outside dimensions of the walled enclosure, in inches?

c) The farmer wants to build a new wall to fit tightly around the outside of the first one. How many concrete blocks are needed? Explain how to get the answer from your answer to part a).

16. Winnipeg's *River Arch* spans 23 m along the Trans-Canada Highway. The approximate area under a symmetrical arch can be calculated using the formula $A = \dfrac{w(H + 4h)}{6}$.

In the formula,
- A represents the area, in square metres
- w represents the width of the arch, in metres
- H represents the height of the centre of the arch, in metres
- h represents the height of the arch measured one quarter of the distance from each end of the arch, in metres

a) Assume the arch is symmetrical. Calculate the approximate area under *River Arch*, in square metres.

b) Would this formula work for imperial units? Explain.

Did You Know?

River Arch symbolizes the past and present of Winnipeg. The arch contains images of bison, wheat, and ploughed fields. *River Arch* is located where two bridges cross the Red River in downtown Winnipeg.

Create Connections

17. a) State three distances that are often measured in SI units and three distances that are often measured in imperial units.

b) For each distance in part a), indicate whether it is more appropriate to use an exact measurement or an approximate measurement. Justify your reasoning.

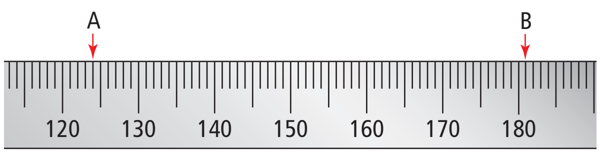
18. Using the conversion factor 1 yd = 0.9144 m, show how to convert from a small imperial unit to a larger SI unit.

1 Review

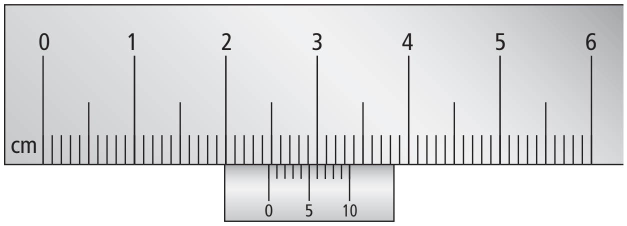
1.1 SI Measurement, pages 8–21

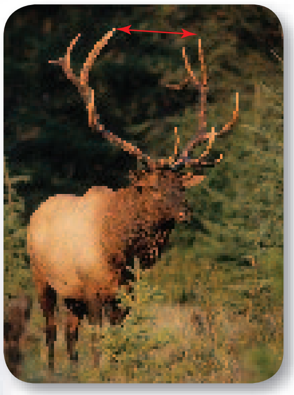
1. Find an object with a curved surface, such as a can. Estimate the distance around the curved object. Explain how to measure the curved distance. Then, measure it and compare your estimate to the actual measurement.

2. Determine the distance from A to B on this SI ruler. Express your answer to the nearest tenth of a centimetre.



3. What reading is shown on this SI caliper? Name an object that could be this length.





4. What is the circumference of the largest circle you could cut from a sheet of paper measuring 30 cm by 20 cm? What area of paper would you cut away?

5. The elk in the photograph stands 210 cm tall, including the antlers. Calculate the distance between the tips of the antlers, to the nearest tenth of a centimetre.

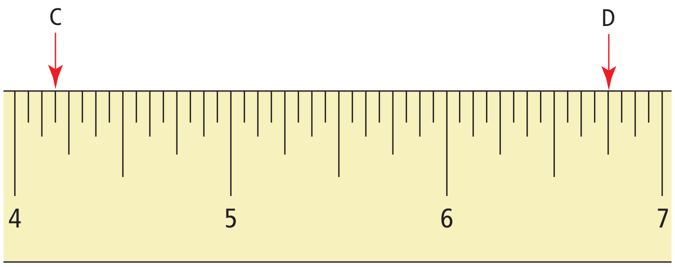
1.2 Imperial Measurement, pages 22–35

6. a) On a plain piece of paper, draw a letter S. As you draw, try to make the line for the letter $4\frac{3}{4}$ in. long.

b) Explain how you could verify that the curve of your S is the required distance.

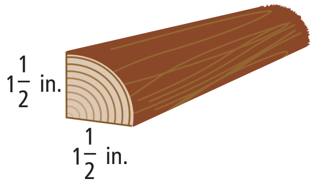
c) Measure your S. If your drawing is out by more than $\frac{3}{4}$ in., try drawing another S.

7. State the reading for point D on this imperial ruler as a mixed number in lowest terms. What is the distance from C to D? Show two ways to determine the answer.

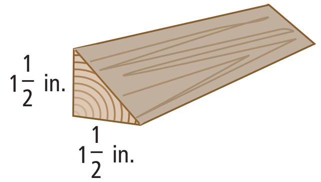


8. At a lumber yard, Jeanette buys lengths of wood with different cross-sections. For each piece of wood, explain how she could calculate the perimeter of the cross-section. Then, calculate each perimeter. Do your answers seem reasonable? Explain.

a)



b)



9. You want to enlarge the photograph of Burrard Inlet, in Vancouver, to fill a 4″ by 6″ frame.

a) Estimate the dimensions of the photograph in imperial units. Then, record the actual measurements.

b) By what scale factor do you need to enlarge the photograph? Will the enlargement need to be cropped to fit in the frame? Justify your reasoning.

1.3 Converting Between SI and Imperial Systems, pages 36–47

10. Convert each measurement to the unit specified.

a) The distance from Calgary to Jasper is 412 km. (miles)

b) Twister is the highest water slide in West Edmonton Mall, at 25.3 m. (feet)

11. The world's tallest man according to *Guinness World Records* was Robert P. Wadlow at 8 ft $11\frac{1}{10}$ in. Suppose his height was stated as 2.7 m. Would this be an approximation or an exact measurement? Justify your answer.

12. Thumbelina, the world's smallest pony, is 17 in. tall. How many times as tall as Thumbelina is the horse in the photograph? Estimate what the height of the horse would be if its head were up. Could you have used different units? Explain.

13. A map of Lesser Slave Lake, AB, is shown.

a) How many kilometres are represented by 1 cm?

b) How many miles are represented by one inch?

c) Estimate the length of Provincial Road 750 from Grouard to Atikameg. Give your answer in miles.

d) Suppose you leave Slave Lake and travel west on Highway 2 for 57 km before turning off. Where might you be going?

1 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. A student measures his pace to be $2\frac{3}{4}$ ft. How many paces will he need to take to walk 1 mi?

A 360

B 640

C 1760

D 1920

2. If 1 yd = 0.9144 m, which is the correct conversion giving the number of inches in a metre?

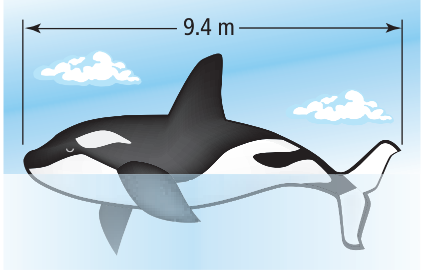
A 1 in. = 25.4 m

B 1 m = 39.37 in.

C 1 m = 254 cm

D 1 m = 100 in.

3. A killer whale has a length of 9.4 m. How many school door heights would it take to exceed the whale's length?



A 1

B 3

C 5

D 7

4. A regulation baseball bat must be $2\frac{3}{4}$ in. or less in diameter and shorter than 42 in. long. Which bat would not be allowed?

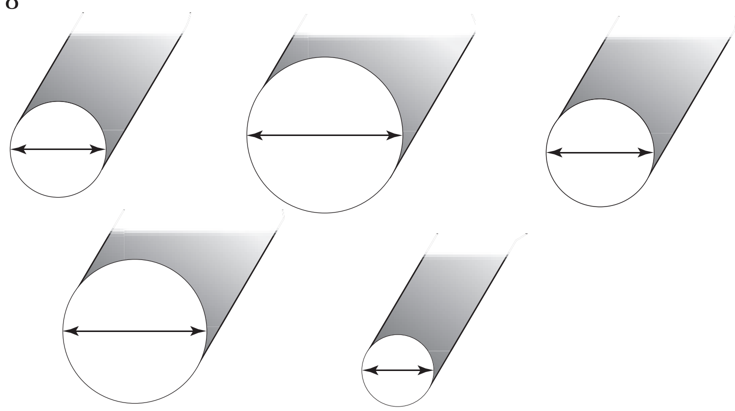
A diameter 6 cm

B length 1.0668 m

C diameter 7 cm

D length 0.9 m

5. How many of these rods can fit through a hole with a diameter of $\frac{5}{8}$ in.?



A 1

B 2

C 3

D 4

Short Answer

6. **a)** State two referents that you could use to measure the length of a van. List three appropriate units that you might measure the length of the van in.

   b) Name two units, one SI and one imperial, that are appropriate for measuring the circumference of a car steering wheel. What is an approximate conversion between these two units?

7. Sketch the approximate size of a Canadian $5 bill. Then, estimate its dimensions in SI units.

8. Janice needs to tighten a nut on her bicycle. She finds two wrenches.
   - The $\frac{1}{4}$ in. wrench is too small.
   - The $\frac{3}{8}$ in. wrench is too large.

   Janice's neighbour suggests that the nut might be an SI size. Assume that the sizes of SI wrenches are only in whole millimetres. What SI wrench sizes are between the two imperial sizes Janice tried?

Extended Response

9. Two identical archways inside the Manitoba provincial legislature are shown. Suppose the space between the archways is 6 ft. Describe how to calculate the perimeter of one opening, including the bottom. Assume that the top of the archway is a semicircle. Calculate the perimeter of one archway, including the bottom. Give your answer in feet and inches.

10. A gymnasium is 40 m long. An instructor asks two students to create lines every 5 m using green masking tape across the floor. When the instructor returns, he notices that the students made the lines 5 yd apart.
    a) How many lines did the students make on the floor?
    b) What is the distance from the last line to the end wall of the gym? Include a diagram.
    c) Members of the football team train by standing at the end wall of the gym, running to the 5 yd line, and returning to the wall. Then, they run to the 10 yd line and return to the wall. They continue this pattern until they reach the opposite end. What is the minimum distance, to the nearest yard, they will run? Justify your answer.
    d) Compare the total distances that the football players would run if the students had placed the lines 5 m apart versus 5 yd apart. Does your answer seem reasonable? Explain.

CHAPTER

2 Surface Area and Volume

You live in a three-dimensional world where ideas such as length, width, and height are not enough for you to understand some objects. To make sense of size in a three-dimensional world, you must understand the concepts of surface area and volume. For example, surface area is used when designing CD inserts for jewel cases. Volume is used when creating PDA, MP3, or cell phone cases.

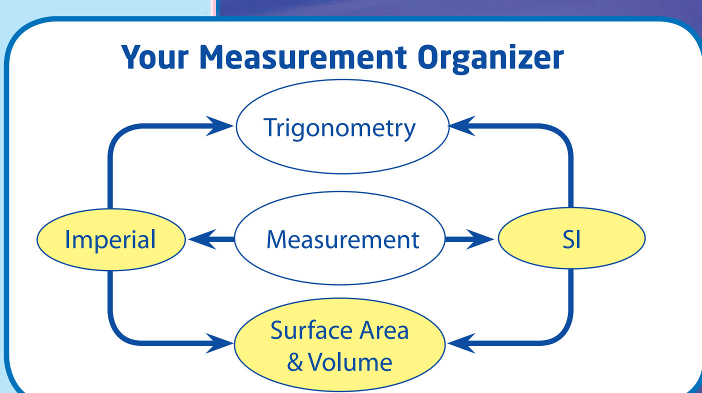
Big Ideas

When you have completed this chapter you will be able to …

- solve problems that involve units of area and volume within SI and imperial systems
- solve problems involving surface areas and volumes of spheres, right cones, right cylinders, right rectangular prisms, and right rectangular pyramids
- solve problems involving square roots and cube roots of numbers

Key Terms

surface area
volume
cylinder
prism
lateral area
cone
slant height
sphere
pyramid
apex



Industrial Designer

Industrial designers are responsible for designing many of the things you see and use every day, from MP3 players to vehicle interiors to electric guitars. In addition to artistic and technical ability, designers often use specialized tools like computer-aided design (CAD) software.

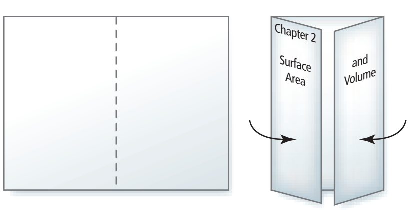
www Web Link

If you are interested in learning more about industrial design, go to www.mhrmath10.ca and follow the links.

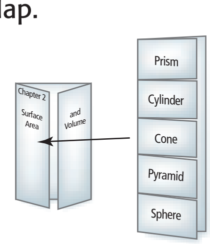
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 2.

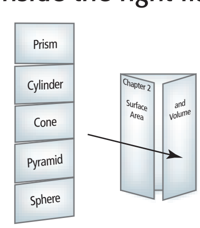
1. Fold an 11 × 17 sheet of paper as shown. On the outside front flaps, add the following labels.



2. Fold and label an 8.5 × 11 sheet of paper as shown. Cut tabs along the lines on the front half. Attach the tabbed page inside the left flap.



3. Fold and label another 8.5 × 11 sheet of paper as shown. Cut tabs along the lines on the front half. Attach the tabbed page inside the right flap.



4. On the back of the Foldable™, write the title What I Need to Work On.

2.1 Units of Area and Volume

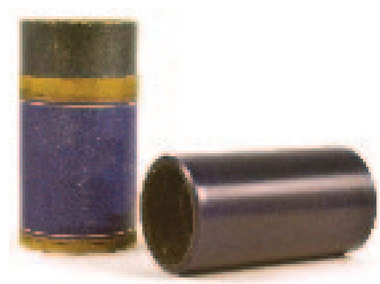
Focus on ...

- solving problems that involve area and volume units within SI and imperial systems
- using mental math to judge the reasonableness of a solution to a problem

Throughout the history of recorded sound, new technologies have been introduced to replace older ones. One example is the progression from sheet music to vinyl records to cassette tapes to CDs to MP3s. As the music industry has advanced, many outdated products have been abandoned. However, there are many music enthusiasts who still prefer the vinyl record over its digital replacements. They believe that vinyl records have a warmer sound than CDs or MP3s. Vinyl records also feature large album covers with imaginative graphics, pullout photos, and liner notes.

Investigate Units of Area

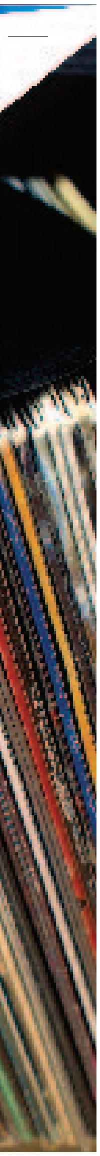
Unit Project



1. By the 1880s, wax cylinders were used to record music. Sound was recorded in the grooves on the outside of hollow cylinders of slightly softened wax. Standard cylinders were about 4 in. long with a diameter of $2\frac{1}{4}$ in. One cylinder could play about two minutes of music or other sound.

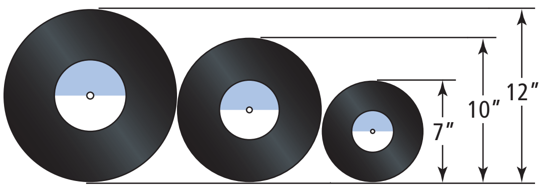
a) Calculate the outside surface area of one of these hollow cylinders.

b) Calculate the rate of the area needed to record the music to the number of minutes of music.



2. In the 1930s, RCA produced the LP. These vinyl records were pressed on a 30 cm diameter flexible plastic disc. Each LP could hold about 45 min of music using both sides.
   a) Calculate the circular area of both sides of an LP.
   b) Calculate the rate of the area needed to record the music to the number of minutes of music.

3. Records evolved into three sizes and three forms of sound reproduction, the 45 rpm, the 78 rpm, and the $33\frac{1}{3}$ rpm (or LP). Newly pressed records were inserted in a paper or cellophane envelope or sleeve, and then slipped into a printed record jacket or album cover.



   a) Choose one size of vinyl record. Calculate the area of a record jacket needed for this vinyl record.
   b) Design an album cover that you would use for the record jacket for your favourite recording artist.

4. **Reflect and Respond**
   a) List possible advantages and disadvantages of vinyl records compared to wax cylinders for recorded music.
   b) Vinyl records have recently made a comeback and sales are on the increase. Discuss with a partner some possible reasons for this increase in popularity.
   c) Brainstorm other advancements in music storage since the early wax cylinders and discuss how technology has changed the storage of music.

Web Link

To learn more about how vinyl records are made, go to www.mhrmath10.ca and follow the links.

Link the Ideas

Sometimes you may know the dimensions in one measurement system, but need to determine the area or volume in the other measurement system. To work with units of area and volume in both measurement systems, you need to understand the relationships between the units of length in each system. Remember that area involves square units while volume involves cubic units.

Example 1 Convert Between SI Units for Area


Cochrane Mural

An art class is creating a mural mosaic. They know that future art classes will have to contribute to the project to complete it. Each person in the class makes a painting on a 15-cm by 15-cm panel. Then, all the panels are assembled into the mural mosaic. There are 25 students in the art class, so the panels they create will be assembled into a square with each side containing five panels. What area is required for this part of the mural?

Solution

Two students calculate the area required for this part of the mural.

Cassy calculates the area of each individual panel.

$A = s^2$
$A = (15)^2$
$A = 225$
Each panel is 225 cm².
There are 25 panels.
$(25)(225) = 5625$
The total area is 5625 cm².

How does Cassy use her knowledge of the area of a square?

Stefan thinks of one large rectangle. Since there are five panels on each side, each measuring 15 cm long, he concludes that each side of the rectangle will measure 75 cm. He converts to 0.75 m and then calculates the area:

$A = lw$
$A = (0.75)(0.75)$
$A = 0.5625$
The area is 0.5625 m².

How does Stefan use his knowledge of the area of a rectangle?

Both students calculated the correct area. When they compared answers, they realized that to convert between square metres and square centimetres, you need to multiply or divide by 10 000.

$(1\text{ m})(1\text{ m}) = 1\text{ m}^2$
$(100\text{ cm})(100\text{ cm}) = 10\,000\text{ cm}^2$
So, $1\text{ m}^2 = 10\,000\text{ cm}^2$

Recall that there are 100 cm in 1 m.

Your Turn

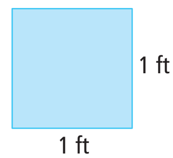
a) Determine the area of a rectangle that is 1.7 m by 2.5 m, in square centimetres.

b) Determine the area of a rectangle that is 50 mm by 25 mm, in square metres.

Example 2 Work With Units for Area

Tiles imported from different countries sometimes have imperial dimensions. A tile layer may need to convert from square feet to square centimetres.

One type of floor tiles is sold in squares measuring 1 ft by 1 ft.



a) What is the area of one tile in square centimetres?

b) The tile layer is working with an area that measures 8 ft by 4 ft. What is the area, to the nearest hundredth of a square centimetre? to the nearest square metre?

The abbreviation for inches is in. or ″.
The abbreviation for feet is ft or ′.
The abbreviation for yards is yd.

Solution

a) Calculate the area of one tile.

$A = s^2$

$A = (30.48)^2$

$A = 929.0304$

Recall that 1 ft = 30.48 cm.

The area of one tile is 929.03 cm², to the nearest hundredth of a square centimetre. You can conclude that 1 ft² is approximately 929.03 cm².

1 ft ≈ 30 cm
A tile that is 1 ft by 1 ft is approximately 30 cm by 30 cm.
(30)(30) = 900
The area of one tile is about 900 cm².

b) The tile layer needs to convert the dimensions of 8 ft by 4 ft to SI units.

The length is 8 ft.
$8(30.48) = 243.84$
Therefore, the length is 243.84 cm.

The width is 4 ft.
$4(30.48) = 121.92$
Therefore, the width is 121.92 cm.

How could you use the value for 8 ft in centimetres to determine the value for 4 ft in centimetres?

$A = lw$

$A = (243.84)(121.92)$

$A = 29\ 728.9728$

The tile area is approximately 29 728.97 cm².

Since 100 cm = 1 m, then 10 000 cm² = 1 m².
To express the area in square metres, divide by 10 000.

$$\frac{29\ 728.9728}{10\ 000} = 2.97289728$$

The tile area is approximately 3 m².

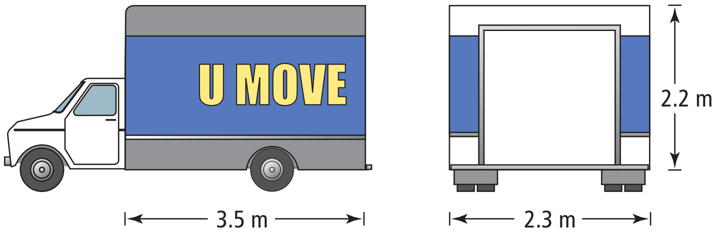
Your Turn

a) Determine the area of a rectangle that is 10 cm by 100 cm, in square feet.

b) Determine the area of a rectangle that is 6 in. by 4 in., in square millimetres.

Example 3 Work With Units for Volume

Sahid has numerous boxes to load onto a moving truck.



What is the volume of the truck, to the nearest cubic foot?

Solution

Convert the dimensions to feet.

The length is 3.5 m.
$3.5(3.281) \approx 11.4835$ 1 m ≈ 3.281 ft
Therefore, the length is approximately 11.4835 ft.

The width is 2.3 m.
$2.3(3.281) \approx 7.5463$
Therefore, the width is approximately 7.5463 ft.

The height is 2.2 m.
$2.2(3.281) \approx 7.2182$
Therefore, the height is approximately 7.2182 ft.

Find the volume of the truck by using the formula $V = lwh$.
$V = lwh$
$V \approx (11.4835)(7.5463)(7.2182)$ How does Sahid use his knowledge of the volume of a rectangular prism?
$V \approx 625.514\ 314$

The volume of the truck is approximately 626 ft³.

Your Turn

Convert the volume of an object that measures 3 cm by 4 cm by 10 cm to cubic inches. 1 cm ≈ 0.3937 in.

Key Ideas

Proportional reasoning can be used to

- solve problems involving area or volume units within SI
- solve problems involving area or volume units within the imperial system
- solve problems requiring the conversion of area or volume within and between the SI and imperial systems using linear dimensions

Convert 0.62 m^2 to square centimetres.

$1 \text{ m} = 100 \text{ cm}$
$1 \text{ m}^2 = (100 \text{ cm})(100 \text{ cm})$
$\quad\;\; = 10\ 000 \text{ cm}^2$
$0.62 \text{ m}^2 = (0.62)(10\ 000 \text{ cm}^2)$
$\quad\quad\;\; = 6200 \text{ cm}^2$
0.62 m^2 is equal to 6200 cm^2.

Calculate the volume of a rectangular prism with dimensions 1 ft by 3 ft by 5 ft in cubic metres.

$1 \text{ ft} = 0.3048 \text{ m}$ $\quad$ $3 \text{ ft} = 3(0.3048 \text{ m}) = 0.9144 \text{ m}$ $\quad$ $5 \text{ ft} = 5(0.3048 \text{ m}) = 1.524 \text{ m}$

$V = (0.3048 \text{ m})(0.9144 \text{ m})(1.524 \text{ m})$
$V = 0.4247526989 \text{ m}^3$

The volume of the prism is approximately 0.42 m^3.

Check Your Understanding

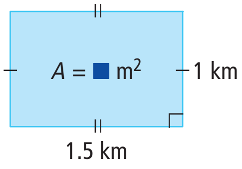
Practise

For help with #1 and #2, you may need to refer to the table of conversion factors.

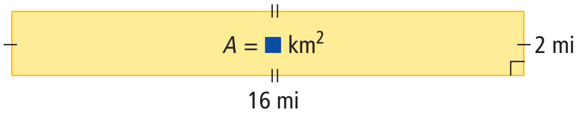
Imperial Unit	SI Unit
1 in.	2.54 cm
1 ft	0.3048 m
1 yd	0.9144 m
1 mi	1.609 km

1. Calculate the following areas to the indicated SI unit. Express your answers to the nearest tenth of a square unit.

a)



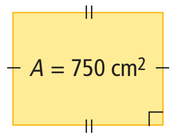
b)



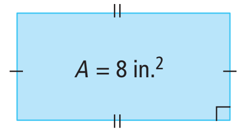
c) the area of a rectangle 35 in. by 10 in., in square centimetres

d) the area of a rectangle 21 ft by 50 ft, in square metres

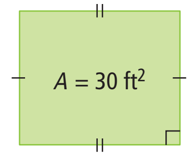
2. Determine possible dimensions for each area. Then, use your dimensions to calculate the area to the indicated equivalent.

a) 

$A = \blacksquare$ mm²

b) 

$A = \blacksquare$ m²

c) 

$A = \blacksquare$ cm²

Did You Know?

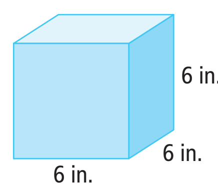
The *Festival du Voyageur* was founded in 1969 by a group of Saint-Boniface entrepreneurs. Originally a 3-day event held in Winnipeg's French Quarter, this event has evolved into a 10-day province-wide celebration. The *Festival du Voyageur* celebrates the *joie de vivre* of the fur traders, who established the Red River Colony and the growing French-Canadian community in western Canada. Held every February, the *Festival du Voyageur's* emphasis is on the beauty of winter, with numerous historical, educational, and entertaining activities.

3. At the *Festival du Voyageur*, an outdoor winter beaverball court is being built. The court measures 9 m by 18 m. Your task as a volunteer is to cover it with a tarp. You find that tarps are sold by the square yard. Find the area required for the tarp in square yards.

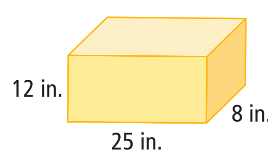
Festival du Voyageur, Winnipeg

4. For each of the following containers, find the volume in cubic centimetres.

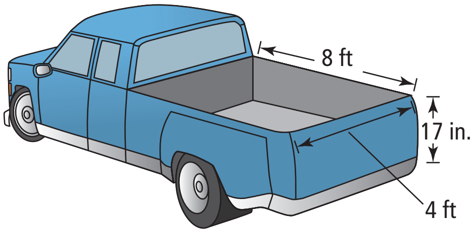
a) a cube with sides measuring 6 in.



b) a right rectangular prism with sides measuring 12 in. by 8 in. by 25 in.

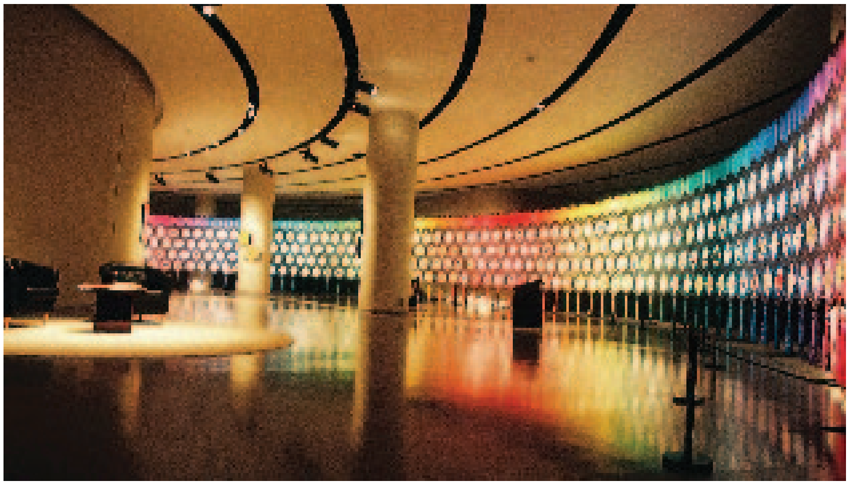


5. A pickup truck has a box that measures 4 ft by 8 ft by 17 in. What is the volume of the box to the nearest tenth of a cubic metre?



Apply

6. The *Quilt of Belonging* is a project created by artist Esther Bryan. The quilt includes 263 handmade, 11-in.-square blocks, representing each immigrant, First Nation, Métis, and Inuit group in Canada. Seventy of these blocks each represent a First Nation, Métis, or Inuit group in Canada. What is the total area of these 70 blocks, in square feet and in square metres?



Did You Know?

The *Quilt of Belonging* is "... the largest and most inclusive work of textile art made about Canada." Esther Bryan, the project artist, says, "The completed quilt, with its many parts, shows that we all can be integrated into the fabric of Canada, living together harmoniously, learning to respect one another for our differences while celebrating what we have in common."

7. Gentry's family is buying a new home. Gentry is concerned about the size of his bedroom in the new home. Gentry's current bedroom measures 10′6″ by 11′3″. The floor plan shows that his new bedroom would have an area of 11.4 m^2.

a) Which room is bigger? by what percent?

b) The carpet in Gentry's new bedroom is worn and needs to be replaced. He finds new carpet he likes for $12.99 per square yard. What is the cost of the new carpet for Gentry's new room, before tax?

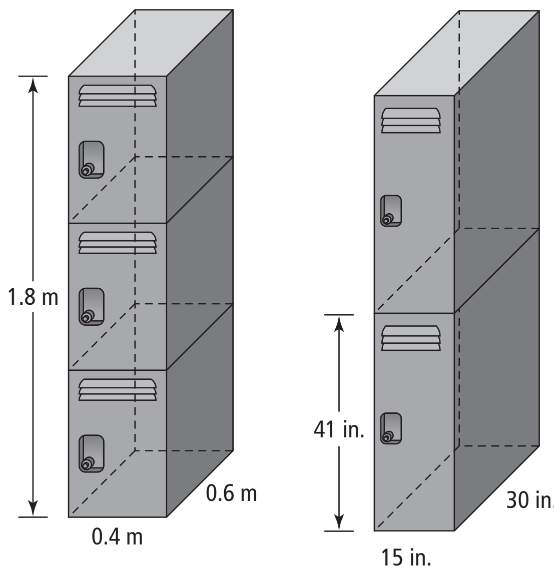
8. Andrea is preparing an installation manual for a cell-phone tower to be used in a European country. The tower specifications are in imperial units, and she must convert them to SI for her client. The specifications state that the signal for the cell-phone tower covers a circular area of radius 2.5 mi. What is this area in SI units?

9. A tile layer has an entranceway to tile. The entrance measures 5′2″ by 3′6″. The tiles each measure 4″ by 4″.

a) What is the area of the entranceway, in square inches?

b) When working on tiling projects, it is recommended that the installer purchase 10% extra material. How many tiles should be purchased for this project?

10. At the local recreation centre, you have a choice between two different types of lockers. You can choose a single locker from a double stack, or a single locker from a triple stack. The dimensions of each stack of lockers are shown below.



a) Which type of locker would give you more space?

b) How much more space, in cubic metres, would you have?

11. Describe how you would determine each volume in the indicated units. What is the volume?

a) a cube with dimensions 1 m by 1 m by 1 m in cubic centimetres

b) a rectangular prism with dimensions 5 cm by 7.1 cm by 10 cm in cubic metres

c) a rectangular prism with dimensions 0.5 m by 1 m by 5 m in cubic millimetres

Extend

12. In the imperial system, large areas of land are measured in acres. For example, the area of Shaw Millennium Skate Park in Calgary, the biggest public skate park in North America, is about 1.85 acres.

a) One acre is the same as 43 560 ft^2. What is the area of Shaw Millennium Skate Park in square feet?

b) Show how you would determine the area you found in part a) in square metres.

c) The SI unit for large areas is the hectare. One hectare is the same as 10 000 m^2. Express the area of Shaw Millennium Park in hectares.

13. The maps in an atlas use scale diagrams. Choose a rectangular area on a map of northern Canada. Using the scale from the atlas, calculate the area of your rectangle in square miles and in square kilometres.

Shaw Millennium Skate Park, Calgary

Create Connections

14. What occupations do you think would use both the imperial and SI measurement systems?

15. Describe your preferred method for converting between SI and imperial measures. Why did you choose this method?

16. Think of your local grocery store. In which departments of the store would you need measurement conversions? Explain.

2.2 Surface Area

Blackfoot Crossing Exhibit Hall

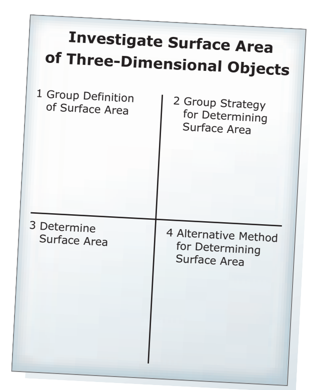
Focus on ...

- solving problems involving the surface area of three-dimensional objects
- finding an unknown dimension of a three-dimensional object given its surface area

Architectural design ideas may evolve from your culture, icons, or everyday life. The exhibit hall at the Blackfoot Crossing Historical Park in southern Alberta is a cultural, educational, and entertainment centre built at the site of the signing of Treaty 7.

The knowledge of surface area of integrated structures is essential when constructing architectural designs like the Blackfoot Exhibit Hall. Architects use surface area to calculate the amount of material needed.

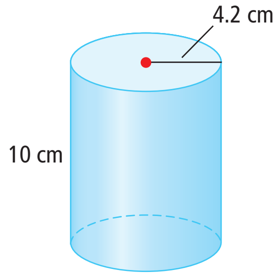
Investigate Surface Area of Three-Dimensional Objects

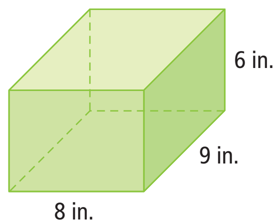


1. As a group, divide a sheet of paper into four quadrants. Label the quadrants with the following:

Quadrant 1: Group Definition of Surface Area
Quadrant 2: Group Strategy for Determining Surface Area
Quadrant 3: Determine Surface Area
Quadrant 4: Alternative Method for Determining Surface Area

2. As a group, choose one of the three-dimensional objects shown below.





Complete quadrant 1 and one of the three remaining quadrants. Then, pass the sheet on to another group.

3. a) Review and discuss the work of the other groups.
   b) Choose and complete one of the remaining quadrants. Then, pass the sheet of paper on to another group.

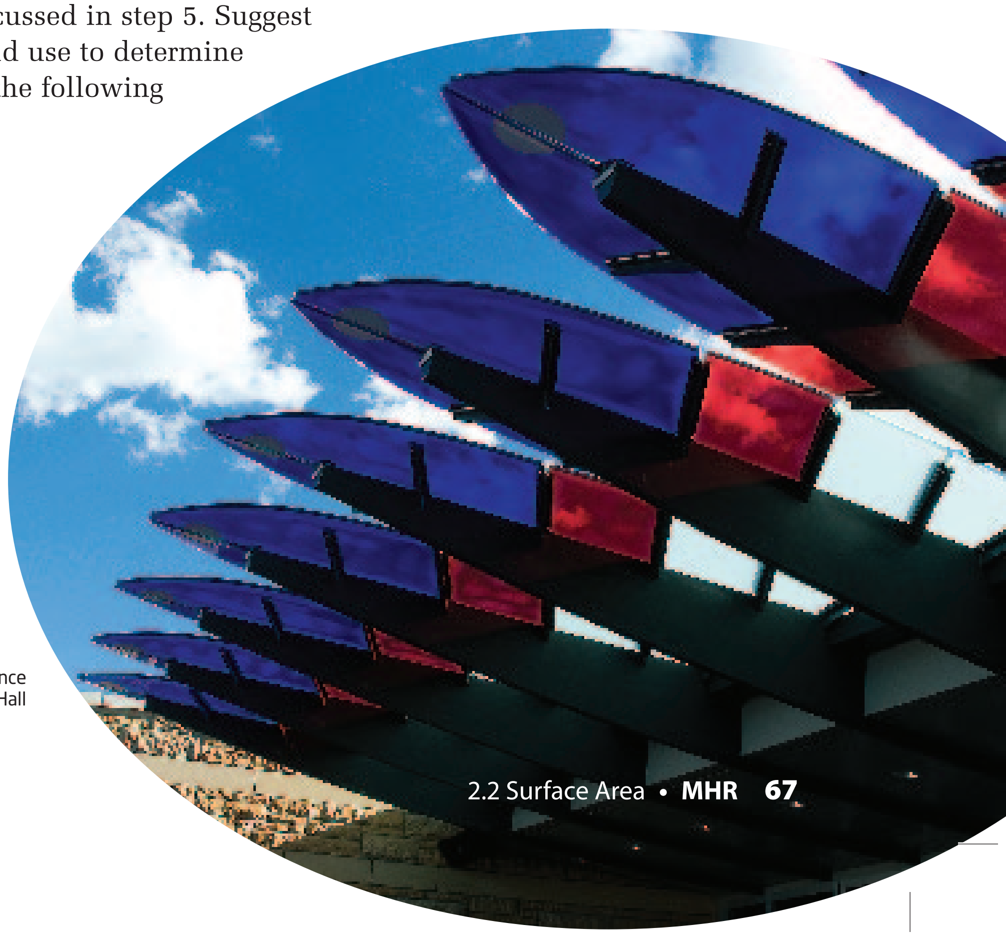
4. Repeat the process in step 3 for the remaining quadrant.

5. **Reflect and Respond**
   a) Did another group's strategy work better than yours? Why or why not?
   b) Did you see any alternative strategies that you preferred? Explain.
   c) In quadrant 3, was there a group strategy that you preferred? Explain.

6. Consider the ideas you discussed in step 5. Suggest possible strategies you could use to determine the surface area of each of the following 3-D objects.
   a) cone
   b) sphere
   c) pyramid

WWW Web Link

To learn more about Blackfoot Crossing Historical Park, go to www.mhrmath10.ca and follow the links.

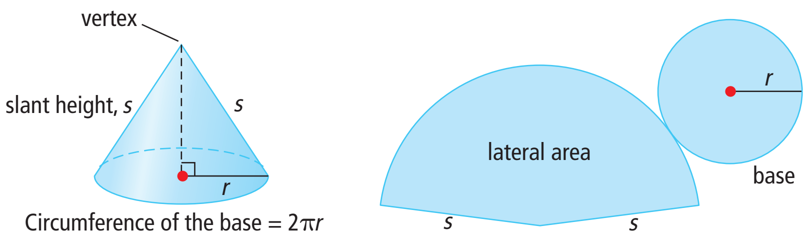


Eagle feather fan over the entrance to Blackfoot Exhibit Hall

Link the Ideas

Surface Area of a Right Cone

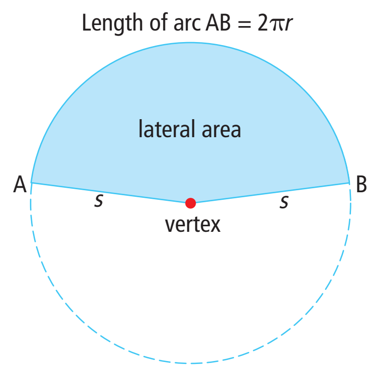
Consider a right **cone** with **slant height** s and base radius r. Construct a net of the cone, including the **lateral area** and the base.



From the lateral area of the net, you can construct a large circle using the vertex of the cone as the centre and the slant height of the cone, s, as the radius.

The lateral area of the base of the cone forms a sector of the large circle.

The circumference of the base of the cone, $2\pi r$, forms the length of the arc AB of the sector.



The circumference of the large circle that is formed from the lateral area is $2\pi s$.

The area of the large circle is πs^2.

To determine the lateral area of the cone you can set up a proportion of corresponding ratios.

$$\frac{\text{lateral area of cone}}{\text{area of large circle}} = \frac{\text{circumference of cone}}{\text{circumference of large circle}}$$

$$\frac{\text{lateral area of cone}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

$$\frac{\text{lateral area of cone}}{\pi s^2} = \frac{r}{s}$$

$$\text{Lateral area of cone} = \left(\frac{r}{\cancel{s}}\right)(\pi \cancel{s^2}^{\,s})$$

$$\text{Lateral area of cone} = \pi rs$$

The lateral area of a right cone with radius r and slant height s is πrs.

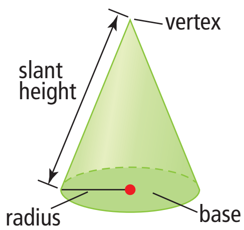
The base of the cone is a circle with radius r, so its area is πr^2.

The total surface area of a right cone is the sum of the areas of the base and the lateral surface.

$$SA_{\text{cone}} = \pi r^2 + \pi rs$$

cone

- a three-dimensional object with a circular base and a curved lateral surface that extends from the base to the vertex

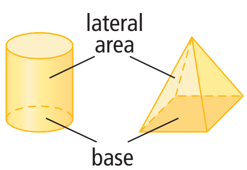


slant height

- the shortest lateral distance from the edge of the base of a cone or pyramid to its highest point

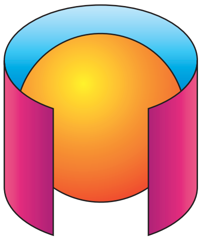
lateral area

- the surface that joins the two bases of a three-dimensional object or that joins the base to the highest point



Surface Area of a Sphere

The formula for the surface area of a **sphere** is linked to the surface area of a right cylinder. Think of wrapping a right cylinder around the sphere.



The diameter of the sphere will be the height of the cylinder.

This height will be $2r$.

The circumference of the sphere will be the circumference of the cylinder. This circumference will be $2\pi r$.

The lateral area of the cylinder, when flattened, forms a rectangle. The area of the rectangle formed by the right cylinder is directly related to the surface area of the sphere.

$A = (length)(width)$

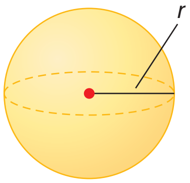
$A = (2\pi r)(2r)$

$A = 4\pi r^2$

$SA_{sphere} = 4\pi r^2$

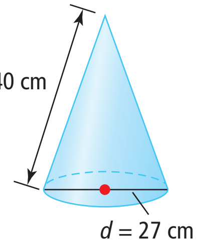
sphere

- a round, ball-shaped object
- a set of points in space that are a given distance (radius) from a fixed point (centre)



Example 1 Calculate the Surface Area of a Right Cone

A right cone has a circular base with diameter 27 cm and slant height 40 cm. Calculate the surface area of the cone, to the nearest tenth of a square centimetre.



Solution

The surface area of a right cone is the sum of the area of the single circular base, B, and the lateral area.

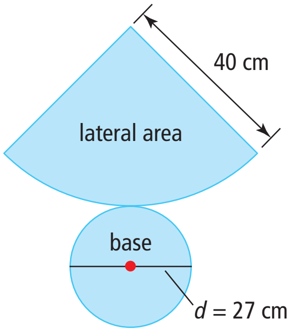
$SA = B + lateral\ area$

$SA = \pi r^2 + \pi rs$

$SA = \pi(13.5)^2 + \pi(13.5)(40)$

$SA = 2269.015\ldots$

Since the diameter of the circle is 27 cm, the radius is 13.5 cm.



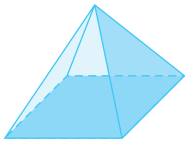
The surface area of the cone is approximately 2269.0 cm^2.

Your Turn

Sketch a right cone with diameter 16 cm and slant height 12 cm. What is its surface area?

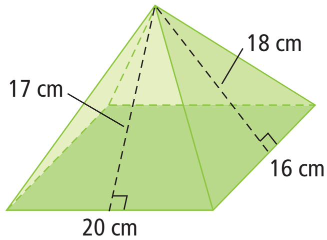
pyramid

- A three-dimensional object with one base and the same number of triangular faces as there are sides on the base



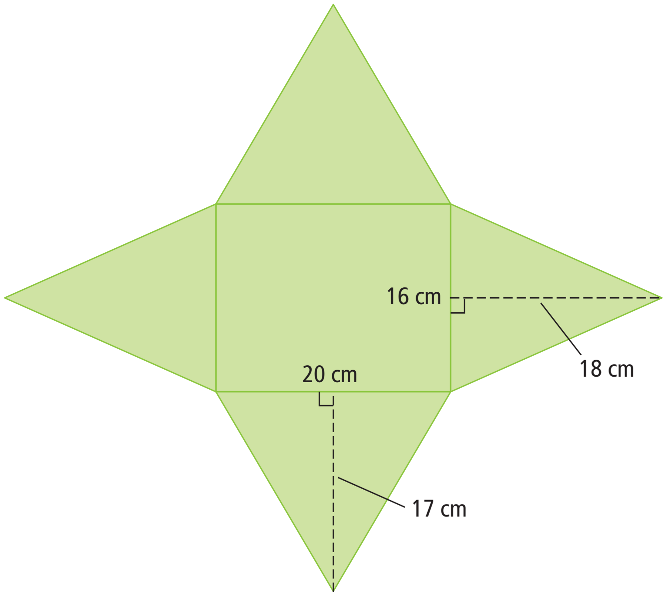
Example 2 Calculate the Surface Area of a Right Pyramid

A right rectangular **pyramid** has a rectangular base measuring 16 cm by 20 cm. The slant height of the triangular face with the shorter base is 18 cm, while the slant height of the triangular face with the longer base is 17 cm. What is the surface area of the pyramid?



Solution

As with a right cone, the surface area of a right pyramid can be calculated as the sum of the area of the base, B, and the lateral area.



$SA = B + \text{lateral area}$

$SA = (\text{length})(\text{width}) + 2[\frac{1}{2}(\text{length})(\text{slant height}_1)] + 2[\frac{1}{2}(\text{width})(\text{slant height}_2)]$

$SA = (20)(16) + 2[0.5(20)(17)] + 2[0.5(16)(18)]$

$SA = 948$

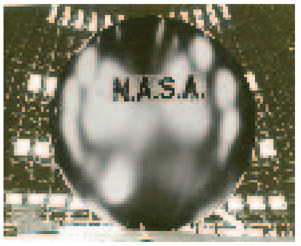
The surface area of the right rectangular pyramid is 948 cm^2.

The lateral area of a right rectangular pyramid is made up of four triangles. The triangles on the opposite faces are the same size.

Your Turn

Sketch a right rectangular pyramid with a square base measuring 10 cm on each side. The slant height of each face is 8.5 cm. What is the surface area of the pyramid?

Example 3 Calculate the Surface Area of a Sphere



A satellite is wrapped with polyester film to protect it during transportation. How much film is required to cover the Echo Satellite, which has a circumference of 95.8 m? Express your answer to the nearest tenth of a square metre.

Solution

The formula for the surface area of a sphere is $SA = 4\pi r^2$, where r is the radius. Calculate the radius from the circumference.

$$C = 2\pi r$$
$$95.8 = 2\pi r$$
$$r = \frac{95.8}{2\pi}$$
$$r = 15.247\ldots$$

Substitute the radius into the formula for the surface area.

$SA = 4\pi r^2$
$SA = 4\pi(15.247\ldots)^2$
$SA = 2921.333\ldots$

Approximately 2921.3 m^2 of film is required to cover the satellite.

Your Turn

Find the surface area of a basketball with diameter 23.85 cm. Express your answer to the nearest hundredth of a square centimetre.

Example 4 Determine a Dimension When the Surface Area is Known

The surface area of an official 5-pin bowling ball varies from approximately 459.96 cm^2 to 506.71 cm^2. What is the variation in the diameter of the bowling ball?

Solution

Substitute into $SA = 4\pi r^2$.

$4\pi r^2 = SA$	$4\pi r^2 = SA$
$4\pi r^2 = 459.96$	$4\pi r^2 = 506.71$
$r^2 = \frac{459.96}{4\pi}$	$r^2 = \frac{506.71}{4\pi}$
$r = \sqrt{\frac{459.96}{4\pi}}$	$r = \sqrt{\frac{506.71}{4\pi}}$
$r \approx 6.05$	$r \approx 6.35$

How do you calculate the diameter from the radius?

The variation in the diameter of an official 5-pin bowling ball is from 12.1 cm to 12.7 cm.

Your Turn

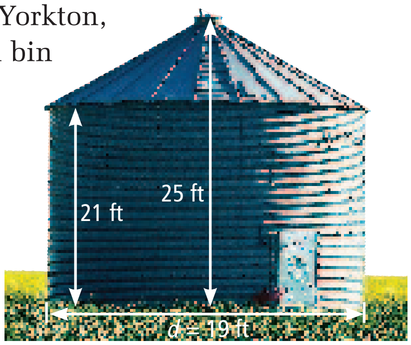
To the nearest millimetre, calculate the radius of a sphere with a surface area of 1 m^2.

Did You Know?

Artifacts from a game similar to bowling were found in the tomb of an ancient Egyptian youth who died in approximately 5200 B.C.E. Ancient Polynesians rolled stones at objects from a distance of 60 ft (18.29 m). This is the same distance as from the foul line to the headpin in modern-day bowling.

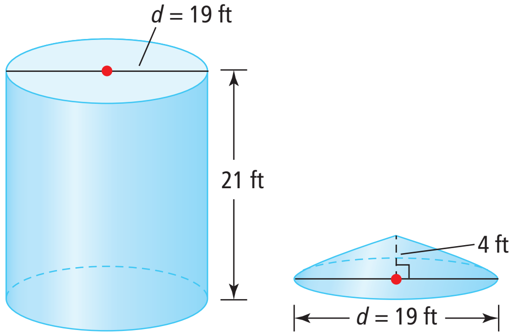
Example 5 Visualize and Find Surface Areas of Composite Objects

A farm equipment manufacturer in Yorkton, SK, has decided to construct a grain bin using galvanized steel. How much steel is required to construct the grain bin as shown? Express your answer to the nearest hundredth of a square foot. Do not include overlap of the steel sheets where they are fastened together.



Solution

Visualize the cylindrical base with a right cone-shaped roof. Sketch the two parts of the structure and add appropriate dimensions.



Remember that the bottom and top of the cylindrical portion and the bottom of the conical roof are not included.

surface area of cylindrical shape $= \pi d \times height$

$$SA_{\text{cylinder}} = \pi(19)(21)$$
$$SA_{\text{cylinder}} = 1253.495\ldots$$

The circumference of a circle can be found using the formula πd or $2\pi r$.

To calculate the surface area of the cone, you need to know the slant height first.
Use the Pythagorean relationship.

$$s = \sqrt{4^2 + 9.5^2}$$
$$s = \sqrt{16 + 90.25}$$
$$s = \sqrt{106.25}$$
$$s = 10.307\ldots$$

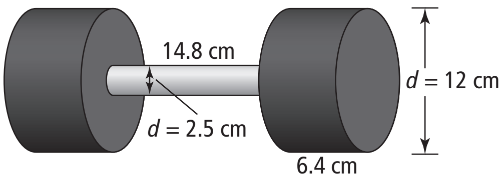
surface area of conical shape $= \pi rs$

$$SA_{\text{cone}} = \pi(9.5)(10.307\ldots)$$
$$SA_{\text{cone}} = 307.636\ldots$$

The combined surface area is $1253.495\ldots + 307.636\ldots = 1561.132\ldots$
The total surface area of the grain bin, to the nearest hundredth of a square foot, is 1561.13 ft^2.
You will need a minimum of 1562 ft^2 of galvanized steel to construct the grain bin.

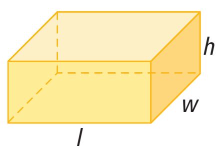
Your Turn

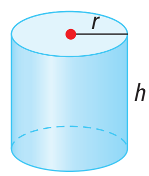
Calculate the surface area of the following composite object, to the nearest tenth of a square centimetre.



Key Ideas

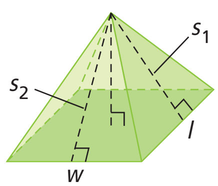
- The surface area of a right cylinder and of a right prism can be calculated using the area of the bases (top and bottom) plus the lateral area.

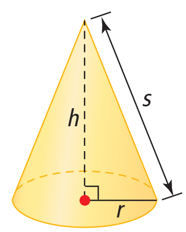




$SA_{prism} = 2lw + 2lh + 2wh$ $\qquad$ $SA_{cylinder} = 2(\pi r^2) + 2\pi rh$

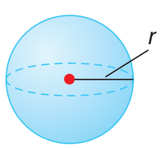
- The surface area of a right pyramid and of a right cone can be calculated using the area of the base plus the lateral area.





$SA_{pyramid} = lw + 2[\frac{1}{2}ls_1] + 2[\frac{1}{2}ws_2]$ $\qquad$ $SA_{cone} = \pi r^2 + \pi rs$

- The surface area of a sphere depends on the radius only.



$SA_{sphere} = 4\pi r^2$

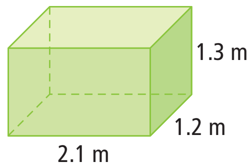
Check Your Understanding

Practise

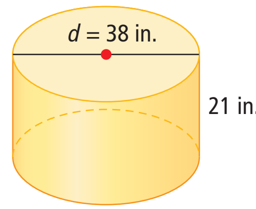
For each of the following, express your answers to the nearest tenth of a unit where necessary.

1. Calculate the surface area of each of the following.

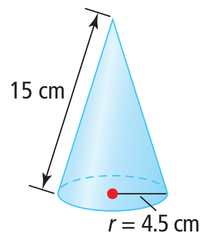
a)



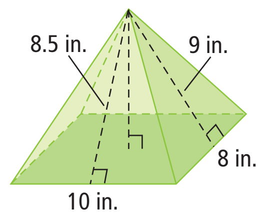
b)



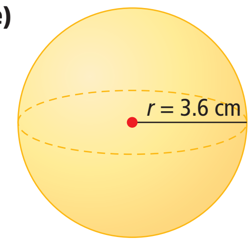
c)



d)



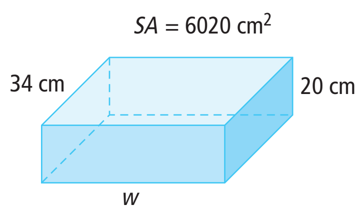
e)



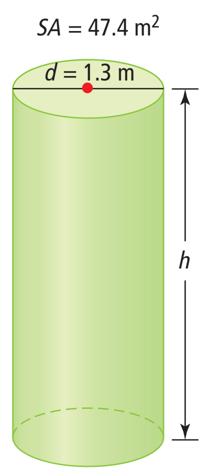
2. Sketch a right pyramid with a square base measuring 16 ft by 16 ft. The slant height is 12 ft. What is the surface area of the pyramid?

3. For each of the following, the surface area is given. Calculate the missing dimension.

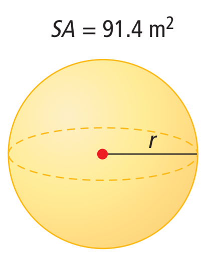
a)



b)

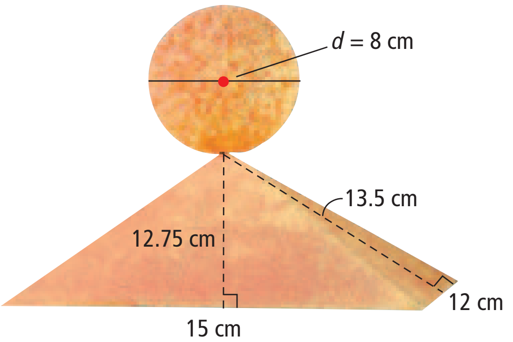


c)



4. A closed box has a surface area of 126 in.^{2}. The base of the box is 5 in. by 3 in. Sketch a diagram and find the height of the box.

5. Calculate the surface area of this object composed of a pyramid and a sphere.



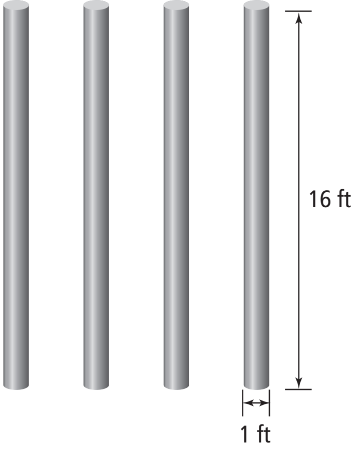
Apply

6. Austin is helping to build the set for a school play. There are four cylindrical pillars standing on the stage that need to be painted. Each pillar is 16 ft high and 1 ft in diameter, as shown below. Austin calculated the surface area to the nearest hundredth of a square foot. His work is shown below.

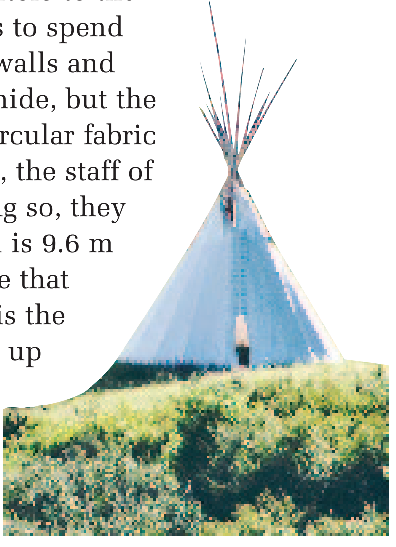
$SA = 2(\pi r^2) + \pi d(h)$
$SA = 2(\pi)(0.5)^2 + \pi(1)(16)$
$SA = 0.5\pi + 16\pi$
$SA = 16.5\pi$
$SA \approx 51.84$
There are four cylinders, so
$SA = 4 \times 51.84$
$SA = 207.36$

The surface area is 207.36 ft^2.

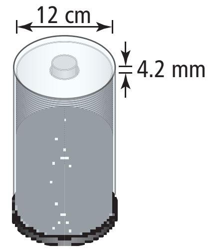
Austin thinks he made an error in his work. Discuss whether Austin actually made an error. What surface area would you paint?



7. One of the activities available to visitors to the Blackfoot Crossing Historical Park is to spend the night in a tipi. Historically, the walls and floor of a tipi were made of buffalo hide, but the current tipi is canvas with a large circular fabric floor. At the beginning of the season, the staff of the park set up the tipis. While doing so, they determine that the diameter of a tipi is 9.6 m and its slant height is 7.3 m. Assume that the tipi approximates a cone. What is the minimum amount of canvas making up the sides of the tipi, to the nearest hundredth of a square metre? Do not include any seam allowances.



8. **Unit Project** Compact discs are sometimes packaged in cylindrical stacks of 100. Each CD has a thickness of 1.2 mm and a diameter of 12 cm.



a) The outside radius of the storage case is 0.7 cm more than that of the CD. The height of the case is 4.2 mm more than that of the stack of 100 CDs. What is the surface area of the storage case, excluding the base, to the nearest square centimetre?

b) If a rectangular CD jewel case holding a single CD is 0.5 cm wider than the CD, 2.5 cm longer than the CD, and 8 times the thickness of the CD, what is the surface area of the jewel case?

9. A designer is working on a line of athletic equipment. He is designing a cylindrical punching bag, similar to the one shown. The designer wants to use a maximum of 1.3 m^2 of material to make the bag, and has determined that the diameter of the bag should be 36 cm. Determine the maximum possible height of the bag, to the nearest tenth of a centimetre.

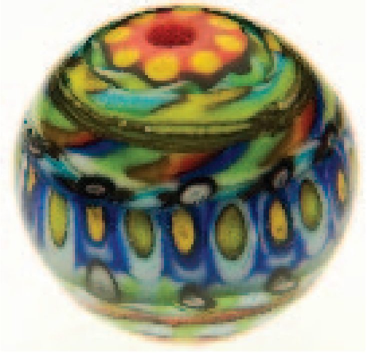
10. Earth has a diameter of approximately 8000 mi. Land forms about 29% of the surface area of Earth. Assume Earth is a sphere. Estimate the area of land on Earth.

11. The photo shows a traditional Haida hand drum that has a diameter of $14\frac{7}{8}$ in. and is 3 in. deep. What is the minimum amount of hide used to make the drum if the hide covers only the top and lateral surfaces? Express your answer to the nearest square inch.

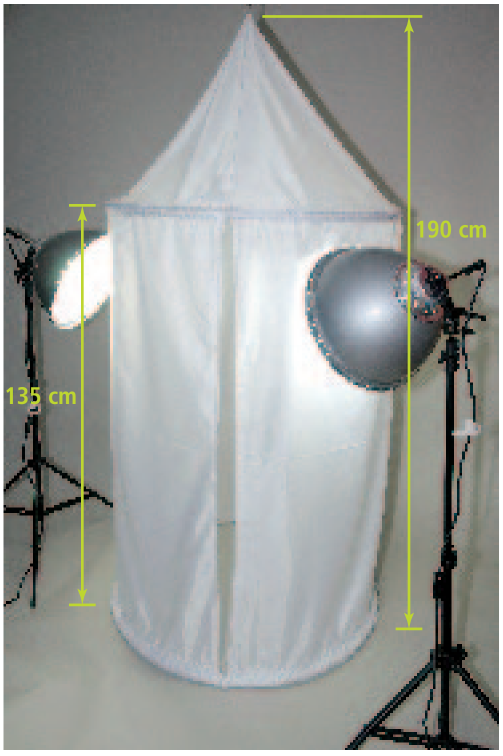
Traditional Haida hand drum showing twin salmon transforming into the next generation. Traditional Haida hand drums are used in ceremony, cultural events such as potlatches, and as artwork. Traditional drums should always be handled with respect following appropriate protocol.

12. The Muttart Conservatory in Edmonton began with a donation from the Muttart family. There are four greenhouses, each in the shape of a right pyramid with a square base. Each of the two largest greenhouses has a base that measures 26 m on each side and has a slant height of 35.4 m. How much glass is needed for each large greenhouse? Express your answer to the nearest square metre.

13. What is the surface area of a glass bead that has a diameter of 11 mm? Express your answer to the nearest square millimetre.



14. Photographers often use a light tent to get the best lighting for items they photograph for museums, catalogues, or online sales. This tent is cylindrical with a conical roof. The diameter of the tent is 1 m, the height of the tent is 190 cm, and the cylindrical wall of the tent is 135 cm high. What is the surface area of the light tent, to the nearest square metre?



Did You Know?

Around the year 1148, the French played *le paume*, meaning "the palm of the hand." This game developed into *jeu de paume*, *real tennis*, *royal tennis*, or, simply *tennis*.

15. Squash is a racquet sport played with a small, soft ball. The game is named for the fact that it is very easy to "squash" the ball. The rules of the game state that the ball must have a diameter of 40 mm ± 0.5 mm. This means that the actual diameter can be up to 0.5 mm more or less than 40 mm.

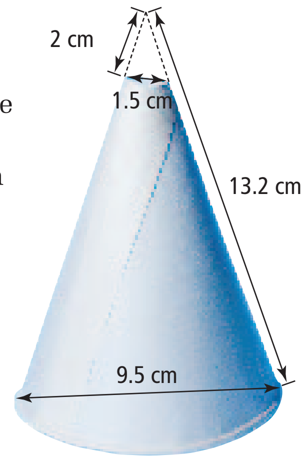
a) What is the minimum and maximum surface area of a regulation squash ball? Express your answers to the nearest square millimetre.

b) Squash balls are often packaged in cubical boxes as shown in the photo. If the box is to be sized so that the ball fits exactly in the box, find the minimum and maximum dimensions of the box.

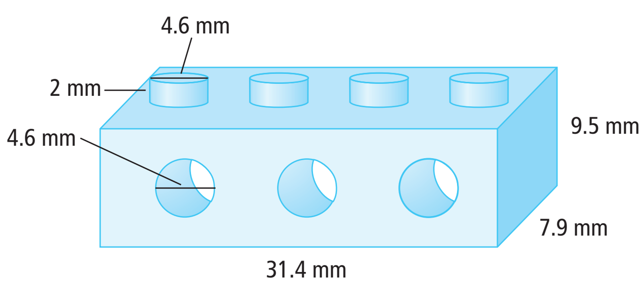
Extend

For each of the following, express your answers to the nearest tenth of a unit where necessary.

16. Gas stations often have a supply of small paper funnels that customers can use to add oil to their vehicle engines without spilling. Each funnel is a right cone with a small hole cut out of the top. The funnel has a slant height of 13.2 cm and the diameter of the large opening is 9.5 cm. The diameter of the small opening is 1.5 cm. Determine the amount of paper in the funnel.



17. A building block is shown below.



The base of the block measures 31.4 mm by 7.9 mm, and it has a height of 9.5 mm. Each right cylinder on top of the block has a diameter of 4.6 mm and a height of 2 mm. Each hole in the block has a diameter of 4.6 mm. Determine the surface area of the block that needs to be painted if you do not paint the bottom of the block.

18. Use spreadsheet software to help investigate how changing the radius of a sphere changes its surface area. Create a spreadsheet like the one shown below.

	A	B	C	D
1	Investigating Changes in Dimensions of a Sphere			
2	Stretch Factor	Radius	Surface Area	Ratio of New SA to Original SA
3	1	2	50.265	1
4	2	4		
5	3			
6	4			
7	5			

a) Use spreadsheet formulas to complete the spreadsheet. Depending on your software, you may need to type "PI" or "PI()" for π. See your spreadsheet's help feature if you need assistance.

b) Compare the stretch factor for the radius to the ratio of the new surface area to the original surface area. What pattern do you notice?

c) Use your pattern to predict the surface area of the sphere if you multiply the radius by 6. Extend your spreadsheet to check your answer.

d) In your own words, express the relationship between a change in the radius of a sphere and the change in its surface area.

Create Connections

19. Sketch an example of a composite object with dimensions in centimetres. Explain how you can find the surface area. Describe how you would convert your answer to an appropriate imperial unit of area.

20. People communicate in different ways, such as orally, in writing, and even using sign language. In your own words, explain why the surface area of any object is expressed in square units.

Did You Know?

American Sign Language is the most commonly used form of sign language in North America.

2.3 Volume

Focus on …

- solving problems involving the volume of three-dimensional objects
- finding an unknown dimension of a three-dimensional object given its volume

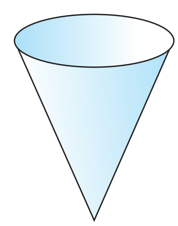
Western Canada is rich in natural resources. To make the most of Canada's natural resources, citizens need to understand the mathematics required to determine what they have, how to access it, and what it is worth. Whether it is recovering diamonds from Nunavut and the Northwest Territories or potash from Saskatchewan, transporting oil, or harvesting wood, measurement is a critical part of responsibly using Canada's resources. For many resources, volume is an important measurement: barrels of oil, board feet of lumber, and bushels of grain are just some of the traditional units of volume.

Investigate Volume

Materials

- conical cup
- paper
- scissors
- tape or glue
- sand, rice, or popcorn

What is the relationship between the volume of a right cone and the volume of a right cylinder?



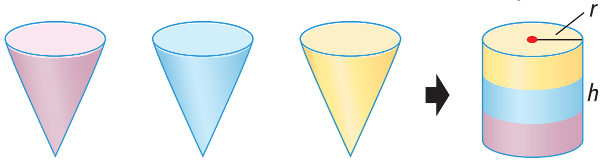
1. Using a right conical cup for reference, create a right cylinder with the same height and the same base area as the cone.
2. If you fill the cup with material (for example, rice, sand, or popcorn), predict how many cups of your material it will take to fill the cylinder.
3. Test your prediction.
4. **Reflect and Respond**
   a) Discuss with a partner the results of your investigation.
   b) Is there a relationship between the amount of material that the cone holds and the amount of material that the cylinder can hold? What is the relationship?

Link the Ideas

Volume of a Right Cone

The volume of a right prism or right cylinder can be found by multiplying the area of its base, B, by its height, h.
$V = Bh$

The volume of a right cone is related to the volume of a right cylinder with the same radius and height. The volume of the cone is one third of the volume of the cylinder.

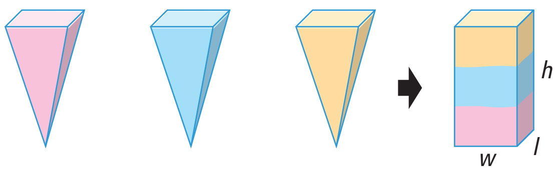


For a right cone with radius r and height h, $V_{\text{cone}} = \frac{1}{3}Bh$.

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

Volume of a Right Pyramid

This relationship is also true for right rectangular prisms and right pyramids. The volume of a right pyramid is one third of the volume of a right prism with the same base and height.



For a right pyramid with a rectangular base of length (l), width (w), and a perpendicular distance (h) from the base of the pyramid to its **apex**, $V_{\text{pyramid}} = \frac{1}{3}Bh$.

$$V_{\text{pyramid}} = \frac{1}{3}lwh$$

Volume of a Sphere

The volume of a sphere is two-thirds the volume of a cylinder with the same radius and a height equal to the diameter of the sphere. If the sphere has a radius r, then the cylinder has a base radius r and a height $2r$.

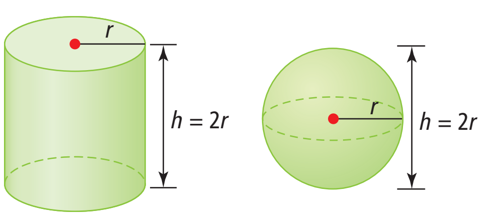
$V_{\text{sphere}} = \frac{2}{3}(\text{volume of cylinder})$

$V_{\text{sphere}} = \frac{2}{3}\pi r^2 h$

$V_{\text{sphere}} = \frac{2}{3}\pi(r^2)(2r)$

$V_{\text{sphere}} = \frac{4}{3}\pi r^3$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$



Web Link

To watch a video showing the relationship between the volume of a right cone and the volume of a right cylinder, go to www.mhrmath10.ca and follow the links.

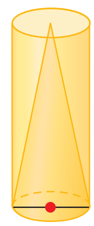
apex

- the highest point of a pyramid

Example 1 Calculate the Volume of a Right Cylinder and a Right Cone

a) Calculate the volume of the right cylinder, to the nearest tenth of a cubic centimetre.

b) Calculate the volume of the right cone, to the nearest tenth of a cubic centimetre.



Solution

a) For the cylinder,

$r = 5 \div 2$ $\quad h = 12$
$\quad = 2.5$

Substitute into the formula $V = \pi r^2 h$.

$V = \pi r^2 h$
$V = \pi(2.5)^2(12)$
$V = 75\pi$
$V = 235.619...$

The exact value is 75π cm^3; an approximate value is 235.6 cm^3.

The volume of the cylinder is approximately 235.6 cm^3.

b) For the cone,

$r = 5 \div 2$ $\quad h = 12$
$\quad = 2.5$

Method 1: Use a Formula

Substitute into the formula $V = \frac{1}{3}\pi r^2 h$.

$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi(2.5)^2(12)$
$V = 25\pi$
$V = 78.539...$

The exact value is 25π cm^3, an approximate value is 78.5 cm^3.

The volume of the cone is approximately 78.5 cm^3.

Method 2: Use Volume Relationships

Since the volume of the cone is one third of the volume of the cylinder, you could divide the volume of the cylinder by three.

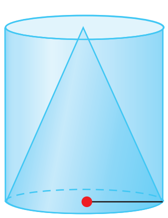
$V = \frac{1}{3}(235.6)$
$V = 78.533...$

The volume of the cone is approximately 78.5 cm^3.

Your Turn

a) What is the volume of the right cylinder, to the nearest cubic centimetre?

b) What is the volume of the right cone, to the nearest cubic centimetre?



Example 2 Calculate the Volume of a Right Pyramid

Many of the operating costs of a greenhouse depend on its volume. For example, the energy used to heat a building depends on the volume of the building. The two large greenhouses at the Muttart Conservatory have square bases measuring 26 m on each side. The apex of each greenhouse is 24 m high. What is the volume of each greenhouse, to the nearest cubic metre?

Solution

Since the base is a square, $l = w = 26$, and $h = 24$.

Substitute into the formula $V = \frac{1}{3}lwh$.

$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(26)(26)(24)$

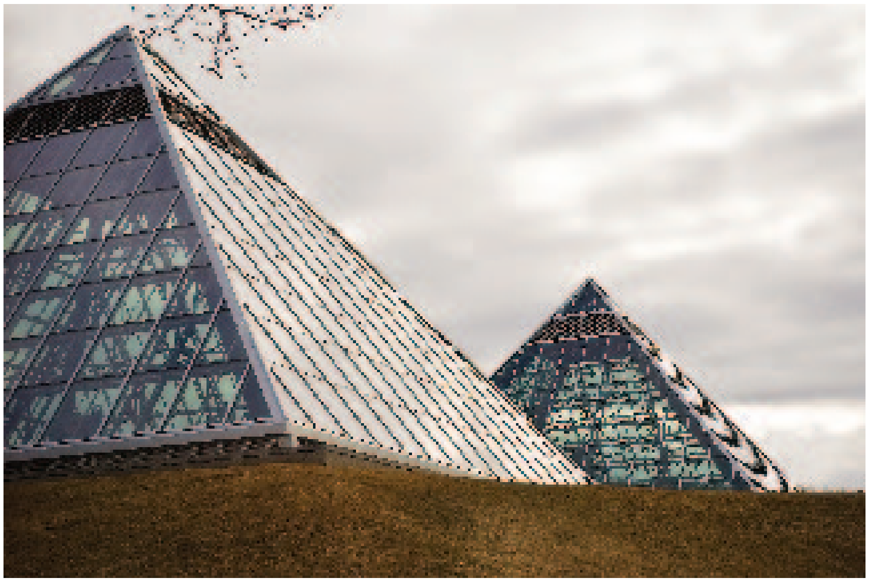
$V = 5408$

The volume of each of the large greenhouses is 5408 m^3.

Your Turn

a) The Muttart Conservatory also has two smaller greenhouses. The base of each greenhouse is a square with side length 19.5 m, and the height of each greenhouse is 18 m. What is the volume of each of the smaller greenhouses?

b) If the smaller greenhouse had been designed as a right rectangular prism with the same size base, what would its height have to be in order for the greenhouse to have the same volume?



Did You Know?

Archimedes is attributed with the discovery of the formula for finding the volume of a sphere. He is considered by most historians of mathematics as one of the greatest mathematicians of all time. Before his death, he requested that his tomb include a monument featuring a stone sphere and cylinder.

Example 3 Calculate an Unknown Dimension When Given a Volume

The volume of an exercise ball is approximately 4188.8 cm^3. What is the diameter of this ball, in centimetres?

Solution

Substitute into the formula for the volume of a sphere.

$$\begin{aligned}
V &= \frac{4}{3}\pi r^3 \\
4188.8 &\approx \frac{4}{3}\pi r^3 \\
3(4188.8) &\approx 3\left(\frac{4}{3}\pi r^3\right) \\
12\,566.4 &\approx 4\pi r^3 \\
\frac{12\,566.4}{4\pi} &\approx \frac{4\pi r^3}{4\pi} \\
1000 &\approx r^3 \\
\sqrt[3]{1000} &\approx \sqrt[3]{r^3} \\
10 &\approx r
\end{aligned}$$

The symbol $\sqrt[3]{\ }$ indicates the cube root of a number. $\sqrt[3]{1000} = 10$ because $10 \times 10 \times 10 = 1000$.

The radius is approximately 10 cm.
Therefore, the diameter of the ball is approximately 20 cm.

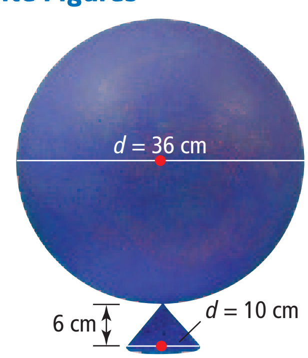
Your Turn

a) Find the cube root of 343.

b) Find the diameter, correct to the nearest millimetre, of a sphere with volume 288π cm^3.

Example 4 Finding the Volume of Composite Figures

Esther is creating a clay sculpture that includes a sphere attached to a right cone. What volume of clay, in cubic centimetres, does she need to make the sculpture?



The diameter is twice the radius. The radius of the sphere is $\frac{36}{2}$ or 18 cm. The radius of the cone is $\frac{10}{2}$ or 5 cm.

Solution

Volume of Sphere	**Volume of Cone**
$V = \frac{4}{3}\pi r^3$	$V = \frac{1}{3}\pi r^2 h$
$V = \frac{4}{3}\pi(18)^3$	$V = \frac{1}{3}\pi(5)^2(6)$
$V = 7776\pi$	$V = 50\pi$

Volume of Entire Sculpture

total volume = volume of sphere + volume of cone

$$= 7776\pi + 50\pi$$
$$= 7826\pi$$

The volume of the sculpture is 7826π cm^3, which is approximately 24 586 cm^3.

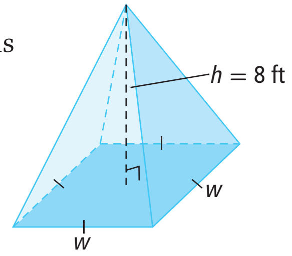
Your Turn

The Dominion Astrophysical Observatory near Victoria, BC, has a cylindrical base with a diameter of 20.1 m and a height of 9.8 m. The dome is half a sphere with the same diameter as the cylindrical base. What is the volume of the observatory?

Key Ideas

- The volume of a right cone is found by calculating one third of the volume of its related right cylinder.
  $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$
- The volume of a right pyramid is found by calculating one third of the volume of its related right prism.
  $V_{\text{pyramid}} = \frac{1}{3}lwh$
- The volume of a sphere is found by using the formula
  $V_{\text{sphere}} = \frac{4}{3}\pi r^3$
- If you know the volume of an object, you can calculate an unknown dimension.

  The volume of the right pyramid with square base is 384 ft^3. Find the dimensions of the base.



$$V = \frac{1}{3}Bh$$
$$384 = \frac{1}{3}w^2(8)$$
$$3(384) = 3\left(\frac{1}{3}w^2(8)\right)$$
$$1152 = w^2(8)$$
$$\frac{1152}{8} = w^2$$
$$144 = w^2$$
$$12 = w$$

The dimensions of the base are 12 ft × 12 ft.

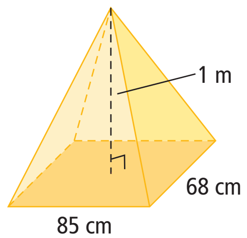
Check Your Understanding

Practise

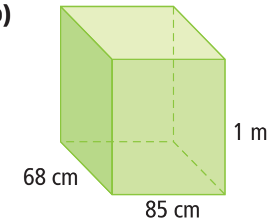
Where necessary, express your answers to the nearest tenth of a unit.

1. Calculate the volume of each of the following.

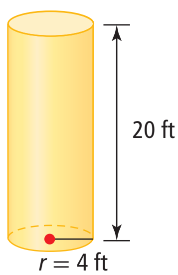
a)



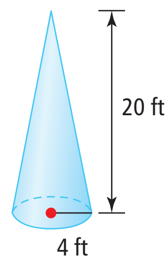
b)



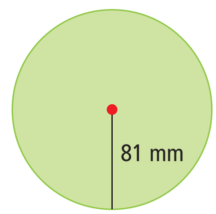
c)



d)

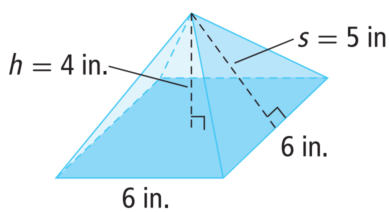


e)

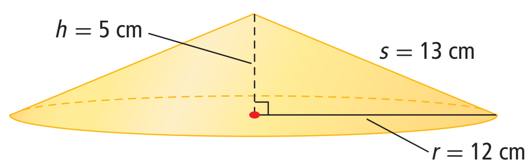


2. Calculate the volume of each solid.

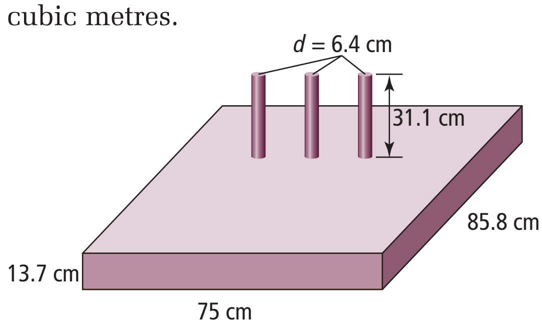
a)



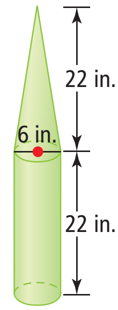
b)



3. Calculate the volume of the following composite object, in cubic metres.



4. Erin and Janine were asked to find the volume of this composite figure. Their work is shown.

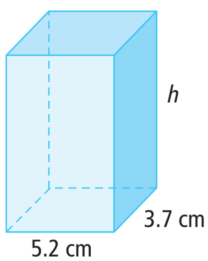


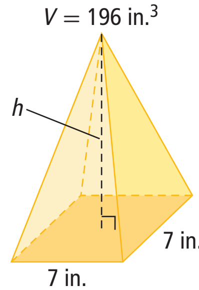
Erin	*Janine*
$V = \frac{1}{3}\pi r^2 h + \pi r^2 h$	$V = \frac{1}{3}(\pi r^2 h + \pi r^2 h)$
$V = \frac{1}{3}\pi(3^2)(22) + \pi(3^2)(22)$	$V = \frac{1}{3}(\pi(3^2)(22) + \pi(3^2)(22))$
$V = \frac{1}{3}198\pi + 198\pi$	$V = \frac{1}{3}(198\pi + 198\pi)$
$V = 66\pi + 198\pi$	$V = \frac{1}{3}(396\pi)$
$V = 264\pi$	$V = 132\pi$
$V \approx 829.38$	$V \approx 414.69$
The volume is 829.38 in.^{3}	The volume is 414.69 in.^{3}

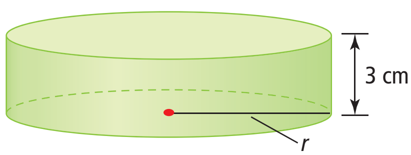
a) Discuss with a partner which student correctly calculated the volume. Justify your answer.

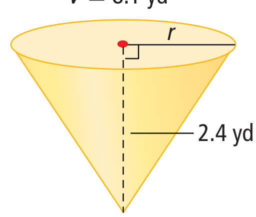
b) Identify which method you would use to calculate the volume of the composite figure.

5. Calculate the missing dimension for each of the following.

a) 

b) 

c) 

d) 

Apply

6. The Alliance Pipeline begins in northeastern British Columbia and is 3017 km long. It is the longest pipeline in North America. It carries crude oil, as well as natural gas, from exploration sites to markets. One of the longest sections of the pipeline is 1221.73 km long, with a diameter of 914 mm. The pipeline does not always run in a straight line. If it was straightened, what is the maximum volume of oil that can be contained in this section of pipeline? Express your answer to the nearest hundredth of a cubic metre.

Alliance Pipeline

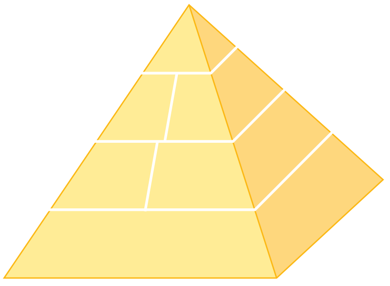
7. A right rectangular prism measures 9 in. × 4 in. × 6 in. What would be the dimensions of a cube with the same volume?

8. Kendra wants to purchase a bead necklace made of jade. The cost of the necklace depends upon the amount of jade in the necklace. Each bead is 7 mm in diameter and there are 100 beads in the necklace. What is the amount of jade in the necklace, in cubic centimetres?

Did You Know?

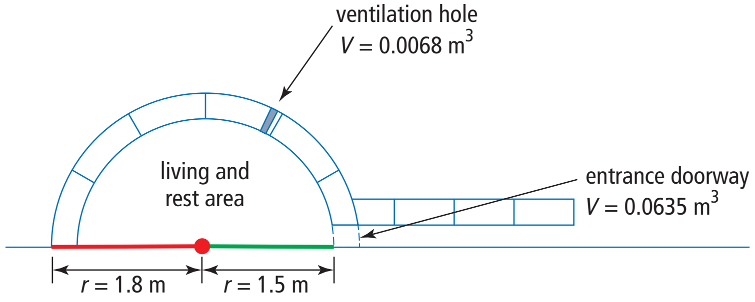
Jade is the official gemstone of British Columbia. It is found in large deposits in the Lillooet and Cassiar regions. Its translucent emerald-green appearance makes it a popular choice for jewellery and other tourism memorabilia.

9. In her art class, Andrea wants to make a locking puzzle in the shape of a pyramid to have on her desk. The base will measure 6 cm by 8 cm, and the height will be 10 cm. Calculate the amount of wood needed to create the pyramid.



10. For each of the following objects, the volume is given. Sketch the object and calculate the unknown dimension.
   a) A right cylinder has a volume of 500 cm^3 and a height of 16 cm. Calculate its radius.
   b) A right cone with a volume of 20 cm^3 has a diameter of 5 cm. Calculate its height.
   c) A sphere has a volume of 48 cm^3. Calculate the radius.

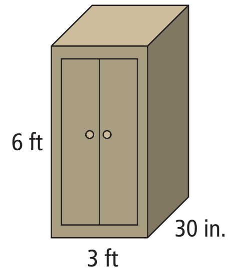
11. Traditionally, the Inuit of northwestern Canada have built domed-shaped family homes called igloos. The volume of snow in an igloo varies depending on the size. Every igloo has a ventilation opening as well as an entrance.



Calculate the volume of snow used to construct the main portion of igloo in the picture, not including the entrance tunnel. Express your answer to the nearest tenth of a cubic metre.

12. The roof of a house is shaped like a right pyramid with a square base. The base of the pyramid measures 32 ft on each side, and the roof must enclose a volume of at least 4096 ft^3 of air. Calculate the minimum height for the apex of the roof.

13. Grady has designed a wooden storage cabinet for his CDs and DVDs. The cabinet has a wooden door that can be closed. The cabinet is a right rectangular prism, as shown.

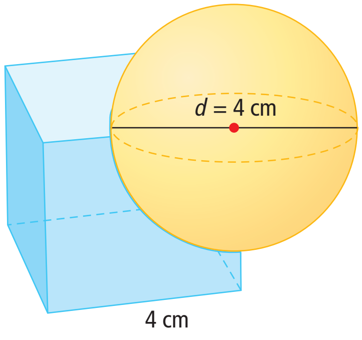


a) Calculate the volume of the cabinet.

b) Grady is concerned that the cabinet is too big for his room. He considers keeping the length and width the same, but reducing the height by one quarter. Estimate the volume of the modified cabinet.

c) Calculate the volume of the cabinet in part b). Was your estimate accurate? Explain how changing one dimension of a right rectangular prism by a factor of k will change the volume of the prism.

14. International Games are being held in your community. Your promotions committee designed a souvenir consisting of a cube with an inserted sphere representing the Earth. Both pieces are constructed from solid crystal. The sphere replaces one quarter of the volume of the cube. Calculate the volume of the souvenir piece.



Did You Know?

The word *igloo*, meaning a house of snow. The igloo shape is semi-spherical because it creates the greatest amount of living space with the least amount of snow.

> **Did You Know?**
>
> The first cellular phones to be created were very large and bulky. The first cell phone came to the market in 1984 from Motorola and weighed 2 lb. It was a DynaTac 8000X and it sold for \$3995.

> **Web Link**
>
> To view a video that describes the cell phone development by Motorola, go to www.mhrmath10.ca and follow the links.

15. **Unit Project** The first cell phones were much larger than present-day cell phones. A typical cell phone now has a volume between 4 in.^{3} and 6 in.^{3}. Using the information shown in the photo, estimate the volume of the first commercial portable cell phone released in 1984 by Motorola.

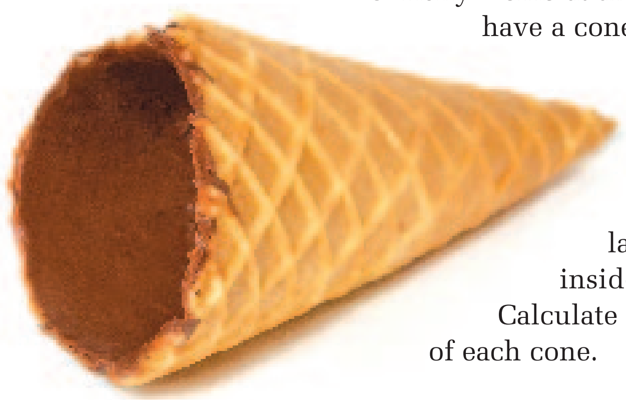
16. **Unit Project** An MP3 player with a memory of 80 GB has a storage capacity of 20 000 average-length songs. A vinyl LP record is 0.11 in. thick and on average can hold 12 songs. If the dimensions of the MP3 player are 4.14 cm wide, 9.15 cm high, and 0.85 cm thick, and the record has a radius 6 in., how many songs per cubic centimetre are there on each storage medium? Express your answers to the nearest hundredth.

Extend

For each of the following, express your answers to the nearest tenth of a unit where necessary.

17. Through your work with volume, you discovered a relationship between the right cone and the right cylinder. Extend your work to describe a relationship between a right cone and a sphere.

18. Kelly works at an ice cream shop. Customers can choose to have a cone that is lined with chocolate. The store buys the chocolate-lined cones, but the owners are wondering if they could save money by lining their own cones. To decide, they ask Kelly to calculate the amount of chocolate used to coat the inside of each cone with a layer of chocolate 1 mm thick. Each cone has an inside diameter of 5.5 cm and a slant height of 13 cm. Calculate the volume of chocolate used to line the inside of each cone.



19. Use spreadsheet software to help investigate how changing the radius of a sphere changes its volume. Create a spreadsheet like the one shown below.

	A	B	C	D
1	Investigating Changes in Dimensions of a Sphere			
2	Stretch Factor	Radius	Volume	Ratio of New Volume to Original Volume
3	1	3	113.1	1
4	2	6		
5	3			
6	4			
7	5			

a) Use spreadsheet formulas to complete the spreadsheet. Depending on your software, you may need to type "PI" or "PI()" for π. See your spreadsheet's help feature if you need assistance.

b) Compare the stretch factor for the radius to the ratio of the new volume to the original volume. What pattern do you notice?

c) Use your pattern to predict the volume of the sphere if you multiply the radius by 6. Extend your spreadsheet to check your answer.

d) In your own words, express the relationship between a change in the radius of a sphere and its volume.

20. Choose another solid that you have studied in this section. Create a table to investigate how the volume of the solid is affected by changing one of the dimensions of the solid.

Create Connections

21. **Unit Project** Work individually or in a small group. Choose a 3-D object related to your Unit 1 project.

a) Estimate its volume in both SI and imperial units. Are your estimates reasonable? Explain.

b) Calculate the volume. Are the units in your answer appropriate for the object?

c) In which measurement system was your estimate more accurate? Why do you think this happened?

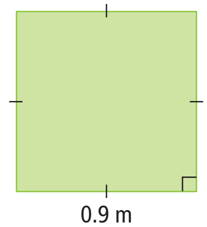
2 Review

2.1 Units of Area and Volume, pages 56–65

Where necessary, express your answers to the nearest hundredth of a unit.

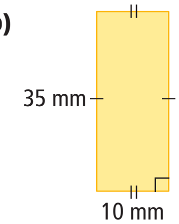
1. Calculate each area using the indicated unit.

a)



$A = $ ■ cm^2

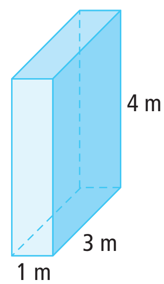
b)



$A = $ ■ in.^2

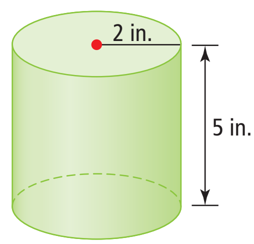
2. Calculate each volume using the indicated unit.

a)



$V = $ ■ ft^3

b)



$V = $ ■ cm^3

3. Describe personal referents that you could use to remember what 1 cm^2, 1 in.^2, and 1 m^2 looks like. Compare your personal referents to those of a classmate.

4. Justin, a landscaper, is fertilizing a new lawn. The directions on the fertilizer are to apply 6 lb of fertilizer per 1000 ft^2. Justin knows that the lawn he is fertilizing has an area of 400 m^2. How much fertilizer should Justin apply to the lawn?

2.2 Surface Area, pages 66–79

Where necessary, express your answers to the nearest hundredth of a unit.

5. Sketch each of the following and calculate the surface area.

a) A cone has radius 15.5 cm and slant height 26.2 cm.

b) A cone has slant height 24 in. and diameter 16 in.

c) A pyramid has a square base with sides 44 cm and slant height 64 cm.

d) A sphere has diameter 3.0 cm.

6. Calculate the missing dimension in each of the following.
   a) A right cone has surface area 12 m^2 and radius 1.3 m.
   b) The square base of a right pyramid has area 2304 cm^2 and surface area 6144 cm^2.
   c) A sphere has surface area 385 cm^2.

7. Carsyn's vehicle has a spare tire that is bolted to the outside of the vehicle. To protect the spare tire, she wants to make or buy a protective cover that she can slip over the tire. The tire has a diameter of 820 mm and a depth of 235 mm. Calculate the minimum amount of material needed to make the cover to the nearest square metre. What assumptions did you make?

8. Kale needs to build a new house for his dog. After researching, he has learned that the width and the length of the doghouse should be about 125% of the length of his dog, and 130% of the height of his dog's head.

   a) Kale's dog is 40 in. long and 28 in. tall. What dimensions should Kale use to make the doghouse if it is a right prism with a square base?
   b) What size do you recommend Kale make the doorway? Explain your reasoning.
   c) Kale realizes that a flat roof is not a good idea for his doghouse. He decides to build a pyramidal roof for the house instead. By experimenting, he decides that the roof will have a slant height of 30 in. Draw a sketch of the doghouse and label all dimensions.
   d) Calculate the total surface area of Kale's doghouse, with the roof.

9. The radius of Earth is 6 370 000 m. Canada has a land area of 9 003 507 km^2. What percent of Earth's surface does Canada represent?

2.3 Volume, pages 80–91

Where necessary, express your answers to the nearest hundredth of a unit.

10. Calculate the volume of each of the following.
   a) A cylinder has radius 15 ft and height equal to its diameter.
   b) A cone has height 14 cm and diameter 4.5 cm.
   c) A rectangular pyramid has base 12 ft by 14 ft and height 18 ft.
   d) A sphere of radius 4.1 cm.

11. Calculate the missing dimension in each of the following.

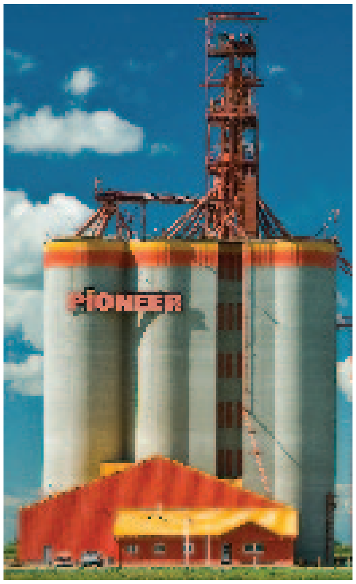
a) A cylinder has volume 1 m^3 and height 0.6 m.

b) A cone has radius 40 cm and volume 7800 cm^3.

c) A square-based pyramid has height 92 cm and volume 5.4 m^3.

d) A sphere has volume 2469 cm^3.

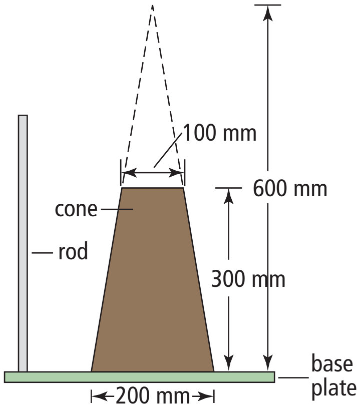
12. Many grain terminals are cylindrical in shape. This one is near Carseland, AB. The height of the terminal is 40 m, and the inside diameter of each grain storage tower is 11.7 m.



a) What is the storage capacity of each tower, to the nearest cubic metre?

b) If a small grain truck has a rectangular box measuring 5 m by 1.5 m by 2.4 m, how many truckloads of grain can one tower hold?

13. When a concrete structure is built, an engineer often performs a slump test on the concrete to ensure that it is of suitable strength and consistency. The engineer fills a truncated cone with bottom diameter 200 mm, top diameter 100 mm, and height 300 mm with concrete. The engineer then places the cone on the ground and lifts off the cone. After a period of time, she measures the amount that the top of the cone has slumped to decide if the concrete is appropriate.



a) The original cone from which the slump-test cone was derived had a height of 600 mm. What was the volume of the original cone? Express your answer to the nearest cubic millimetre.

b) What is the volume of the slump-test cone? Express your answer to the nearest cubic millimetre.

2 Practice Test

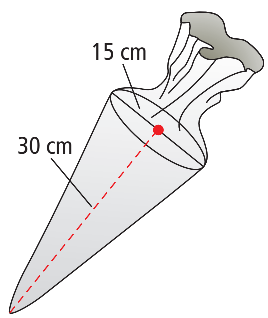
Multiple Choice

For #1 to #5, choose the best answer.

1. Fertilizer application instructions say to apply 0.5 L of fertilizer per 100 m^2. Zack knows that his lawn has dimensions of 15 m by 20 m. How many litres of fertilizer will Zack require?

A 1.5 L **B** 4.5 L **C** 5.7 L **D** 6.8 L

2. A cake-decorating bag has the shape of a cone. To the nearest cubic centimetre, how much frosting will fit into a cake-decorating bag that has a diameter of 15 cm and a height of 30 cm?



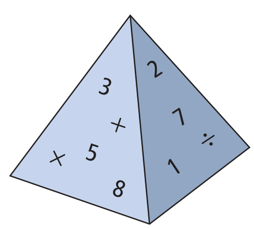
A 1767 cm^3 **B** 5301 cm^3

C 7068 cm^3 **D** 21 206 cm^3

3. An official major league baseball has a diameter of 7.4 cm. Correct to the nearest tenth of a unit, the amount of leather needed to cover the ball is

A 1697.4 cm^2 **B** 688.1 cm^2 **C** 212.2 cm^2 **D** 172.0 cm^2

4. A magnetic paperweight has the shape of a right pyramid with slant height 8.5 cm. The square base of the pyramid has side length 10 cm. The surface area of the pyramid is



A 270 cm^2 **B** 283 cm^2

C 440 cm^2 **D** 850 cm^2

5. A drink manufacturer is designing the can for a new type of drink. The can is to have a capacity of between 250 mL and 285 mL. The height of the can is set at 14 cm. If 1 mL is equivalent to 1 cm^3, what range of values is possible for the diameter of the can?

A 2.4 cm to 2.5 cm **B** 4.8 cm to 5.0 cm

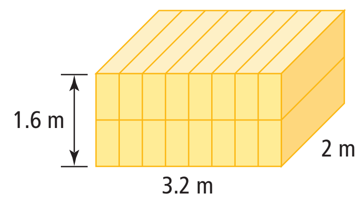
C 7.5 cm to 7.9 cm **D** 15.0 cm to 15.9 cm

Short Answer

6. Calculate the volume and surface area of a sphere with diameter 28 cm. Express your answers to the nearest whole unit.

7. A right conical paper cup has a height of 65 mm and a diameter of 38 mm. Calculate the volume of the cup. Express your answer to the nearest tenth of a cubic millimetre.

8. The Yukon Quest is an annual dogsled race between Whitehorse, YT and Fairbanks, Alaska. The race distance is 1645 km. Racers must have between 8 and 14 dogs for this challenging race. Many racers have built kennels to transport the dogs to the race. One competitor has built a trailer that is a right rectangular prism measuring 2 m wide by 1.6 m high by 3.2 m long. He has divided this trailer into 16 identical kennels for his dog team, as shown in the diagram. How much material is needed to build the kennels?



9. A theatre owner has two different right cylindrical popcorn containers. One is 18 cm high with a diameter of 14 cm, and the other is 16 cm high with a diameter of 16 cm. Which container should be for a small order of popcorn, and which should be the large? Justify your answer.

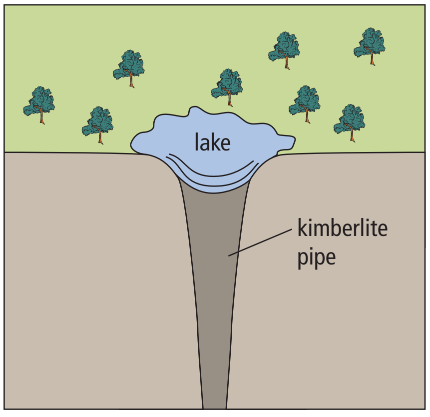
Extended Response

10. You have 500 cm^2 of material available to cover different shapes. What is the maximum volume of each object that you can make with this amount of material?

a) a cube **b)** a sphere

11. A tennis ball has a diameter of $2\frac{9}{16}$ in. A manufacturer packages three tennis balls stacked on top of one another in right cylindrical cans. Sketch the tennis ball container. Label the diameter and height of the container. What are the volume and surface area of the container?

12. Canada is one of the top three producers of diamonds worldwide. Most of these diamonds are mined in Nunavut and the Northwest Territories. Diamonds are recovered from kimberlite pipes, long narrow regions beneath Earth's surface.



One kimberlite deposit has an area of 50 000 m^2 at ground level and a depth of 2 km. Estimate the volume of the kimberlite deposit. What assumptions did you make?

13. A recycling depot in Yellowknife has a machine that bales the recycled goods into bales measuring 5 ft by 4 ft by 3 ft.

a) A new machine is being considered that will make right cylindrical bales. If the bales contain the same amount of material as the original ones and have a height of 5 ft, what is their diameter?

b) Which type of bale has the smaller surface area? Justify your answer.

CHAPTER

3 Right Triangle Trigonometry

Suppose you need to calculate the distance across a river for the construction of a bridge or the height of a building or monument. Each of these distances can be calculated using the properties of right triangles, similar triangles, and trigonometry. Trigonometry is the branch of mathematics that studies the relationships between angles and the lines that form them in triangles. It was first developed for use in astronomy and geography. Today, trigonometry is used in surveying, navigation, engineering, construction, and the sciences to explore the relationships between the side lengths and angles of triangles.

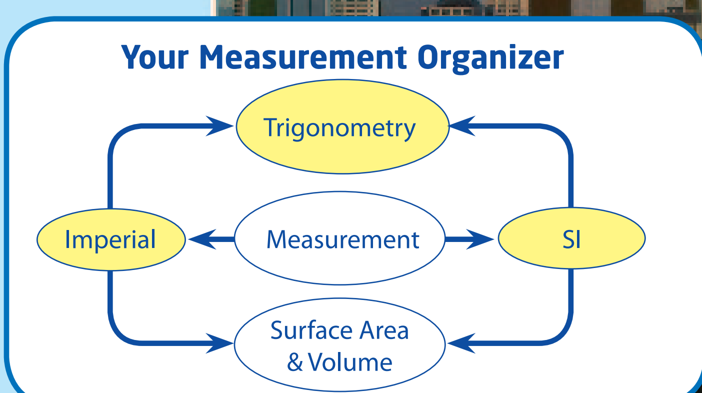
Big Ideas

When you have completed this chapter, you will be able to …

- apply the Pythagorean theorem and primary trigonometric ratios to solve problems involving right triangles
- solve problems involving indirect and direct measurement
- solve right triangles

Key Terms

hypotenuse
opposite side
adjacent side
tangent ratio
sine ratio
cosine ratio
primary trigonometric ratios



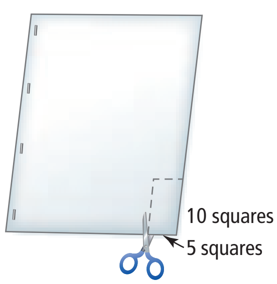
Astronomer

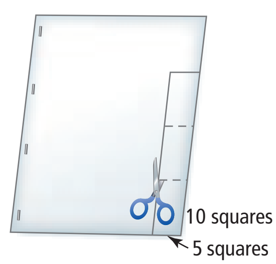
Astronomers study matter in outer space and the celestial bodies. They study their compositions, motions, and origins. Astronomers usually focus their work on planetary science, solar astronomy, the origin and evolution of stars, and the formation of galaxies. Extragalactic astronomy is the study of distant galaxies. It includes studying how distant galaxies move, when they collide, and how they transform as a result of this interaction.

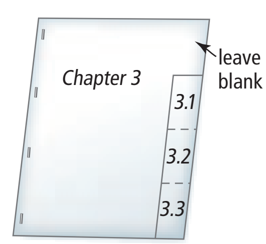
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 3.

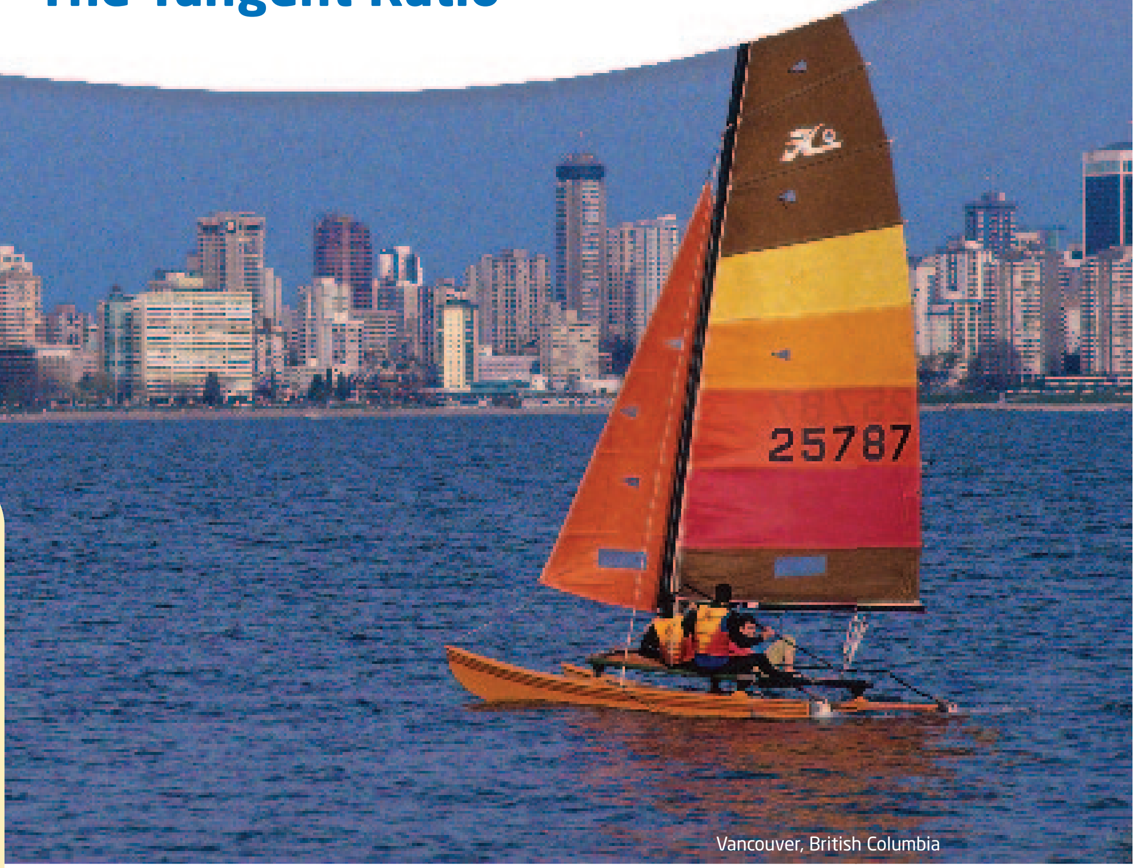
1. Staple four sheets of single-sided grid paper together, along the left edge. Make sure the grid sides face down.
2. Make a mark ten squares up from the bottom right edge of the top sheet. Cut through the top three sheets about five squares in from this mark as shown.
3. Cut through the top two sheets up ten more squares. As you do this, you will form tabs along the right side. Continue, until you have four tabs.
4. Label the Foldable™ as shown. On the back of the Foldable™, write the title What I Need to Work On.







3.1 The Tangent Ratio



Vancouver, British Columbia

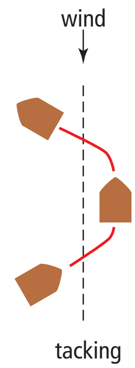
Focus on ...

- explaining the relationships between similar triangles and the definition of the tangent ratio
- identifying the hypotenuse, opposite side, and adjacent side for a given acute angle in a right triangle
- developing strategies for solving right triangles
- solving problems using the tangent ratio

In addition to the Pacific Ocean, there are many lakes in Western Canada that are ideal for sailing. One important aspect of boating is making sure you get where you want to go. Navigation is an area in which trigonometry has played a crucial role; and it was one of the early reasons for developing this branch of mathematics.

People have used applications of trigonometry throughout history. The Egyptians used features of similar triangles in land surveying and when building the pyramids. The Greeks used trigonometry to tell the time of day or period of the year by the position of the various stars. Trigonometry allowed early engineers and builders to measure angles and distances with greater precision. Today, trigonometry has applications in navigating, surveying, designing buildings, studying space, etc.

Investigate the Tangent Ratio



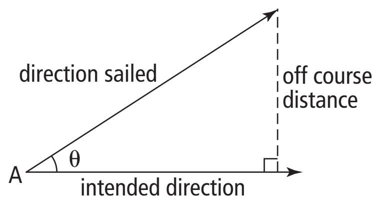
Materials
- grid paper
- protractor
- ruler

Sailing is a very popular activity. One of the limitations of sailing is that a boat cannot sail directly into the wind. Using a technique called *tacking*, it is possible to sail in almost any direction, regardless of the wind direction. When sailing *on a tack*, you are forced to sail slightly off course and then compensate for the distance sailed when you change direction. You can use trigonometry to determine the distance a boat is off course before changing direction.

1. a) On a sheet of grid paper draw a horizontal line 10 cm in length to represent the intended direction.

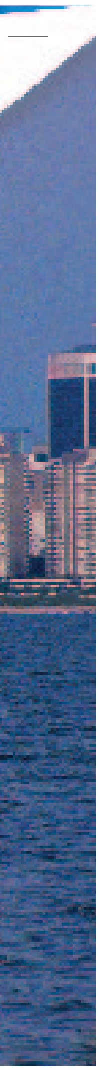
b) Draw a tacking angle, θ, of 30°.

c) Every two centimetres, along your horizontal line, draw a vertical line to indicate the off course distance. Label the five triangles you created, $\triangle$ABC, $\triangle$ADE, $\triangle$AFG, $\triangle$AHI, and $\triangle$AJK.



2. Measure the base and the height for each triangle. Complete the following table to compare the off course distance to the intended direction. In the last column, express the ratio, $\frac{\text{off course distance}}{\text{intended direction}}$, to four decimal places.

Triangle	Intended Direction	Off Course Distance	$\frac{\text{Off Course Distance}}{\text{Intended Direction}}$
$\triangle$ABC			
$\triangle$ADE			
$\triangle$AFG			
$\triangle$AHI			
$\triangle$AJK			



Did You Know?

- vertices of a triangle are commonly labelled with uppercase letters, for example △ABC
- angles of a triangle are commonly labelled with Greek letter variables
- some common Greek letters used are theta, θ, alpha, α, and beta, β.

3. a) The diagram you drew in step 1c) forms a series of nested similar triangles. How do you know the triangles are similar?

b) Use your knowledge of similar triangles to help describe how changing the side lengths of the triangle affects the ratio $\frac{\text{off course distance}}{\text{intended direction}}$.

4. a) Use your calculator to determine the tangent ratio of 30°. To calculate the tangent ratio of 30°, make sure your calculator is in the degree mode.

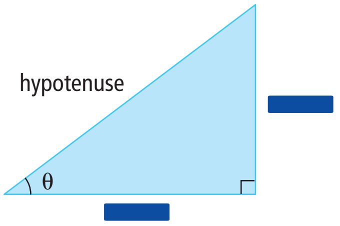
Press C TAN 30 =.

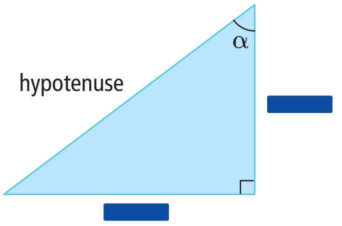
b) How does the value on your calculator relate to the data in step 2?

hypotenuse

- the side opposite the right angle in a right triangle

5. In the two right triangles shown, the **hypotenuse** is labelled and an angle is labelled with a variable. Copy each triangle. Use the words **opposite** and **adjacent** to label the side opposite the angle and the side adjacent to the angle.





opposite side

- the side across from the acute angle being considered in a right triangle
- the side that does not form one of the arms of the angle being considered

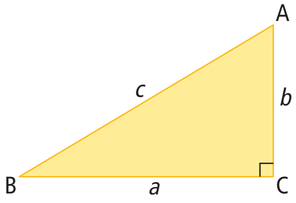
adjacent side

- the side that forms one of the arms of the acute angle being considered in a right triangle, but is not the hypotenuse

6. Reflect and Respond

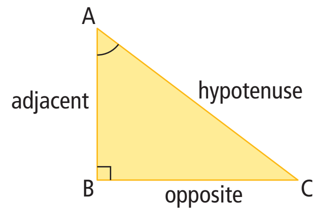
a) Use your results from steps 1 to 4 and the terminology from step 5 to describe a formula you could use to calculate the tangent ratio of any angle.

b) Use your formula to state the tangent ratios for ∠A and ∠B in the following diagram.



Link the Ideas

A trigonometric ratio is a ratio of the measures of two sides of a right triangle.



One trigonometric ratio is the **tangent ratio**.

The short form for the tangent ratio of angle A is tan A.

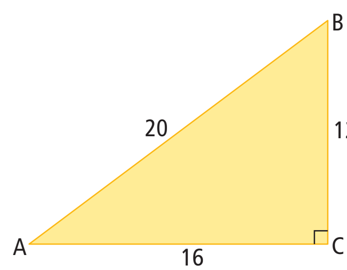
$$\text{tangent A} = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

tangent ratio
- for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side adjacent
- $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

Example 1 Write a Tangent Ratio

Write each trigonometric ratio.

a) tan A
b) tan B



Solution

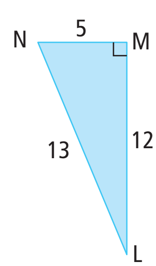
a)
$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan A = \frac{BC}{AC}$$
$$\tan A = \frac{12}{16}$$
$$\tan A = \frac{3}{4}$$

b)
$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan B = \frac{AC}{BC}$$
$$\tan B = \frac{16}{12}$$
$$\tan B = \frac{4}{3}$$

Your Turn

Calculate each trigonometric ratio.

a) tan L
b) tan N



Example 2 Calculate a Tangent and an Angle

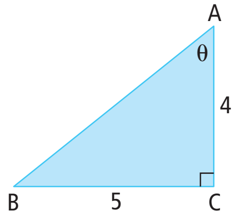
a) Calculate tan 25° to four decimal places.

b) Draw a triangle to represent $\tan \theta = \frac{5}{4}$. Calculate the angle θ to the nearest tenth of a degree.

Solution

a) $\tan 25° \approx 0.4663$

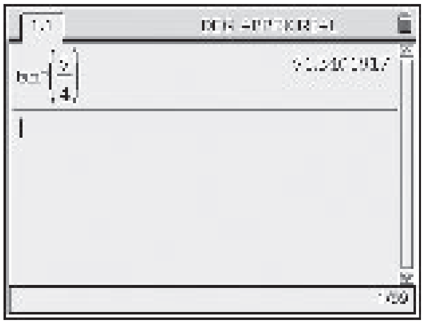
b) Since $\tan \theta = \frac{5}{4}$, the side opposite the angle θ is labelled 5 and the side adjacent to the angle θ is labelled 4.



The inverse function on a calculator allows you to apply the tangent ratio in reverse. If you know the ratio, you can calculate the angle whose tangent this ratio represents.

$$\tan \theta = \frac{5}{4}$$
$$\theta = \tan^{-1}\left(\frac{5}{4}\right)$$
$$\theta = 51.340\ldots°$$

The angle θ is 51.3°, to the nearest tenth of a degree.



Your Turn

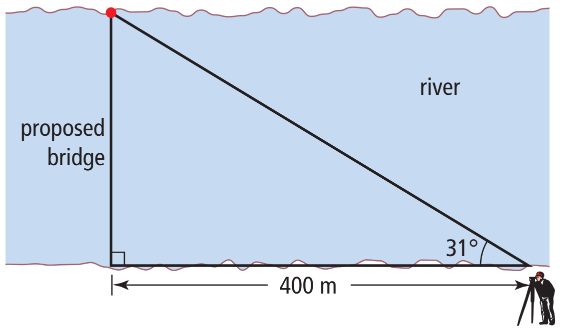
Explore your particular calculator to determine the sequence of keys required. Then, calculate each tangent ratio and angle.

θ	Tan θ
27°	
45°	
57°	

θ	Tan θ
	0.5095
	0.5543
	1.4653

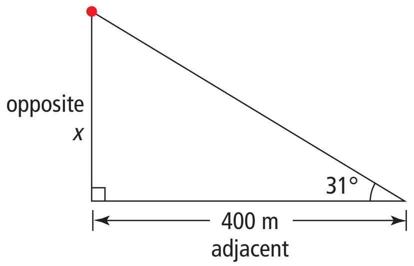
Example 3 Determine a Distance Using the Tangent Ratio

A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor uses a theodolite to measure angles. The surveyor measures a 31° angle to the bridge site across the river. What is the width of the river, to the nearest metre?



Solution

Let x represent the distance across the river.
Identify the sides of the triangle in reference to the given angle of 31°.

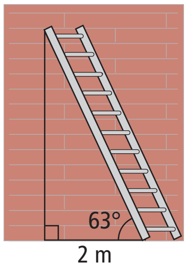


$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 31° = \frac{x}{400}$$
$$400(\tan 31°) = x$$
$$240.344... = x$$

To the nearest metre, the width of the river is 240 m.

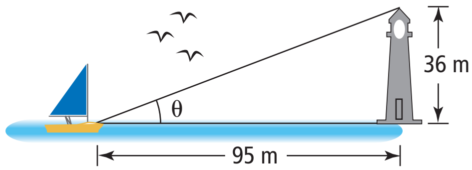
Your Turn

A ladder leaning against a wall forms an angle of 63° with the ground. How far up the wall will the ladder reach if the foot of the ladder is 2 m from the wall?



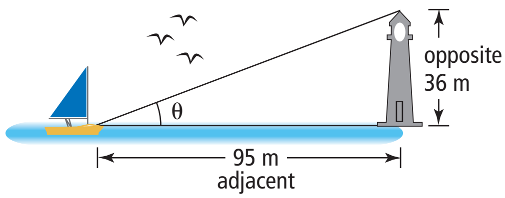
Example 4 Determine an Angle Using the Tangent Ratio

A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Calculate the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.



Solution

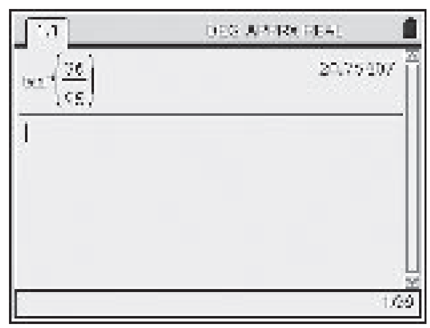
Identify the sides of the triangle in reference to the angle of θ.



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \frac{36}{95}$$

$$\theta = 20.754\ldots$$



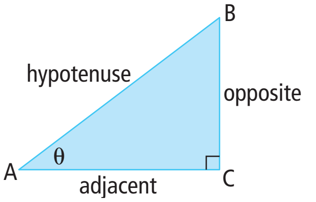
The angle from the boat to the top of the lighthouse is approximately 21°.

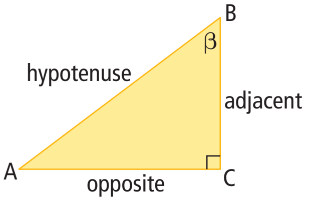
Your Turn

A radio transmission tower is to be supported by a guy wire. The wire reaches 30 m up the tower and is attached to the ground a horizontal distance of 14 m from the base of the tower. What angle does the guy wire form with the ground, to the nearest degree?

Key Ideas

- In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal.
- The sides of a right triangle are labelled according to a reference angle.





- The tangent ratio compares the length of the side opposite the reference angle to the length of the side adjacent to the angle in a right triangle.

$$\tan \theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta}$$

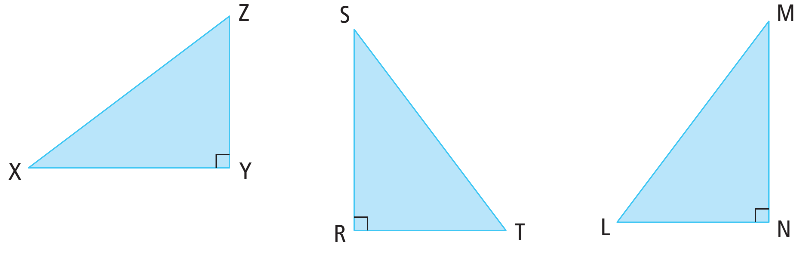
- You can use the tangent ratio to
  - determine the measure of one of the acute angles when the lengths of both legs in a right triangle are known
  - determine a side length if the measure of one acute angle and the length of one leg of a right triangle are known

Check Your Understanding

Practise

1. Identify the hypotenuse, opposite, and adjacent sides associated with each specified angle.

a) $\angle X$ **b)** $\angle T$ **c)** $\angle L$



2. Draw right $\triangle DEF$ in which $\angle F$ is the right angle.

a) Label the leg opposite $\angle D$ and the leg adjacent to $\angle D$.

b) State the tangent ratio of $\angle D$.

3. Determine each tangent ratio to four decimal places using a calculator.

   a) tan 74°
   b) tan 45°
   c) tan 60°
   d) tan 89°
   e) tan 37°
   f) tan 18°

4. Determine the measure of each angle, to the nearest degree.

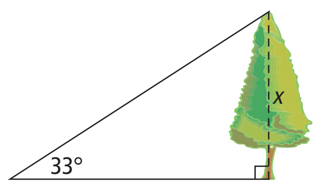
   a) $\tan A = 0.7$
   b) $\tan \theta = 1.75$
   c) $\tan \beta = 0.5543$
   d) $\tan C = 1.1504$

5. Draw and label a right triangle to illustrate each tangent ratio. Then, calculate the measure of each angle, to the nearest degree.

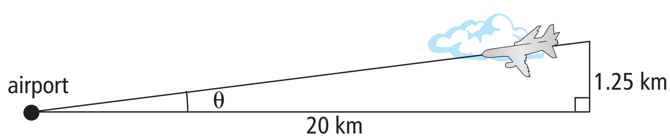
   a) $\tan \alpha = \frac{2}{3}$
   b) $\tan B = \frac{5}{2}$

6. Determine the value of each variable. Express your answer to the nearest tenth of a unit.

   a)



   b)

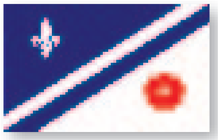


7. Kyle Shewfelt, from Calgary, AB, was the Olympic floor exercise champion in Athens in 2004. Gymnasts perform their routines on a 40-ft by 40-ft mat. They use the diagonal of the mat because it gives them greater distance to complete their routine.

   a) Use the tangent ratio to determine the angle of the gymnastics run relative to the sides of the mat.
   b) To the nearest foot, how much longer is the diagonal of the mat than one of its sides?

Apply

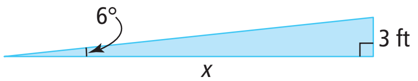
8. Claudette wants to calculate the angles of the triangle containing the fleur-de-lys on the Franco-Albertan flag. She measures the legs of the triangle to be 154 cm and 103 cm. What are the angle measures?



Did You Know?

The Franco-Albertan flag was created by Jean-Pierre Grenier. The flag was adopted by the Association canadienne-française de l'Alberta in March 1982.

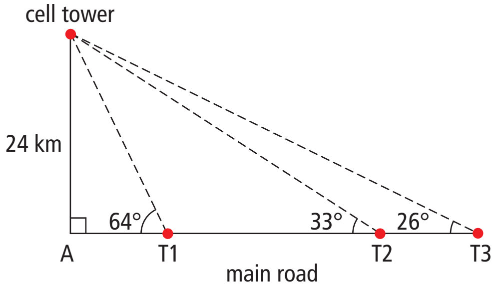
9. A ramp enables wheelchair users and people pushing wheeled objects to more easily access a building.



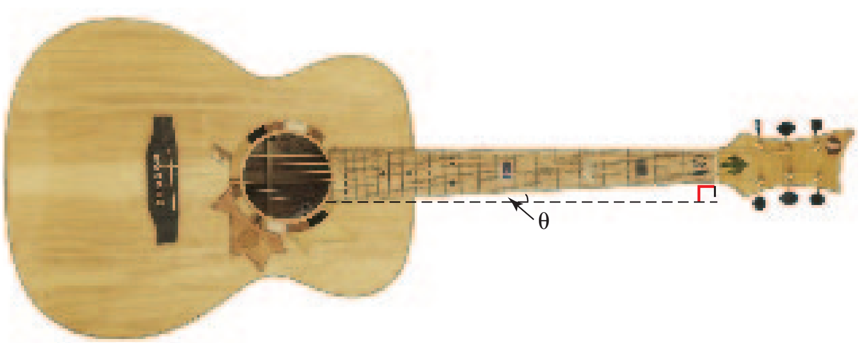
a) Determine the horizontal length, x, of the ramp shown. State your answer to the nearest foot.

b) For a safe ramp, the ratio of vertical distance : horizontal distance needs to be less than 1 : 12. Would the ramp shown be considered a safe ramp? Explain.

10. **Unit Project** A satellite radio cell tower provides signals to three substations, T1, T2, and T3. The three substations are each located along a stretch of the main road. The cell tower is located 24 km down a road perpendicular to the main road. A surveyor calculates the angle from T1 to the cell tower to be 64°, from T2 to the cell tower to be 33°, and from T3 to the cell tower to be 26°. Calculate the distance of each substation from the intersection of the two roads. Express your answers to the nearest tenth of a kilometre.



11. In the construction of a guitar, it is important to consider the tapering of the strings and neck. The tapering affects the tone that the strings make. For the *Six String Nation Guitar* shown, suppose the width of the neck is tapered from 56 mm to 44 mm over a length of 650 mm. What is the angle of the taper for one side of the guitar strings?



Did You Know?

The *Six String Nation Guitar*, nicknamed *Voyageur*, is made from 63 pieces of history and heritage, from every part of Canada. It represents many different cultures, communities, and characters. The guitar is made from pieces of wood, bone, steel, shell, and stone from every province and territory. It literally embodies Canadian history.

12. When approaching a runway, a pilot needs to maneuver the aircraft, so that it can approach the runway at a constant angle of 3°. A pilot landing at Edmonton International Airport begins the final approach 30 380 ft from the end of the runway. At what altitude should the aircraft be when beginning the final approach? State your answer to the nearest foot.

13. The Idaà Trail is a traditional route of the Dogrib, an Athapaskan-speaking group of Dene. It stretches from Great Bear Lake to Great Slave Lake, in the Northwest Territories. Suppose a hill on the trail climbs 148 ft vertically over a horizontal distance of 214 ft.

 a) Calculate the angle of steepness of the hill.

 b) How far would you have to climb to get to the top of the hill?

Extend

Did You Know?

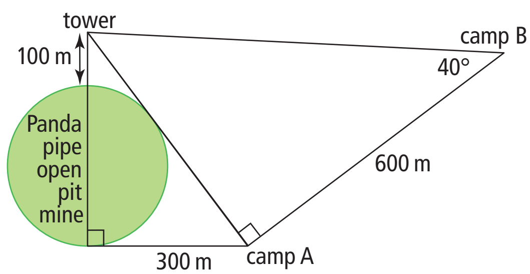
Ekati mine is Canada's first diamond mine. It is located 200 km south of the Arctic Circle in the Northwest Territories. Diamond mines contain pipes, which are cylindrical pits where diamonds are founds.

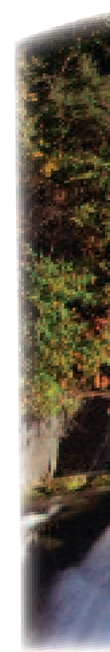
14. One of the Ekati mine's pipes, called the Panda pipe, has northern and southern gates. A communications tower stands 100 m outside the north gate. The tower can be seen from a point 300 m east of the south gate at camp A.

 a) The distance between camp A and camp B is 600 m. Calculate the diameter of the Panda pipe.

 b) Calculate the distance from camp B to the tower.


Panda Pipe

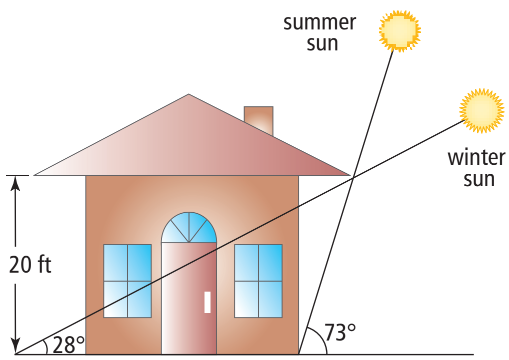




15. Habitat for Humanity Saskatoon has designed a home that provides passive solar features. The idea is to keep the sun off the outside south wall during the summer months and to have the wall exposed to the sun as much as possible during the winter months. The highest angle of the sun during the summer months is 73°.

a) Suppose the wall of the house is 20 ft tall. How much overhang on the roof trusses should be provided so that the shadow of the noonday sun reaches the bottom of the wall during the summer months?

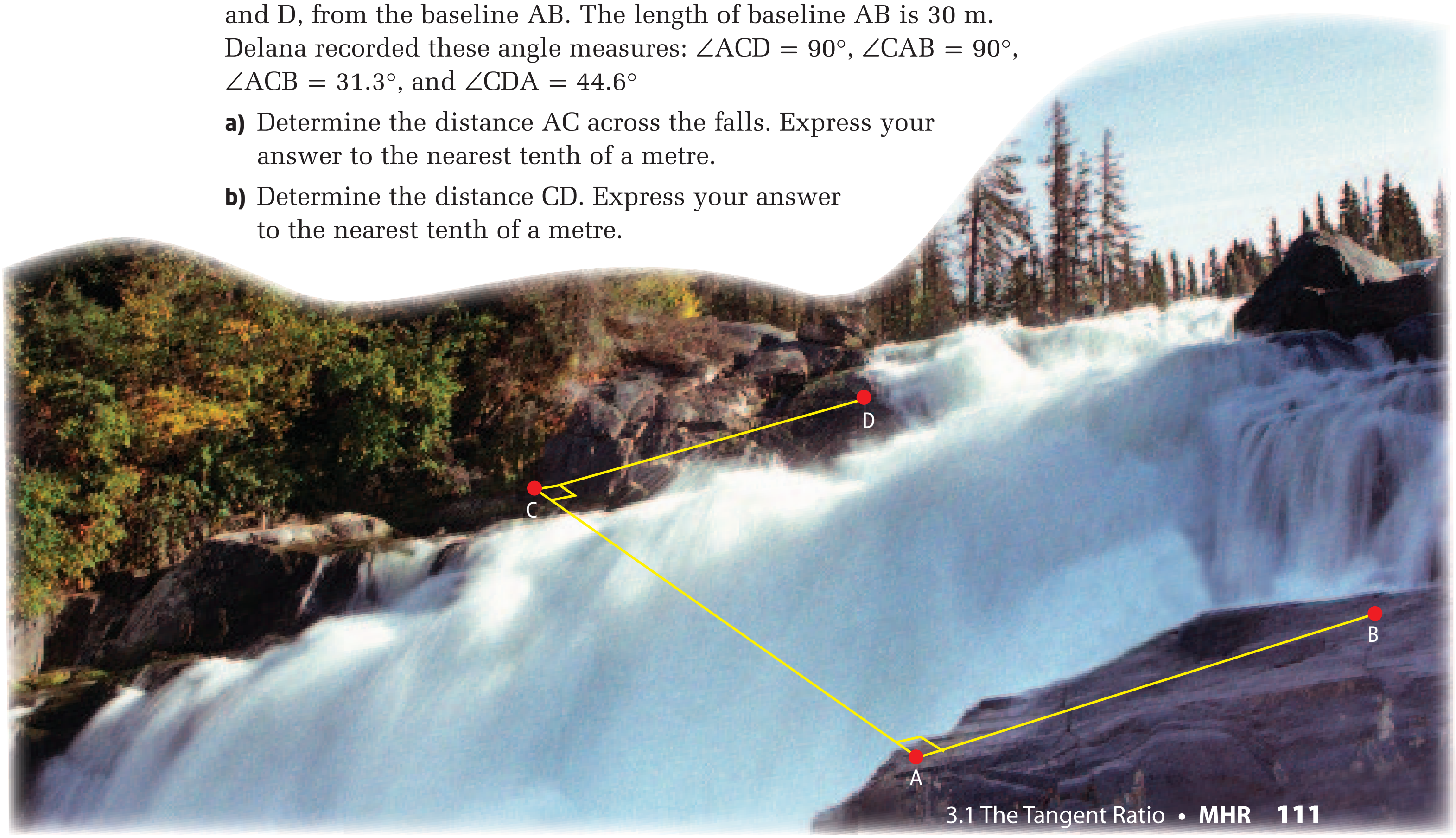
b) The lowest angle of the sun during the winter months is 28°. What height of the wall will be in direct sunlight during the winter months?



16. Nistowiak Falls, located in Lac LaRonge Provincial Park is one of the highest waterfalls in Saskatchewan. Delana, a surveyor, needs to measure the distance across the falls. She sighted two points, C and D, from the baseline AB. The length of baseline AB is 30 m. Delana recorded these angle measures: $\angle ACD = 90°$, $\angle CAB = 90°$, $\angle ACB = 31.3°$, and $\angle CDA = 44.6°$

a) Determine the distance AC across the falls. Express your answer to the nearest tenth of a metre.

b) Determine the distance CD. Express your answer to the nearest tenth of a metre.

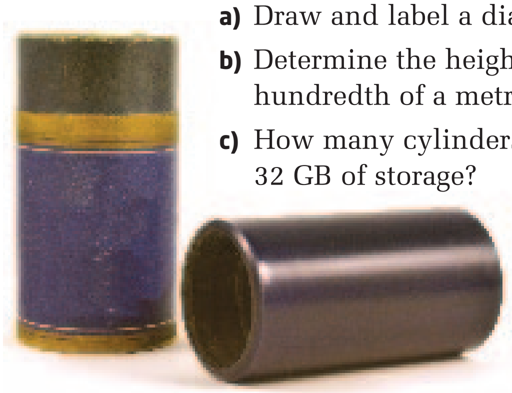


17. **Unit Project** The first sound recordings were done on wax cylinders that were 5 cm in diameter and 10 cm long. Wax cylinders were capable of recording about 2 min of sound. Modern music storage devices can have tremendous memory and store thousands of songs. Janine calculated the number of wax cylinders needed to match a 32 GB storage capacity. Imagine that these cylinders are stacked one on top of another. From a distance of 10 m, the angle of elevation to the top of the stack would be 89.5°.

a) Draw and label a diagram to represent the situation.

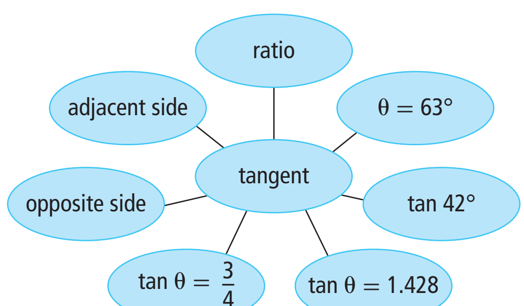
b) Determine the height of the stack of cylinders, to the nearest hundredth of a metre.

c) How many cylinders would need to be stacked to match 32 GB of storage?



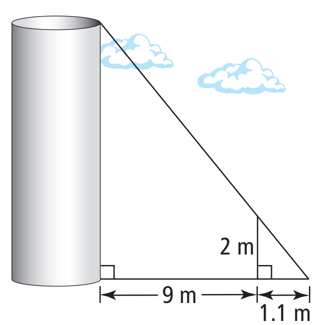
Create Connections

18. Copy the following graphic organizer. For each item, describe its meaning and how it relates to the tangent ratio.



19. Draw a right triangle in which the tangent ratio of one of the acute angles is 1. Describe the triangle.

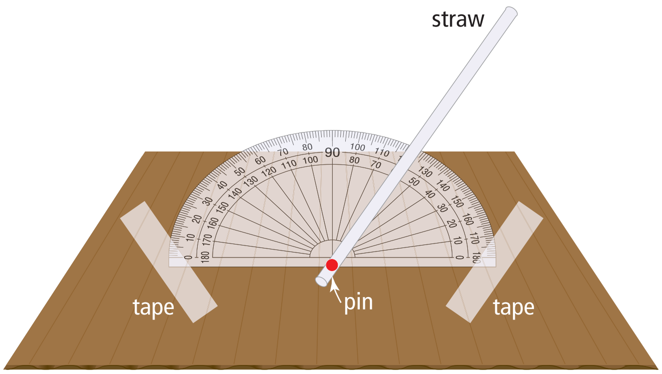
20. Devin stores grain in a cylindrical granary. Suppose Devin places a 2-m-tall board 9 m from the granary and 1.1 m away from a point on the ground. Describe how Devin could use trigonometry to calculate the angle formed with the ground and the top of the granary. Then, determine this angle.



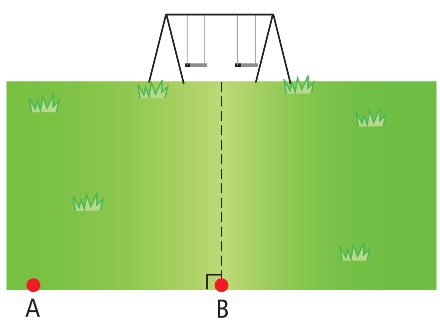
21. **MINI LAB** When measuring inaccessible distances, a surveyor can take direct measurements using a transit. A transit can measure both horizontal and vertical angles.

Materials
- piece of cardboard
- large protractor
- drinking straw
- tape
- pin
- measuring tape

Step 1 Construct a transit as shown in the diagram. Pin the straw at the centre of the protractor.



Step 2 Explain how a transit could be used to assess the distance to an object. Hint: You will need to draw and measure a baseline. This is the line from A to B in the diagram.



Step 3 To calculate the distance to some objects in your schoolyard, use your transit to measure the required angles.

Object	Length of Baseline AB	Measure of ∠A	Distance to the Object

3.2 The Sine and Cosine Ratios

Focus on ...

- using the sine ratio and cosine ratio to solve problems involving right triangles
- solving problems that involve direct and indirect measurement

The first suspension bridge in Vancouver was built in 1889 by George Mackay. He had built a cabin along the canyon wall and needed a bridge to conveniently access his cabin. Mathematical tools, such trigonometry, can enable you to calculate distances that cannot be measured directly, such as the distance across a river canyon.

In section 3.1, you learned about the tangent ratio. This ratio compares the opposite and adjacent side lengths in reference to an acute angle in a right triangle. There are two other trigonometric ratios that compare the lengths of the sides of a right triangle. These ratios, called the sine ratio and cosine ratio, involve the hypotenuse.

Materials

- protractor
- ruler

Investigate Trigonometric Ratios

1. Choose an angle between 10° and 80°. This will be your reference angle.

2. a) Draw right triangle ABC, using your reference angle.

   b) Draw three right triangles similar to △ABC using the same reference angle.

3. Write the equivalency statements that show the similarity of each triangle to $\triangle ABC$.

4. Label the sides of each triangle. Use the terms hypotenuse, opposite, and adjacent according to the reference angle.

5. Measure the sides of each triangle. You may wish to record the measurements in a table similar to this one or using spreadsheet software. Express each ratio to four decimal places.

Triangle	Length of Hypotenuse	Length of Opposite Side	Length of Adjacent Side	Ratio of Opposite to Adjacent	Ratio of Opposite to Hypotenuse	Ratio of Adjacent to Hypotenuse
$\triangle$						

6. Complete a similar table using the other acute angle in each triangle as your reference angle.

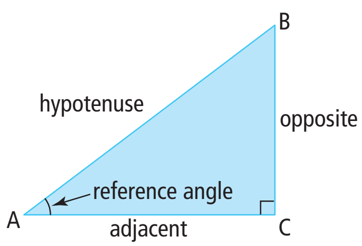
7. **Reflect and Respond** Discuss with a partner the results of the calculations of the ratios. Describe any similarities or patterns that you notice.

8. What relationships do you observe among the ratios for the angles between the two tables?

9. What conclusions can you make about how the ratios relate to your reference angle?

Link the Ideas

The short form for the **sine ratio** of angle A is sin A. The short form for the **cosine ratio** of angle A is cos A.



$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

sine ratio

- for an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse
- $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

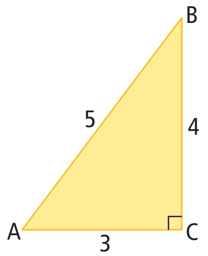
cosine ratio

- for an acute angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse
- $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

Example 1 Write Trigonometric Ratios

Write each trigonometric ratio.

a) sin A **b)** cos A
c) sin B **d)** cos B



Solution

a) $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin A = \dfrac{BC}{AB}$

$\sin A = \dfrac{4}{5}$

b) $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos A = \dfrac{AC}{AB}$

$\cos A = \dfrac{3}{5}$

c) $\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin B = \dfrac{AC}{AB}$

$\sin B = \dfrac{3}{5}$

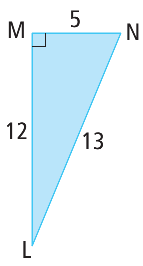
d) $\cos B = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos B = \dfrac{BC}{AB}$

$\cos B = \dfrac{4}{5}$

Your Turn

Write each trigonometric ratio.



a) sin L **b)** cos N
c) cos L **d)** sin N

Example 2 Evaluate Trigonometric Ratios

The **primary trigonometric ratios** and their inverses can be evaluated using technology.

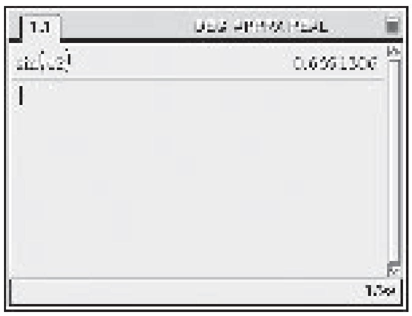
a) Evaluate each ratio, to four decimal places.
$\sin 42°$ $\cos 68°$

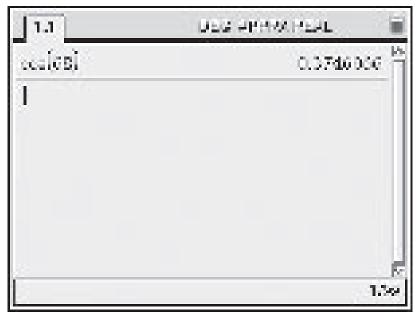
b) Determine each angle measure, to the nearest degree.
$\sin \theta = 0.4771$ $\cos \beta = 0.7225$

primary trigonometric ratios
- the three ratios, sine, cosine, and tangent, defined in a right triangle

Solution

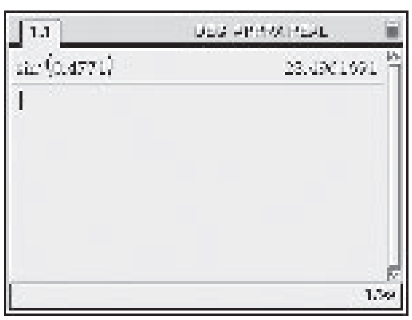
a) $\sin 42° \approx 0.6691$ $\cos 68° \approx 0.3746$

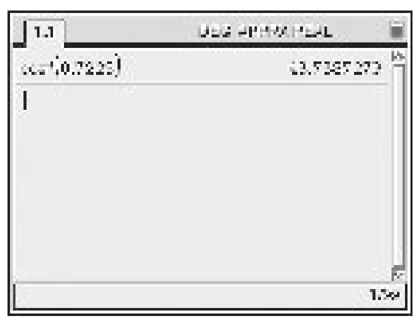




b) $\sin \theta = 0.4771$
$\theta = \sin^{-1}(0.4771)$
$\theta \approx 28°$

$\cos \beta = 0.7225$
$\beta = \cos^{-1}(0.7225)$
$\beta \approx 44°$





Your Turn

a) Evaluate each trigonometric ratio, to four decimal places.
$\sin 60°$ $\sin 30°$ $\cos 45°$

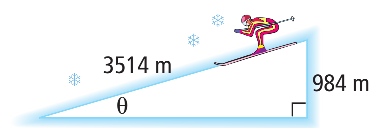
b) What is the measure of each angle, to the nearest degree?
$\sin \beta = 0.4384$ $\cos \theta = 0.2079$

Example 3 Determine an Angle Using a Trigonometric Ratio

In the World Cup Downhill held at Panorama Mountain Village in British Columbia, the skiers raced 3514 m down the mountain. If the vertical height of the course was 984 m, determine the average angle of the ski course with the ground. Express your answer to the nearest tenth of a degree.

Solution

Visualize the problem by sketching a diagram to organize the information.



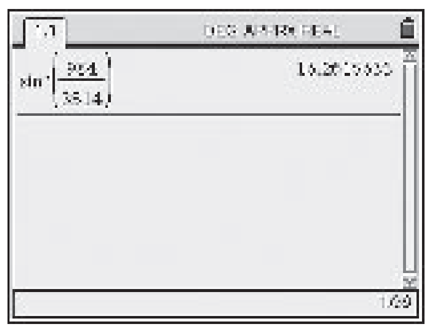
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

For the unknown angle, the lengths of the opposite side and hypotenuse are known. So, use the sine ratio.

$$\sin\theta = \frac{984}{3514}$$

$$\theta = \sin^{-1}\left(\frac{984}{3514}\right)$$

$$\theta = 16.2615\ldots°$$



The average angle of the ski course is 16.3°, to the nearest tenth of a degree.

Your Turn

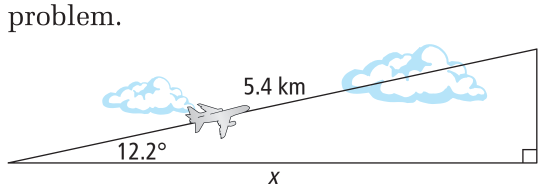
A guy wire supporting a cell tower is 24 m long. If the wire is attached at a height of 17 m up the tower, determine the angle that the guy wire forms with the ground.

Example 4 Determine a Distance Using a Trigonometric Ratio

A pilot starts his takeoff and climbs steadily at an angle of 12.2°. Determine the horizontal distance the plane has travelled when it has climbed 5.4 km along its flight path. Express your answer to the nearest tenth of a kilometre.

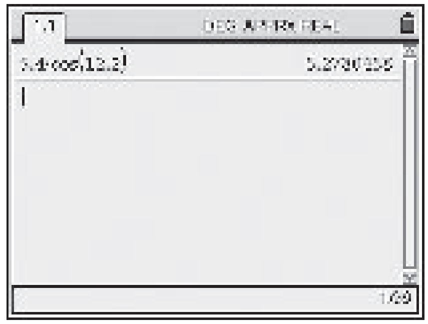
Solution

Organize the information by sketching a diagram to illustrate the problem.



$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 12.2° = \frac{x}{5.4}$$
$$5.4(\cos 12.2°) = x$$
$$5.278... = x$$

How do you decide which trigonometric ratio to use?



The horizontal distance travelled by the airplane is approximately 5.3 km.

Your Turn

Determine the height of a kite above the ground if the kite string extends 480 m from the ground and makes an angle of 62° with the ground. Express your answer to the nearest tenth of a metre.

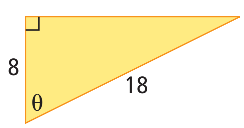
Key Ideas

- The sine ratio and cosine ratio compare the lengths of the legs of a right triangle to the hypotenuse.

  $$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- The sine and cosine ratios can be used to calculate side lengths and angle measures of right triangles.
- Visualizing the information that you are given and that you need to find is important. It helps you determine which trigonometric ratio to use and whether to use the inverse trigonometric ratio.

  Determine the value of θ, to the nearest degree.



$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos \theta = \frac{8}{18}$$
$$\theta = \cos^{-1}\left(\frac{8}{18}\right)$$
$$\theta = 63.6122...°$$

Angle θ is approximately 64°.

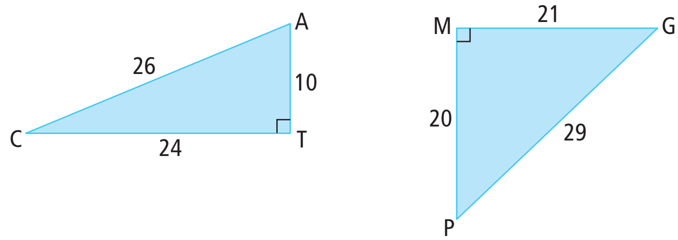
Check Your Understanding

Practise

1. Evaluate each trigonometric ratio to four decimal places.

a) cos 34°
b) cos 56.4°
c) sin 62.9°
d) sin 19.6°
e) sin 90°
f) cos 80°

2. Write each trigonometric ratio in lowest terms.



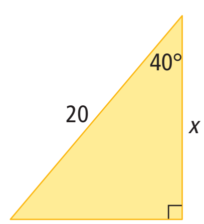
a) sin A
b) sin C
c) cos C
d) cos G
e) sin P
f) cos P

3. Calculate the measure of each angle, to the nearest degree.

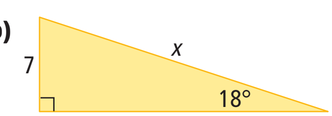
a) $\cos A = 0.4621$
b) $\cos \theta = 0.6779$
c) $\sin \beta = 0.5543$
d) $\sin C = 1.232$
e) $\sin \alpha = \frac{1}{2}$
f) $\cos B = \frac{3}{4}$

4. Determine each length of x. Express your answer to the nearest tenth of a unit.

a)

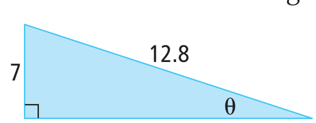


b)



5. Determine the measure of each angle θ. Express your answer to the nearest tenth of a degree.

a)



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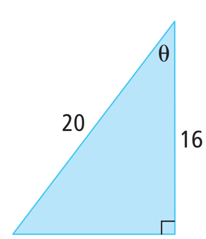
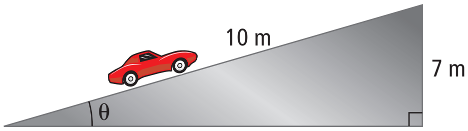


6. Determine the value of each variable. Express each answer to the nearest tenth of a unit.

a)



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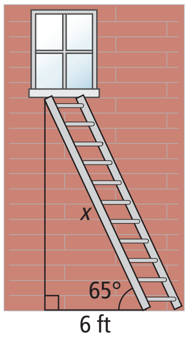
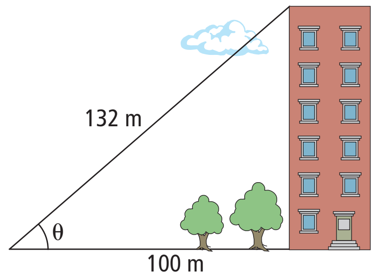
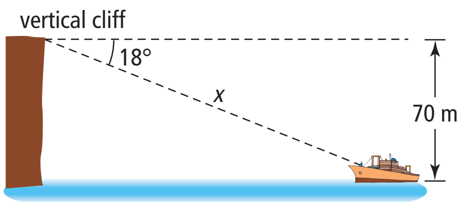


c)

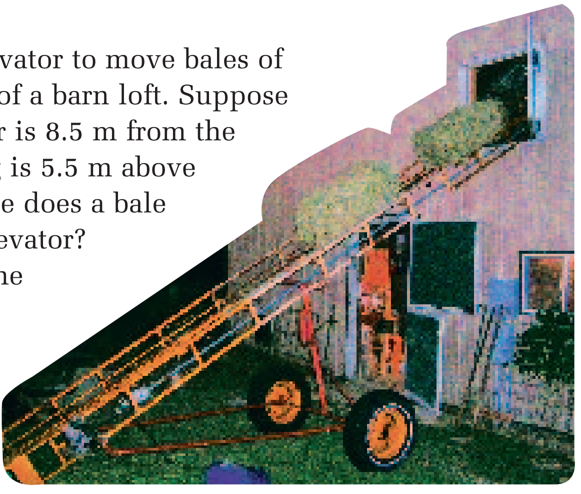


d)



Apply

7. Some farms use a hay elevator to move bales of hay to the second storey of a barn loft. Suppose the bottom of the elevator is 8.5 m from the barn and the loft opening is 5.5 m above the ground. What distance does a bale of hay travel along the elevator? Express your answer to the nearest tenth of a metre.



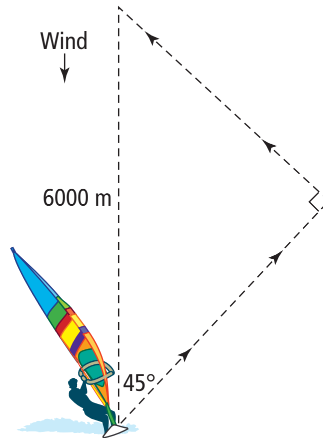
8. A 30-m-long line is used to hold a helium weather balloon. Due to a breeze, the line makes a 75° angle with the ground.

   a) Draw a right triangle to model the problem. Label the measurements you know. Use variables to represent the unknown measurements.

   b) Use trigonometry to determine the height of the balloon. Express your answer to the nearest tenth of a metre.

9. Oil rigs are found throughout Alberta. They play a crucial role in the search for crude oil and natural gas products. Determine the height of a rig if a 52-m-long guy wire is attached to the top of the rig and forms an angle of 50° with the ground. Express your answer to the nearest tenth of a metre.

10. Gerry is windsurfing at Squamish Pit, just north of Vancouver, BC. In order to get upwind 6000 m, Gerry sails at a 45° angle to the wind and then turns 90° and heads toward his original destination. How far would he have to sail to get directly upwind the 6000 m? Express your answer to the nearest tenth of a metre.



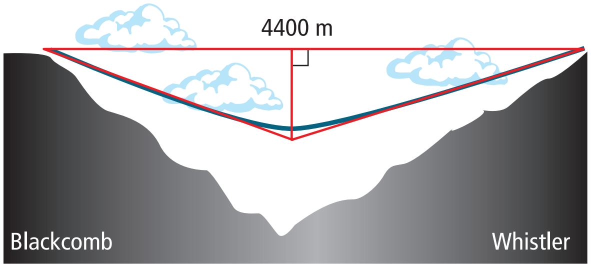
11. Toonik Tyme is Nunavut's biggest spring festival, celebrating the return of spring. To set up one of the holes for ice golf, the organizers cleared a track in the form of a right angle. The distance from the teeing area to the vertex of the right angle is 180 yd.

    a) The angle from the teeing area to the flag at the other end of the track is 34°. Draw a diagram of the ice golf hole.

    b) Determine the direct distance from the teeing area to the flag, to the nearest yard.

    c) How much shorter would the direct distance be than following the track?

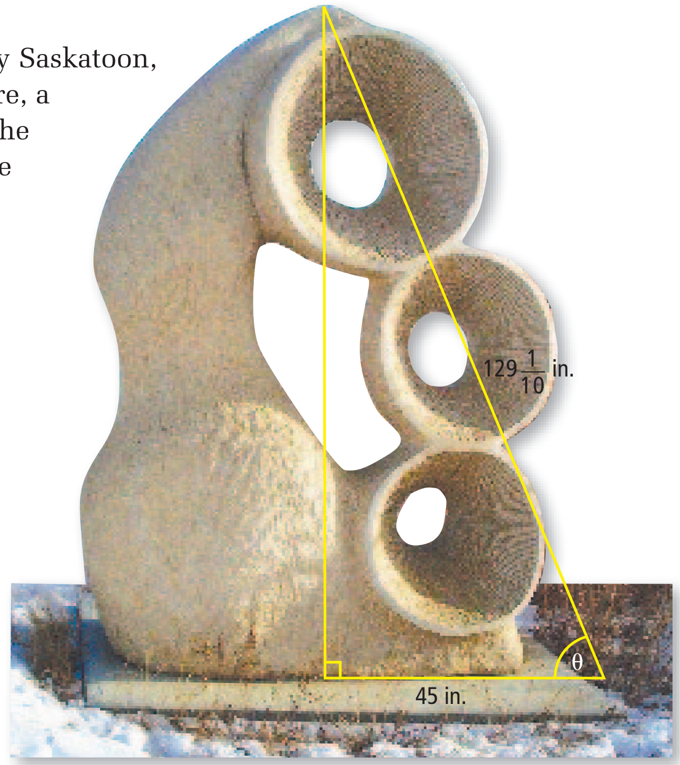
12. The PEAK 2 PEAK Gondola connects two mountain ski resorts, Whistler Mountain and Blackcomb Mountain, near Vancouver, BC. The straight-line distance between the two peaks is 4400 m. The gondola travels 4600 m along a cable that sags in the centre. Determine the approximate angle that the cable makes with the horizontal, to the nearest degree.



13. *Dream Maker* is a dolomite sculpture by Saskatoon, SK, artist Floyd Wanner. In the sculpture, a line can be drawn that passes through the centre of the two upper circles. Suppose this line is $129\frac{1}{10}$ in. long and the base line is 45 in. Describe how you might calculate

a) the height of *Dream Maker*

b) the angle between the baseline and the line through the two upper circles



Did You Know?

Dolomite is a rock consisting mainly of calcium and magnesium carbonate. It is mined around the world, including in western Canada. Dolomite is used to improve garden soil. It is also used as an ornamental stone, and in construction materials.

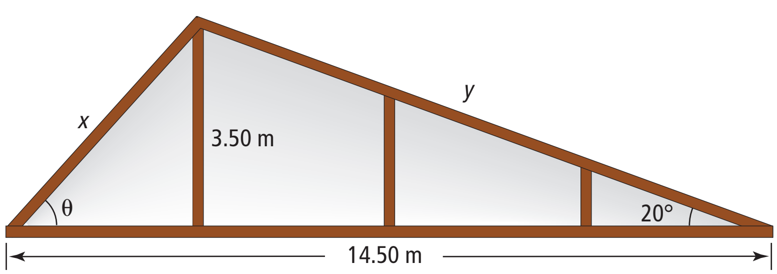
14. At Wapiti Valley Ski Area in Saskatchewan, the beginner slope is inclined at an angle of 11.6° from the horizontal and the advanced slope at an angle of 26.9° from the horizontal.

a) Suppose Francis skis 1200 m down the advanced slope while Barbara skis the same distance down the beginner slope. Predict who will cover a greater horizontal distance. Justify your prediction.

b) Calculate the difference between the horizontal distances for the two skiers, to the nearest tenth of a metre.

Extend

15. Michael is building a cabin at Cold Lake, AB. He has drawn a diagram to design his roof truss. Determine the values of x, y, and θ.



16. An equilateral triangle is inscribed in a circle. Determine the side length of the triangle if the diameter of the circle is 200 cm.

Create Connections

Materials
- 1 m of foam pipe insulation, cut lengthwise
- marble or small steel ball
- eight to ten thick books or bricks or a chair
- masking tape
- measuring tape
- table

17. **MINI LAB** Work with a partner or in small groups to explore how varying the angle of a ramp in ski jumping changes the launch angle and duration of flight.

Step 1 Build a ramp similar to the one shown. Place the edge of the ramp at the end of the table. Make a sketch of the right triangle formed by the pipe insulation, books, and table. Include measurements of the length of each leg of the triangle. Determine the angle formed between the pipe insulation and the table.

Step 2 Place a marble at the top of the ramp. Without pushing, let it roll. Observe the flight path. Mark the place where the marble first lands on the floor, using masking tape. Repeat this step two more times and record the horizontal distance the marble lands from the edge of the table. You may wish to complete a chart similar to this one.

Sketch of the Triangle	Measure of the Angle (°)	Distance Measured		
		Trial 1	Trial 2	Trial 3

Step 3 Adjust the ramp so that it curves downward to the table and runs flat along the table for about 20 cm before it reaches the end. Roll the marble down the track and record the distances.

Step 4 Add a book to the end of the ramp, so that the ramp curves upward as it nears the end. Roll the marble and record your measurements.

a) Describe how changing the launch angle of the ramp affects the distance travelled by the marble. Explain why.

b) Would changing the angle of the ramp with the table affect the distance the marble travels? Explain.

3.3 Solving Right Triangles

Aurora borealis above Churchill, Manitoba

Focus on ...

- explaining the relationships between similar right triangles and the definitions of the trigonometric ratios
- solving right triangles, with or without technology
- solving problems involving one or more right triangles

The polar aurora is one of the most beautiful and impressive displays of nature. There have been various attempts to explain the phenomenon of these northern lights. Carl Stormer, a Norwegian scientist, used a network of cameras that simultaneously photographed the aurora. He used the photos to measure the parallax angle shifts and then calculate the height of the aurora.

Investigate Estimation of Distance

Materials

- metre stick or measuring tape

In this investigation you will use the method of parallax to help you estimate the distance to an object.

1. Have a partner stand a distance away from you. Then, mark the floor where each of you is standing using a small piece of paper or other identifying item, such as masking tape. Stretch out your arm with your thumb pointed upward and close your right eye. Line your thumb up with your partner.

Did You Know?

If you stretch your arm out in front of your face with your thumb pointing upward, and then close one eye, your thumb appears to shift slightly. This shift is known as *parallax*. Your brain uses this information to figure out how far away from you objects are.

2. Open your right eye and close your left eye. Do not move your outstretched arm. Have your partner move to his or her right until he or she is in line with your thumb again. Then, mark the new location where your partner is standing.

3. Use a metre stick to measure the distance from you to your partner and the distance between your partner's locations.

4. Reflect and Respond

a) What is the relationship between the distance to your partner and the distance between your partner's locations? Hint: You may wish to repeat your measurements to help you examine the pattern.

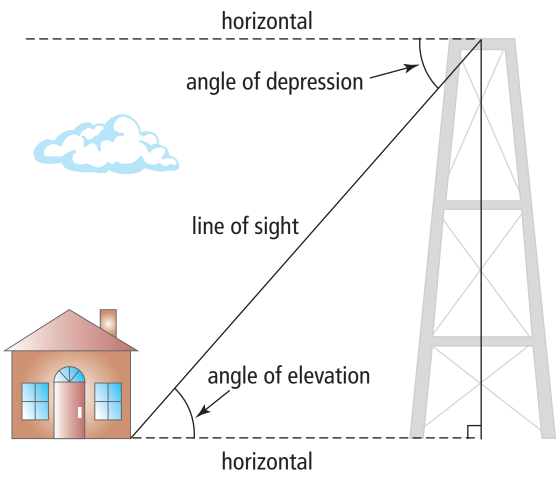
b) Explain how this relationship can help you estimate your distance to an object.

Link the Ideas

The line of sight is the invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation and an angle of depression.

- An angle of elevation is the angle formed by the horizontal and a line of sight *above* the horizontal.
- An angle of depression refers to the angle formed by the horizontal and a line of sight *below* the horizontal.

Measure the angle of elevation and the angle of depression in the diagram. How are the measures of two angles related?



Example 1 Use Angle of Elevation to Calculate a Height

Sean wants to calculate the height of the First Nations Native Totem Pole. He positions his transit 19.0 m to the side of the totem pole and records an angle of elevation of 63° to the top of the totem pole. If the height of Sean's transit is 1.7 m, what is the height of the totem pole, to the nearest tenth of a metre?

Solution

Let x represent the height from the transit to the top of the totem pole.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 63° = \frac{x}{19.0}$$
$$x = 19.0(\tan 63°)$$
$$x = 37.289\ldots$$

Height of totem pole
= height of transit + height from transit to top of pole
= 1.7 + 37.289...
= 38.989...

The height of the First Nations Native Totem Pole is 39.0 m, to the nearest tenth of a metre.

Did You Know?

The First Nations Native Totem Pole is in Beacon Hill Park, in Victoria, BC. The totem pole was erected in 1956 and is one of the world's tallest totem poles.

Your Turn

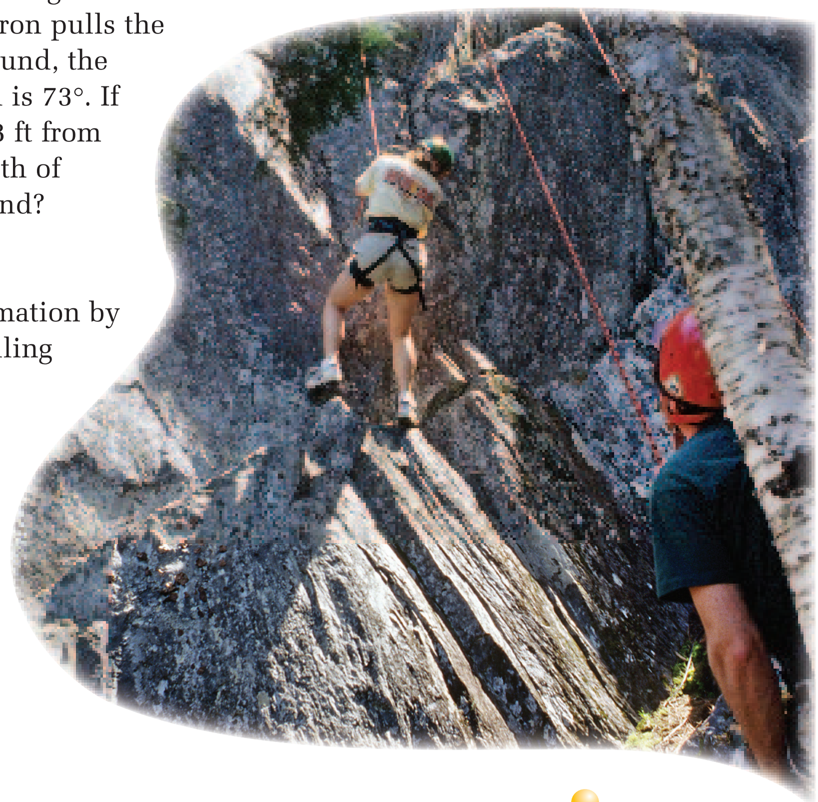
A surveyor needs to determine the height of a large grain silo. He positions his transit 65 m from the silo and records an angle of elevation of 52°. If the height of the transit is 1.7 m, determine the height of the silo, to the nearest metre.

Did You Know?

A *belayer* is the person on the ground who secures a climber who is rock climbing. The belayer and climber each wear a harness that attaches to a rope. The belayer controls how much slack is in the rope. It takes skill and concentration to be a successful belayer.

Example 2 Calculate a Distance Using Angle of Depression

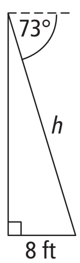
Natalie is rock climbing and Aaron is belaying. When Aaron pulls the rope taut to the ground, the angle of depression is 73°. If Aaron is standing 8 ft from the wall, what length of rope is off the ground?



Solution

Visualize the information by sketching and labelling a diagram.

Let h represent the length of rope that is off the ground.



Use the properties of angles to determine the angle measure of one of the acute angles inside the right triangle.

$\theta = 90° - 73°$
$\theta = 17°$

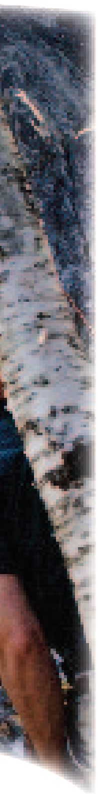
The angle that the rope makes at the top with the vertical is 17°.

$$\sin 17° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 17° = \frac{8}{h}$$
$$h = \frac{8}{\sin 17°}$$
$$h = 27.362\ldots$$

The rope off the ground is approximately 27 ft long.

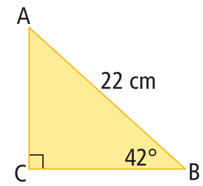
Your Turn

A balloonist decides to use an empty football field for his landing area. When the balloon is directly over the goal post, he measures the angle of depression to the base of the other goal post to be 53.8°. Given that the distance between goal posts in a Canadian football field is 110 yd, determine the height of the balloon.



Example 3 Solve a Right Triangle

Solve the triangle shown. Express each measurement to the nearest whole unit.



Solution

To *solve* a triangle means to determine the lengths of all unknown sides and the measures of all unknown angles. To solve this triangle, you need to determine the lengths of sides AC and CB and the measure of ∠A.

$\angle A = 180° - (90° + 42°)$ **What is the sum of the angles in a triangle?**

$\angle A = 48°$

Using ∠B as the reference angle and knowing the length of the hypotenuse, apply the cosine ratio to calculate the length of side CB.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 42° = \frac{CB}{22}$$
$$CB = 22(\cos 42°)$$
$$CB = 16.349\ldots$$

Calculate the length of side AC.

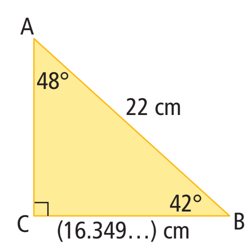
Method 1: Apply a Trigonometric Ratio

Since all angles are known, any of the primary trigonometric ratios could be applied.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$ **How will you decide which ratio to use?**
$$\sin 42° = \frac{AC}{22}$$
$$AC = 22(\sin 42°)$$
$$AC = 14.720\ldots$$

Method 2: Apply the Pythagorean Theorem

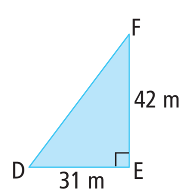
$$AB^2 = AC^2 + CB^2$$
$$22^2 = AC^2 + (16.349\ldots)^2$$
$$484 = AC^2 + 267.295\ldots$$
$$216.704\ldots = AC^2$$
$$\sqrt{216.704\ldots} = AC$$
$$14.720\ldots = AC$$



Angle A measures 48°. Side CB is about 16 cm long and side AC is about 15 cm long.

Your Turn

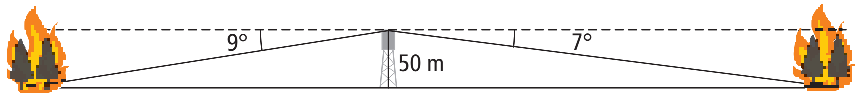
Solve the triangle shown. Express each measurement to the nearest whole unit.



What information are you given? Use the given information as much as possible in your calculations.

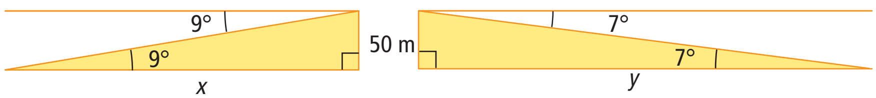
Example 4 Solve a Problem Using Trigonometry

From a height of 50 m in his fire tower near Francois Lake, BC, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9°. The other fire is due east at an angle of depression of 7°. What is the distance between the two fires, to the nearest metre?



Solution

Model the problem using right triangles.
Let x and y represent the lengths of the bases of the triangles.



$$\tan 9° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 9° = \frac{50}{x}$$
$$x = \frac{50}{\tan 9°}$$
$$x = 315.687\ldots$$

Use the given angles to find the measure of one acute angle in each right triangle.

$$\tan 7° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 7° = \frac{50}{y}$$
$$y = \frac{50}{\tan 7°}$$
$$y = 407.217\ldots$$

Add to determine the distance between the fires.
$315.687\ldots + 407.217\ldots = 722.904\ldots$

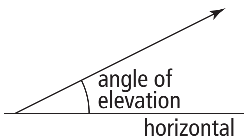
The distance between the fires, to the nearest metre, is 723 m.

Your Turn

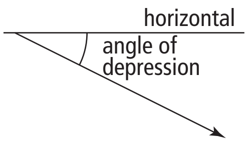
From his hotel window overlooking Saskatchewan Drive in Regina, Ken observes a bus moving away from the hotel. The angle of depression of the bus changes from 46° to 22°. Determine the distance the bus travels, if Ken's window is 100 m above street level. Express your answer to the nearest metre.

Key Ideas

- An angle of elevation is the angle between the line of sight and the horizontal when an observer looks upward.



- An angle of depression is the angle between the line of sight and the horizontal when the observer looks downward.



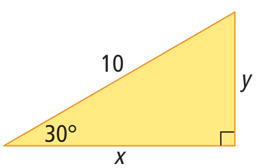
- To solve a triangle means to calculate all unknown angle measures and side lengths.

Check Your Understanding

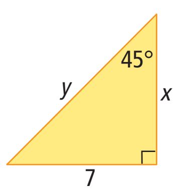
Practise

1. Solve each triangle, to the nearest tenth of a unit.

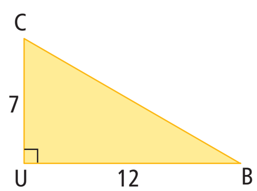
a)



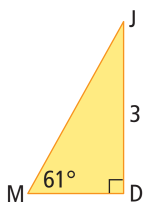
b)



c)

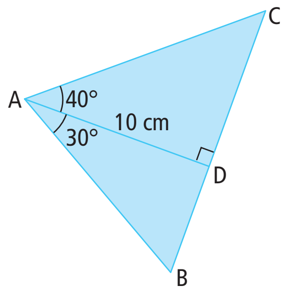


d)

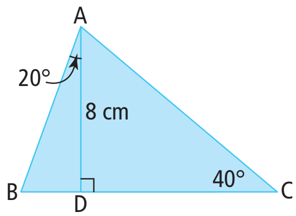


2. Calculate the length of BC, to the nearest tenth of a centimetre.

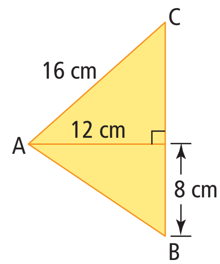
a)



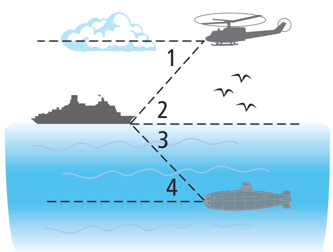
b)



3. Determine the measure of ∠CAB, to the nearest degree.



4. Describe each angle as it relates to the diagram.



a) ∠1

b) ∠2

c) ∠3

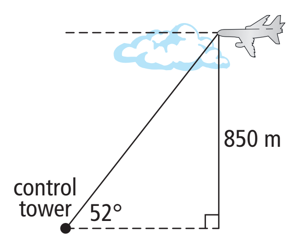
d) ∠4

5. The heights of several tourist attractions are given in the table. Determine the angle of elevation from a point 100 ft from the base of each attraction to its top.

	Attraction	Location	Height
a)	World's largest fire hydrant	Elm Creek, MB	29 ft
b)	World's largest dinosaur	Drumheller, AB	80 ft
c)	Saamis Tipi	Medicine Hat, AB	215 ft
d)	World's largest tomahawk	Cut Knife, SK	40 ft
e)	Igloo church	Inuvik, NT	78 ft

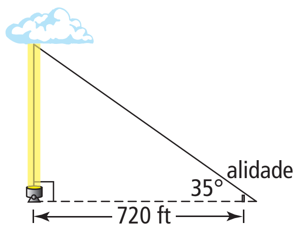
Apply

6. An airplane is observed by an air traffic controller at an angle of elevation of 52°. The airplane is 850 m above the observation deck of the tower. What is the distance from the airplane to the tower? Express your answer to the nearest metre.



7. Cape Beale Lighthouse, BC, is on a cliff that is 51 m above sea level. The lighthouse is there to warn boats of the danger of shallow waters and the possibility of rocks close to the shore. The safe distance for boats from this cliff is 75 m. If the lighthouse keeper is 10 m above ground and observes a boat at an angle of depression of 50°, is the boat a safe distance from the cliff? Justify your conclusion.

8. At night, it is possible to make precise measurements of cloud height using a search light. An alidade is set 720 ft away from the search light. It measures the angle of elevation to the place where the light strikes the cloud to be 35°. What is the altitude of the cloud? Express your answer to the nearest foot.



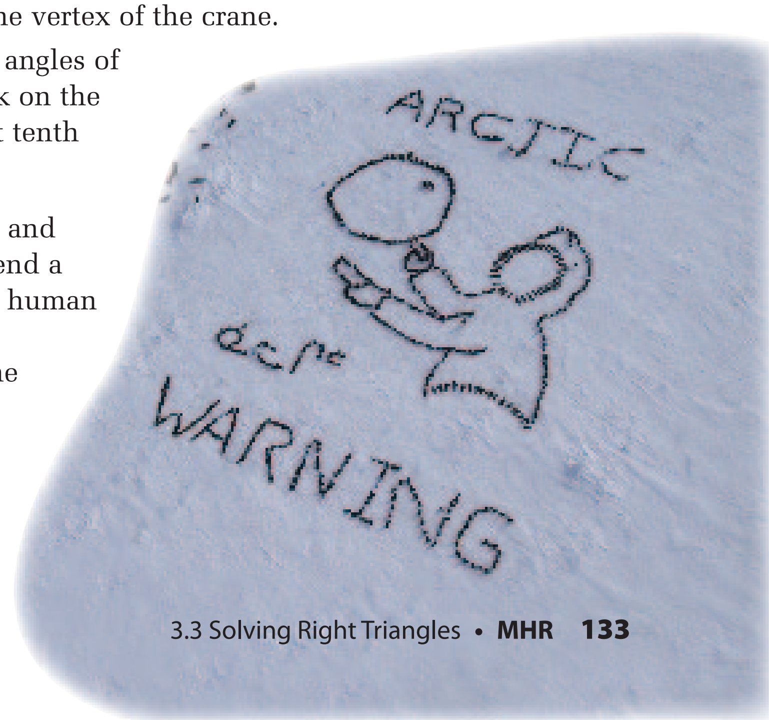
alidade

9. The working arm of a tower crane is 192 m high and has a length of 71.6 m. Suppose the hook reaches the ground and moves along the arm on a trolley.

a) Determine the maximum distance from the hook to the operator when the trolley is fully extended at 71.6 m and the minimum distance when the trolley is closest to the operator at 8.1 m. Hint: The operator is located at the vertex of the crane.

b) Determine the maximum and minimum angles of depression from the operator to the hook on the ground. State your answer to the nearest tenth of a degree.

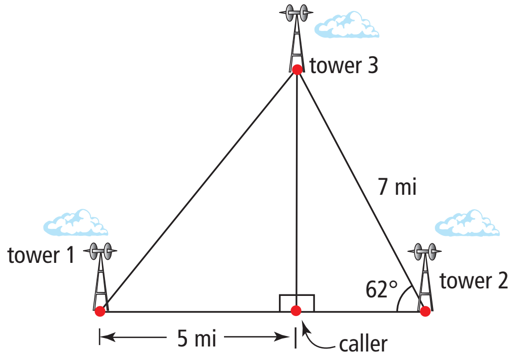
10. Arctic Wisdom involved children, parents, and Elders gathering on Baffin Island, NU, to send a message. To achieve the best picture of the human image on the sea ice, an aerial photograph was taken. The angle of depression from the helicopter was 58° and the height of the helicopter was 140 m. How far away from the image was the helicopter?



Did You Know?

For the 2010 Olympic Games in Vancouver, the Millennium Water Project involved building 1100 condominiums. This project made use of eight tower cranes that lifted steel, concrete, large tools, and generators. The cranes often rise hundreds of feet into the air and can reach out just as far.

11. **Unit Project** A cell phone can be used to send music, but as your location changes, you move in and out of range from one *cell* to the next. Three or more cellular towers may pick up a cell phone's signal. A cell phone signal has been located 5 mi from tower 1.



a) What is the distance from the caller to tower 3?

b) How far is tower 1 from tower 3?

12. The Disabled Sailing Association had its first sessions at the Jericho Sailing Centre in Vancouver, BC. At a recent regatta, a television news team tracked two sailboats from a helicopter 800 m above the water. The team observed the sailboats on the left and right sides of the helicopter at angles of depression of 58° and 36°, respectively.

a) Which boat is located closer to the helicopter? Explain.

b) Determine the distance between the two boats. Express your answer to the nearest metre.

13. Two tourists stand on either side of the Veterans Pole, honouring Canadian Aboriginal war veterans, in Victoria, BC. One tourist measures the angle of elevation of the top of the pole to be 21°. To the other tourist, the angle of elevation is 17°. If the height of the pole is 5.5 m, how far apart are the tourists? Express your answer to the nearest tenth of a metre.

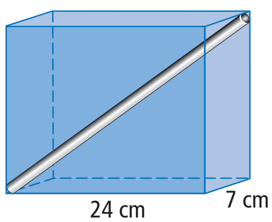
Extend

14. From the top of a 35-m-tall building, an observer sees a truck heading toward the building at an angle of depression of 10°. Ten seconds later, the angle of depression to the truck is 25°.

a) Determine the distance that the truck has travelled. Express your answer to the nearest metre.

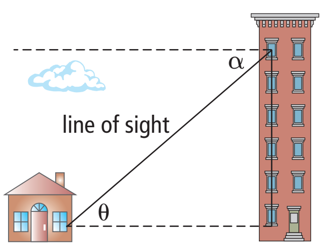
b) If the speed limit for the area is 40 km/h, is the truck driver following the speed limit? Explain.

15. A rectangular prism has base dimensions of 24 cm by 7 cm. A metal rod is run from the bottom corner diagonally to the top corner of the prism. If the rod forms an angle of 40° with the bottom of the box, calculate the volume of the box.



Create Connections

16. From her apartment, Jennie measures the angle of depression to Mike's house. At the same time, Mike measures the angle of elevation to Jennie's apartment.



a) Mike's brother Richard observes Mike and states that Mike made an error, because the angle of elevation must be greater than the angle of depression. Is Richard correct? Explain your reasoning.

b) In order to calculate the measure of angle θ, you can be given any of the following measurements:
- the height of Jennie's window
- the horizontal distance between buildings
- the length of line of sight
- the measure of angle α

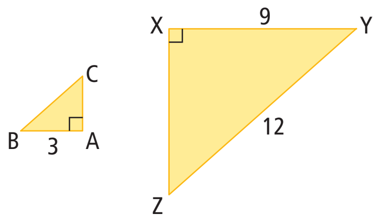
Which measurement(s) would you prefer to be given? Explain how you would use these measurements to calculate θ.

3 Review

3.1 The Tangent Ratio, pages 100–113

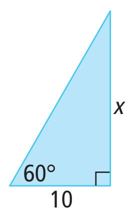
Where necessary, express your answers to the nearest tenth of a unit.

1. Triangles ABC and XYZ are similar. Calculate the lengths of the unknown sides.

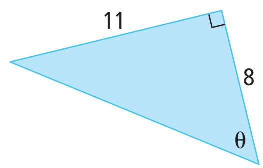


2. Determine the value of the variable in each triangle.

a)



b)



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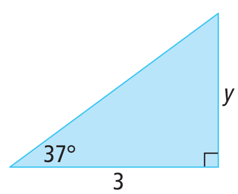


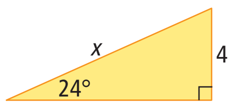
3. A group of conservationists needs to calculate the angle of elevation of the river bank of the North Saskatchewan River. They set up a right triangle using two measuring poles. If they measure the vertical height to be 64 cm and the horizontal distance to be 50 cm, what is the angle of elevation of the river bank?

3.2 The Sine and Cosine Ratios, pages 114–124

Where necessary, express your answers to the nearest tenth of a unit.

4. Determine the value of the variable in each triangle.

a)



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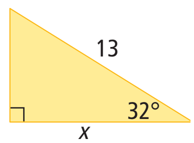

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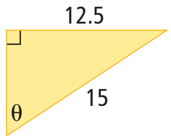


5. Augers are used to move grain into storage bins. Suppose an auger is 67 ft long and the granary is 44 ft high. Determine the angle formed by this auger and the ground.

6. A 14-ft ladder leans against the bottom of a window and makes an angle of 64° with the ground. What is the height to the bottom of the window?

3.3 Solving Right Triangles, pages 125–135

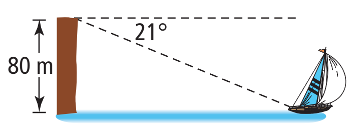
Where necessary, express your answers to the nearest tenth of a unit.

7. In $\triangle ABC$, $BC = 7.4$ km, $\angle B = 90°$, and $\angle A = 38°$.

a) Draw and label the triangle.

b) Solve $\triangle ABC$.

8. The angle of depression from the top of an 80-m-high cliff to a sailboat is 21°. Determine the distance from the base of the cliff to the sailboat.



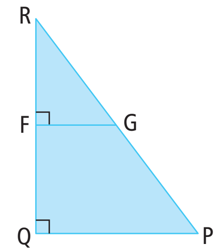
9. A lifeguard sitting on a platform that is 14 ft high observes someone swimming. The first sighting of the swimmer is at an angle of depression of 60°. The angle of depression becomes 30° the next time the lifeguard looks at the swimmer. Explain whether the swimmer is moving toward or away from the lifeguard. Use a diagram to support your answer. Then, determine the distance that the swimmer has travelled.

3 Practice Test

Multiple Choice

For #1 to #4, choose the best answer.

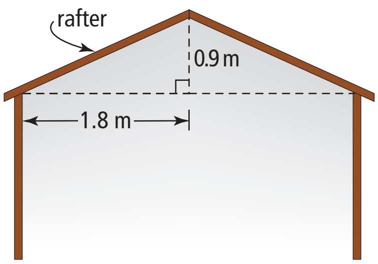
1. For the similar triangles shown, which expression is true?



A $\frac{FG}{QP} = \frac{RG}{PG}$ **B** $\frac{PQ}{GR} = \frac{RG}{QF}$

C $\frac{RF}{QR} = \frac{GR}{RP}$ **D** $\frac{GF}{RF} = \frac{QP}{RP}$

2. Madeleine's dad is designing a garage to build beside their house. He wants a 30-cm overhang on each side. How long should each rafter be?



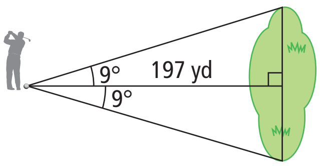
A 2.0 m **B** 2.3 m

C 3.8 m **D** 4.4 m

3. A gardener uses topsoil to improve garden soil for his Regina customers. He purchased a special pickup truck that acts like a dump truck. If the 80-in. truck bed is raised to a 40° angle, how high is the upper end of the truck box above the wheels?

A 51 in. **B** 61 in. **C** 67 in. **D** 80 in.

4. The 17th hole at the Rivershore Golf Course near Kamloops, BC, is 197 yd from the teeing area to the centre of the green. Suppose the largest angle at which you can drive the golf ball to the left or right and still land on the green is 9°. What is the width of the green, to the nearest yard?



A 15 yd **B** 31 yd **C** 47 yd **D** 62 yd

Short Answer

5. During the annual Windscape Kite Festival in Swift Current, SK, Yves and Lucian's kite got caught in the top of a tree. Yves wants to use similar triangles to calculate the height of the tree. A nearby 9-m flagpole casts a shadow that is 6 m long. Yves and Lucian estimate the shadow of the tree to be 3.5 m long. What is the height of the tree, to the nearest tenth of a metre? Include a diagram of the situation.

6. Evaluate each trigonometric ratio, to four decimal places.

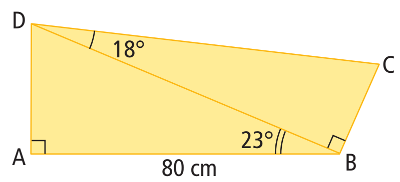
a) tan 17° **b)** sin 68° **c)** cos 23°

7. Calculate the measure of each angle, to the nearest degree.

a) $\sin \theta = 0.2588$ **b)** $\tan \alpha = 5.6713$ **c)** $\cos \theta = 0.7431$

8. Zachary was calculating the length of side CD in the figure. His partial solution is shown.

$$\cos 23^\circ = \frac{80}{BD}$$
$$BD = \frac{80}{(\cos 23^\circ)}$$
$$\sin 18^\circ = \frac{BD}{CD}$$
$$CD = \frac{BD}{\sin 18^\circ}$$

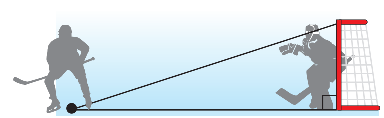


Before Zachary completed his work, he realized that he had made an error. Identify Zachary's error. Explain a strategy to help him avoid making this error again.

Extended Response

9. The Quikcard Edmonton Minor Hockey Week is one of the largest hockey tournaments in North America. The tournament has grown to include more than 480 teams from Alberta.

a) Suppose the goalie's shoulder rises to 40 in., and a player takes a shot 20 ft from the net. Through what angle of elevation of the puck's flight will the goalie make the save? Give your answer to the nearest tenth of a degree.



b) The height of the net is 48 in. A player takes a shot over the right shoulder of the same goalie from part a) at an angle of elevation of 8.5°. If the puck travels a distance of 29 ft, will the player score a goal? Explain why.

1 Unit Connections

Unit 1 Project

Use your answers to the unit project questions throughout chapters 1, 2, and 3, as well as your own research, to prepare a presentation on music distribution. Your presentation should include the following:

- research on the history of music recording
- a comparison of various storage devices
- a description of the impact technology has had on music distribution

To complete your presentation, predict what the next technological advance in music distribution might be. Include answers to the following questions:

- Describe what you think the next advance in music distribution might look like. Provide measurements for length, area, and volume of the new equipment in both SI and imperial units.
- How might this equipment work?
- What impact might the advance have on how you access music?
- How might this equipment distribute music to people around the world?

Unit Review

Chapter 1 Measurement Systems

1. Identify referents that could be used for the following linear measurements.

millimetre	centimetre	metre
inch	foot	yard

2. For each total length, choose a comparable unit of measurement in the SI system. Determine the length to the nearest tenth of a unit. Justify your choice of units.

a) A table-tennis ball has a diameter of the width of two fingers. Each finger is half an inch wide.

b) Samantha often wears her hair in a ponytail. Her ponytail is 5 hand-widths long. The width of her hand is 3 in.

c) It takes Everrett 14 steps to leave the classroom. One of his paces measures half a yard.

3. Convert each measurement to the indicated unit.

a) 3500 mm = ■ cm **b)** 3.5 ft = ■ in.

c) 8723 m = ■ km **d)** 4.25 ft = ■ m

e) 67 cm = ■ in. **f)** 14 km = ■ mi

4. A rectangular oak table measures 5 ft 10 in. by 3 ft 9 in. What is the perimeter of the table, in feet and inches?

5. An artist sculpts a $10\frac{1}{4}$-in. tall clay model of a horse. If the scale used for the sculpture is 1:6, how tall would the actual horse be, in feet and inches?

Chapter 2 Surface Area and Volume

6. Calculate the area of each figure, as indicated.

a) a rectangle with dimensions 250 cm by 180 cm, in square metres

b) a square of side length 4 mi in square yards

7. Melody is helping prepare a cake for a banquet in Nanaimo, BC. She needs to know the amount of icing needed for the initial covering of the cake before she adds the final decorations. Assume that Melody does not ice the bottom of any layer.

a) What surface area does Melody need to ice for the top three layers, in square centimetres, if the top three layers are square and have the following dimensions:

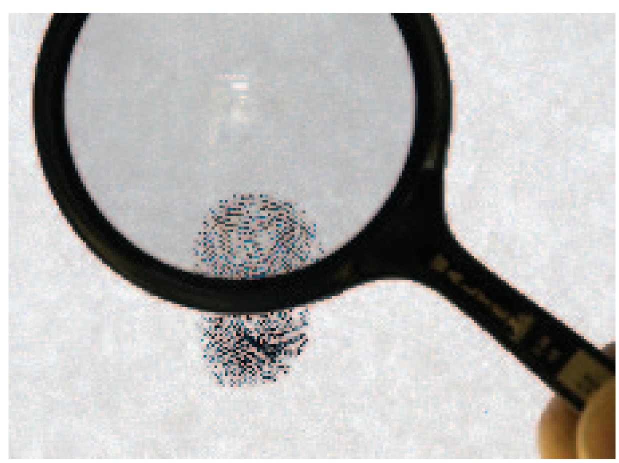
Top layer: side length of 10 cm and a height of 7 cm

Second layer: side length of 14 cm and a height of 8.5 cm

Third layer: side length of 18 cm and a height of 9 cm

b) The volume of cake used for the bottom layer is 4000 cm^3. The bottom square layer has a height of 10 cm. What surface area, in square centimetres, does Melody need to ice on the bottom layer?

8. Nalze went on a field trip with his class to the RCMP detachment in Yellowknife, NT. In the Henry Larson Building, there was a display on crime investigation. Nalze used a magnifying glass to look at a fingerprint.



a) From the photo, estimate the diameter of the magnifying glass Nalze used.

b) If the glass is approximately 0.3 cm thick, what is the volume of glass used?

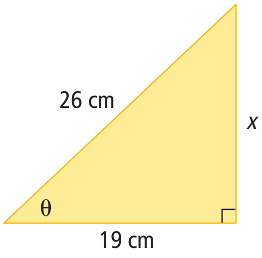
9. Saskatchewan artist Jacqueline Berting created *The Glass Wheatfield - A Salute to Canadian Farmers*. It is made up of 11 000 individually crafted waist-high stalks of glass wheat mounted in a steel base. The average cylindrical stem is 40 in. tall with a diameter of $\frac{1}{8}$ in. Each head of wheat contains the equivalent amount of glass as a cone that is 4 in. long with a base diameter of $\frac{3}{4}$ in. Approximately how much glass did Jacqueline use for the sculpture?


The Glass Wheatfield
Regina Plains Museum

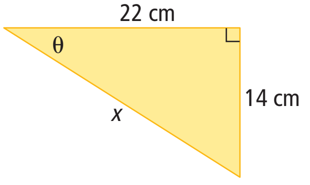
Chapter 3 Right Triangle Trigonometry

10. Determine the measurements of each unknown side and unknown angle. State side lengths to the nearest tenth of a unit and angles to the nearest degree.

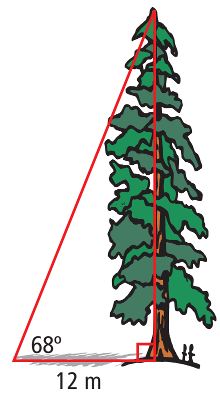
a)



b)



11. A tree casts a shadow 12 m long. The angle measured to the top of the tree from the end of the shadow is 68°. What is the height of the tree? Express your answer to the nearest tenth of a metre.



12. An oil rig is held vertical by two guy wires of unequal lengths on opposite sides of the oil rig. One of the wires makes an angle of 45° with the platform. The other wire is 90 ft long and makes an angle of 55° with the platform. Both wires are attached 8 ft down from the top of the rig.

a) Sketch and label a diagram of this situation.

b) Calculate the height of the oil rig, to the nearest foot.

c) Do you think the length of the unknown wire is greater than the 90-ft wire? Justify your prediction. Then, determine the measurement, to the nearest half of a foot.

d) Determine the distance on the platform between the two guy wires, to the nearest half of a foot.

13. A 15-m-long ladder is placed in a driveway between two buildings. The ladder leans against one building and reaches 12 m up the side. If the ladder is rotated to lean on the other building, it reaches 8 m up the side. How wide is the driveway between the two buildings?

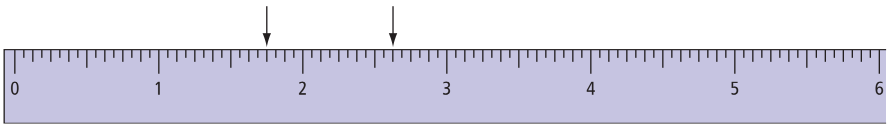
14. Neighbourhoods A and B are situated on opposite sides of a mountain that stands 780 m high. The angles of elevation from each neighbourhood to the top of the mountain are 67° and 54°. What would be the length of a tunnel from neighbourhood A to neighbourhood B? Express your answer to the nearest tenth of a metre.

1 Unit Test

Multiple Choice

For #1 to #4, choose the best answer.

1. What is the distance measured between the two arrows on this imperial ruler?



A $\frac{7}{8}$ in. **B** $\frac{16}{14}$ in. **C** $\frac{7}{16}$ in. **D** $\frac{15}{16}$ in.

2. Elijah is helping install baseboards in a bedroom in the basement. He knows that one of his paces is approximately equal to 1 yd. If he walks 15 paces along the width of the room and 18 paces along the length, what is the approximate perimeter of the room, in feet?

A 99 ft **B** 198 ft **C** 270 ft **D** 792 ft

3. Carrie was asked to calculate the slant height of a right cone. She is given that the surface area is 251.3 cm^2 and the diameter is 10 cm. Her work is shown below.

Step 1 $SA = \pi r^2 + \pi rs$

Step 2 $251.3 = \pi(5^2) + \pi(5)s$

Step 3 $\frac{251.3}{(25\pi + 5\pi)} = s$

Step 4 $2.7 = s$

When Carrie examined her work, she realized that she made her first error in

A Step 1 **B** Step 2 **C** Step 3 **D** Step 4

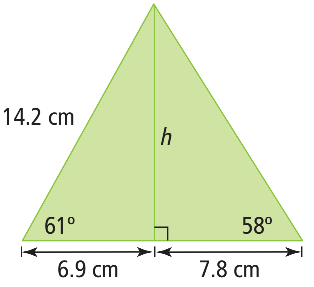
4. The equation that could be used to calculate the value of h in the diagram is

A $\cos 58° = \frac{h}{7.8}$

B $\tan 58° = \frac{h}{7.8}$

C $\cos 61° = \frac{h}{14.2}$

D $\sin 61° = \frac{h}{6.9}$



Numerical Response

Complete the statements in #5 to #7.

5. Jett measures the diagonal of the television screen in his family room to be 117 cm. Laura measures the diagonal of her television screen to be 54 in. Laura's television is ■ in. larger than Jett's television, expressed to the nearest inch.

6. A glass paperweight is in the shape of a sphere and has a volume of 356 818 mm^3. The radius of the paperweight is ■ mm.

7. Your school is installing a wheelchair ramp outside the front doors. The current stairs reach a height of 0.7 m. If the ramp is 8 m long, the horizontal distance to the end of the ramp, to the nearest tenth of a metre, is ■ m.

Written Response

8. Alicia found a unique gift for her friend's birthday. She bought a purse that is in the shape of a right pyramid with a square base. The dimensions of the base are 12.0 cm by 12.0 cm, and the slant height is 16.16 cm.

a) Determine the height of the purse.

b) How much space is inside the purse?

c) Alicia wants to place the purse in a gift box with a lid. She has gift boxes of the following volumes:
- 2100 cm^3
- 2200 cm^3

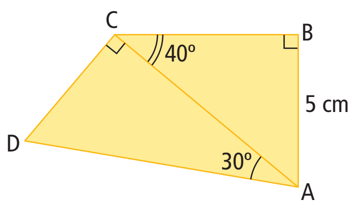
For each size of gift box, explain whether the purse will fit inside.

9. Given △ACD is adjacent to △ABC.

a) Write an equation that could be used to calculate the length of AC.

b) Calculate the length of AC.

c) Calculate the length of DC, to the nearest centimetre.

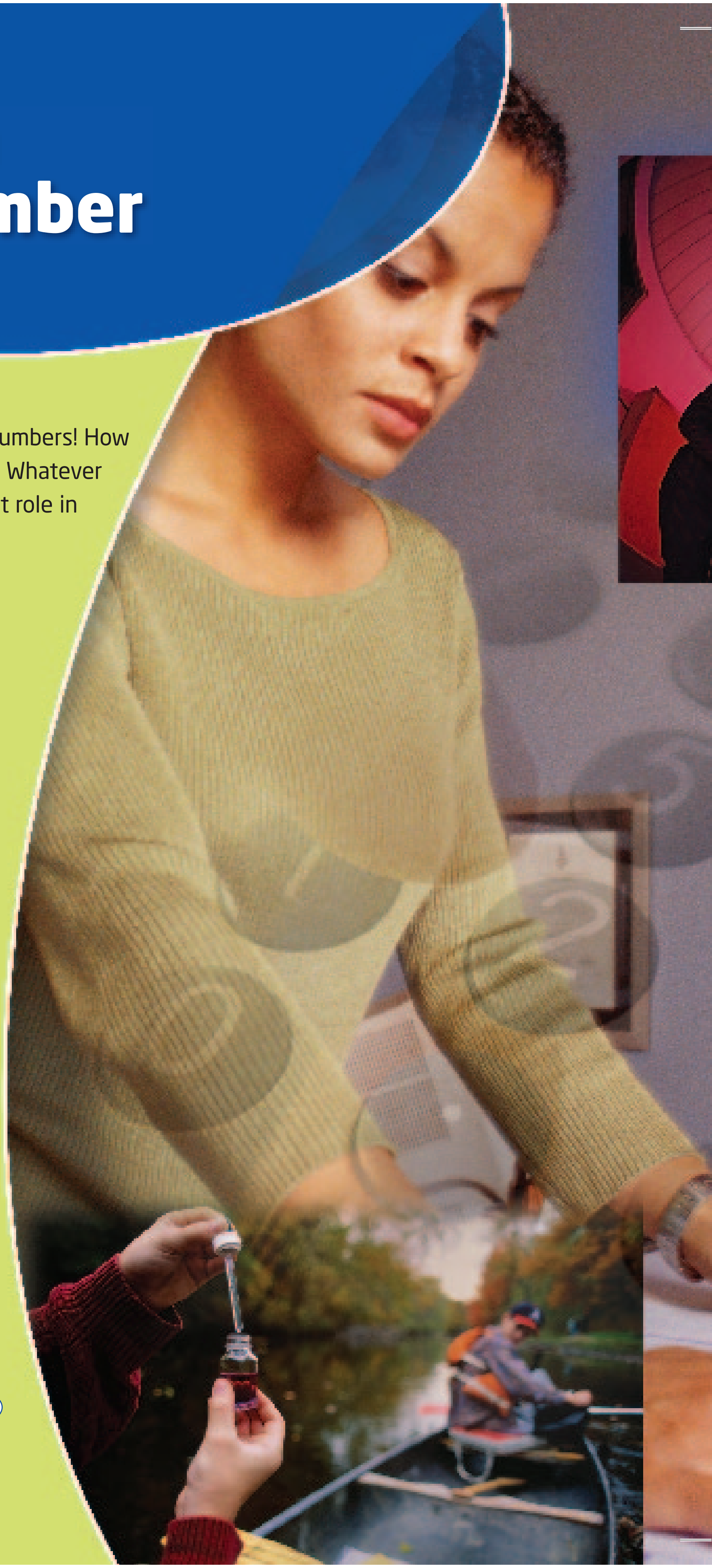


10. As a spectator at a hockey game, Brennan is sitting 40 m horizontally from the goal net. His seat is 10 m above ice level.

a) At what angle of depression is Brennan watching the goalie make a save?

b) A seat becomes available directly below Brennan, so he moves 3 m down. Will the angle of depression from Brennan to the goalie increase or decrease? Justify your answer.

UNIT 2 Algebra and Number

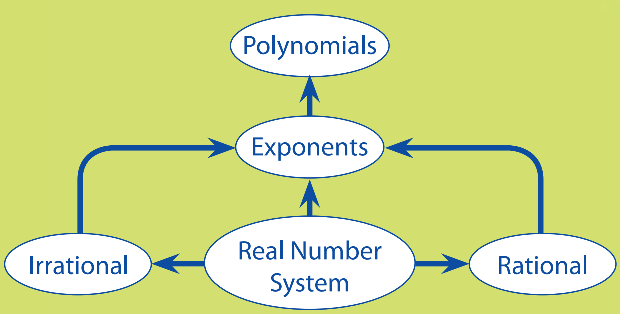


Consider how much of your daily life involves numbers! How do numbers relate to the careers you see here? Whatever you aspire to be, numbers will play an important role in what you do.

Number, and more specifically the real number system, forms the foundation for mathematics. One field of mathematics, algebra, includes the multiplication of polynomials and factoring of trinomials. In this unit, you will use square roots, cube roots, and irrational numbers along with exponents to solve problems in a variety of contexts. You will also develop and use algebraic skills to solve problems involving the multiplying and factoring of polynomials.

Your Algebra and Number Organizer

You can use this algebra and number organizer to see how the concepts in this unit are connected. You will see this organizer on the first page of each chapter. The concepts covered in the chapter will be highlighted.



Looking Ahead

In this unit, you will solve problems involving …

- square roots and cube roots
- integral and rational exponents
- irrational numbers, including radicals
- multiplying polynomials
- factoring polynomials

Unit 2 Project

Math in Art

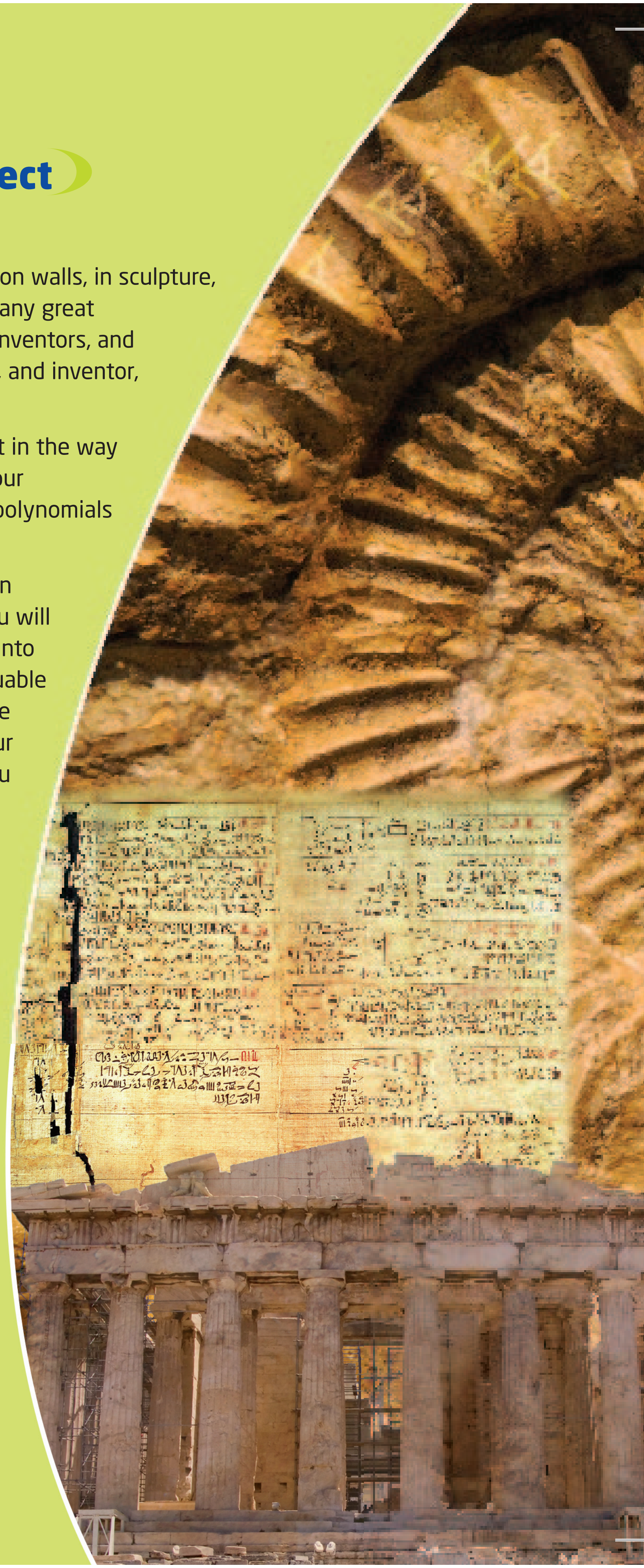
Art takes many different forms. It can be seen on walls, in sculpture, in architecture, on clothing, and even in nature. Many great mathematicians were also philosophers, historians, inventors, and artists. Leonardo da Vinci was a mathematician, artist, and inventor, who incorporated mathematics into his art.

In the Unit 2 project, you will discover that there is art in the way that numbers interact with each other. You will use your understanding of irrational numbers, exponents, and polynomials to incorporate math into your own work of art.

Unit Project questions and activities are included in Chapters 4 and 5. As you move through Chapter 4, you will discover how mathematics historically has been built into art. As you move through Chapter 5, you will gain valuable insights needed to create your own piece of art. At the end of Chapter 5, you will create your piece of art. Your final presentation will include a description of how you incorporated mathematics in your art.

While completing your project, you will ...

- discover relationships among irrational numbers, exponents, polynomials, and art (Chapters 4 and 5)
- apply your understanding of radicals to analyse the golden ratio (Chapter 4)
- create your own art and explain how you used mathematics in its development (Chapter 5)




BABYLONIAN NUMERALS

CHAPTER

4 Exponents and Radicals

Exponents have been used to help model and solve problems since the time of the Babylonians, about 4000 years ago. For example, Plimpton 322 is a stone tablet written in Babylonian script (about 1900 to 1600 B.C.E.). The tablet includes sets of three positive numbers, such as 3, 4, and 5. These can be the measurements of the sides of a right triangle. These numbers are known as Pythagorean triples.

Today, we use exponents for solving problems that range from calculating interest earned on savings to estimating how fast bacteria can grow.

Big Ideas

When you have completed this chapter, you will be able to ...

- solve problems that involve square roots and cube roots
- solve problems involving powers with integral and rational exponents
- represent, identify, and simplify irrational numbers

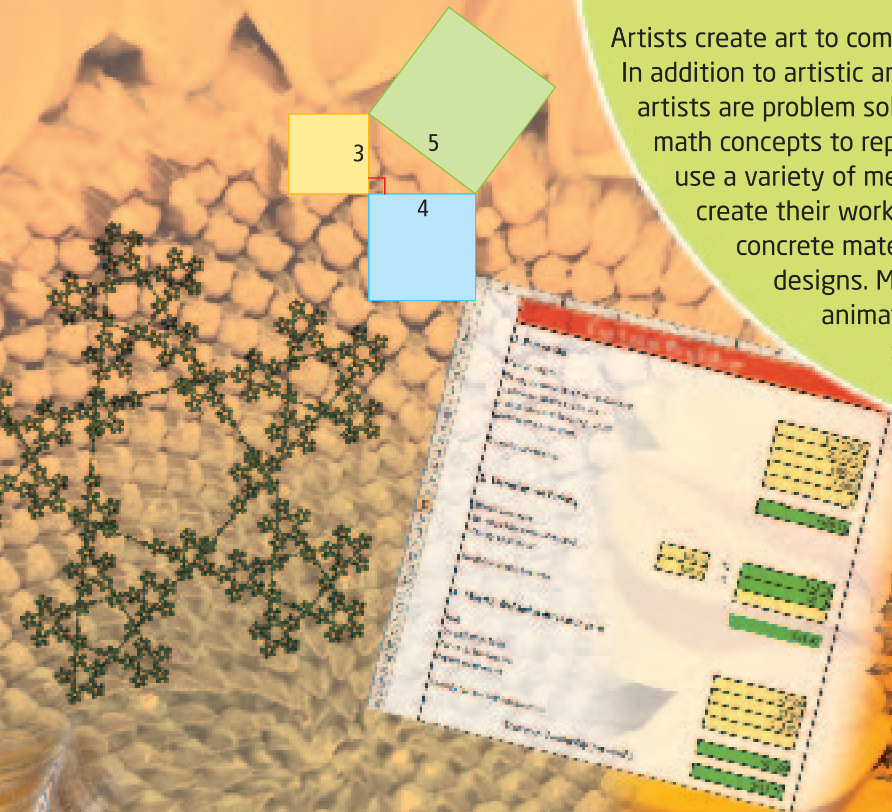
Key Terms

perfect square
square root
perfect cube
cube root
prime factorization
irrational number
radical
radicand
index
mixed radical
entire radical

Your Algebra and Number Organizer

Polynomials
↑
Exponents
↑
Irrational ← Real Number System → Rational

(Irrational → Exponents; Rational → Exponents)



Artist

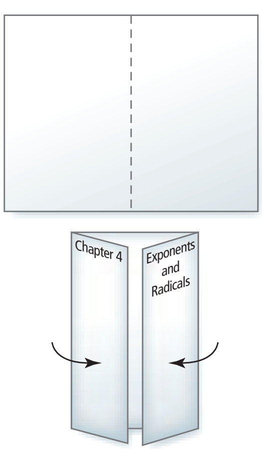
Artists create art to communicate ideas. In addition to artistic and technical skills, artists are problem solvers who often use math concepts to represent reality. Artists use a variety of methods and materials to create their works. Some artists use concrete materials to create their designs. Multimedia artists and animators use computer design software to model objects in 3-D.

Web Link

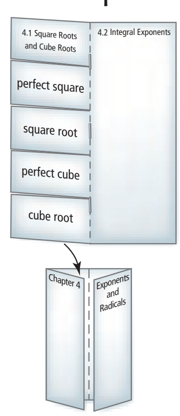
If you are interested in learning more about artists, go to www.mhrmath10.ca and follow the links.

Make the following Foldable™ to take notes on what you will learn in Chapter 4.

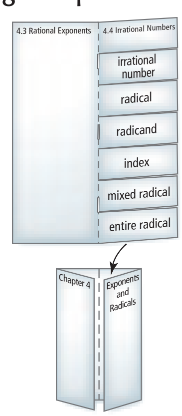
1. Fold a sheet of 11 × 17 paper as shown. On the outside front flaps, add the labels shown below.



2. Fold and label a sheet of 8.5 × 11 paper as shown. Cut tabs along the lines on the left half. Attach the tabbed page inside the left flap.



3. Fold and label another sheet of 8.5 × 11 paper as shown. Cut tabs along the lines on the right half. Attach the tabbed page inside the right flap.



4. On the centre panel of the Foldable™, write the title Exponent Laws. On the back, write the title What I Need to Work On.

4.1 Square Roots and Cube Roots

Focus on ...
- determining the square root of a perfect square and explaining the process
- determining the cube root of a perfect cube and explaining the process
- solving problems involving square roots or cube roots

Workers apply what they know about surface area and volume when working with square shapes and cubes.

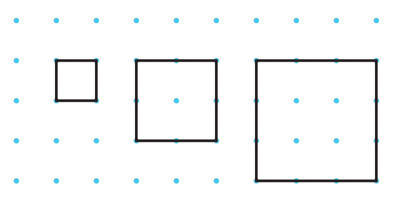
A house painter must calculate the surface area of the walls of a house when preparing a cost estimate. If you know the area of a square wall, how could you calculate the side lengths?

A designer must calculate the size of the case required to enclose a speaker for a sound system. If you know the volume of a cube-shaped box, how could you calculate the edge lengths?

Materials
- square dot paper
- isometric dot paper

Investigate Square Roots and Cube Roots

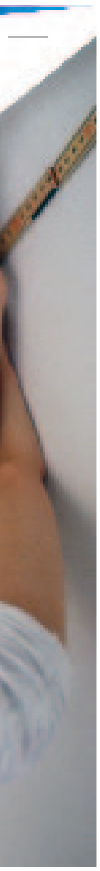
1. a) Determine the area of each square shown. Record the information in a table.



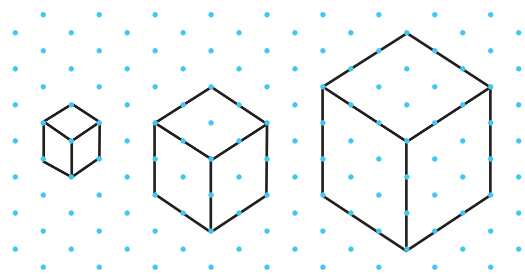
Side Length	Area in Exponential Form	Area

b) Extend the pattern for squares with dimensions of 4, 5, and 6 units.

c) What is the relationship between the side length of a square and the area of the square?



2. a) Determine the volume of each cube shown. Record the information in a table.



Edge Length	Volume in Exponential Form	Volume

b) Extend the pattern for cubes with dimensions of 4, 5, and 6 units.

c) What is the relationship between the edge length of a cube and the volume of the cube?

3. **Reflect and Respond**

Discuss with a partner.

a) What strategy could you use to find the side length of a square if you were given the area?

b) What strategy could you use to find the edge length of a cube if you were given the volume?

c) Explain, using a diagram, how you could predict
- the side length of a square with an area of 64 square units
- the edge length of a cube with a volume of 343 cubic units

Link the Ideas

Perfect squares and **square roots** are related to each other. The number 25 is a perfect square. It is formed by multiplying two factors of 5 together.

$(5)(5)$ or $5^2 = 25$

The symbol for square root is $\sqrt{\ }$.

The square root of 25 is 5, or $\sqrt{25} = \sqrt{(5)(5)}$
$= \sqrt{5^2}$
$= 5$

perfect square
- a number that can be expressed as the product of two equal factors
- for example, $16 = (4)(4)$ or 4^2

square root
- one of two equal factors of a number
- for example, $\sqrt{49} = \sqrt{(7)(7)}$
$= 7$

perfect cube

- a number that is the product of three equal factors
- for example, $64 = (4)(4)(4)$ or 4^3

cube root

- one of three equal factors of a number
- for example, $\sqrt[3]{512} = \sqrt[3]{(8)(8)(8)}$
  $= 8$

Perfect cubes and **cube roots** are related to each other. The number 27 is a perfect cube. It is formed by multiplying three factors of 3 together.

$(3)(3)(3)$ or $3^3 = 27$

The symbol for cube root is $\sqrt[3]{\ }$.

The cube root of 27 is 3, or $\sqrt[3]{27} = \sqrt[3]{(3)(3)(3)}$
$= \sqrt[3]{3^3}$
$= 3$

Some numbers are both perfect squares and perfect cubes.

$64 = (8)(8)$ and $64 = (4)(4)(4)$
$= 8^2$ $\quad = 4^3$

Therefore, 64 is a perfect square and a perfect cube.

Example 1 Identify Perfect Squares and Perfect Cubes

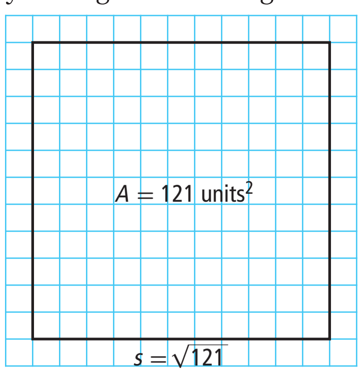
State whether each of the following numbers is a perfect square, a perfect cube, both, or neither.

a) 121 **b)** 729 **c)** 356

Solution

a) To decide whether 121 is a perfect square you might use a diagram.

$10^2 = 100$ Too low
$12^2 = 144$ Too high
$11^2 = 121$ Correct!



A square with side lengths of 11 units has an area of 121 units^2.

$(11)(11) = 121$.

Therefore, 121 is a perfect square.

To decide whether 121 is a perfect cube, you could use guess and check.

No whole number cubed results in a product of 121.

$4^3 = 64$ Too low
$5^3 = 125$ Too high

Therefore, 121 is not a perfect cube.

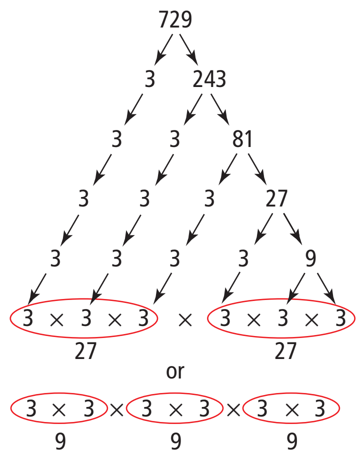
WWW Web Link

To learn more about perfect squares and square roots, go to www.mhrmath10.ca and follow the links.

To learn more about perfect cubes and cube roots, go to www.mhrmath10.ca and follow the links.

b) For 729, you might use **prime factorization**. Prime factorization involves writing a number as the product of its prime factors. A factor tree helps organize the prime factors.

Record the prime factorization for 729. Then, identify the factors that can be squared or cubed to form the product 729.



These two groups indicate the square root of 729.

These three groups indicate the cube root of 729.

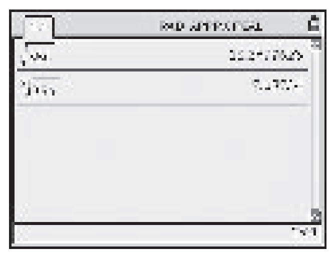
You can write 729 as the product of $(27)(27) = 27^2$. Therefore, 729 is a perfect square.

You can write 729 as the product of $(9)(9)(9) = 9^3$. Therefore, 729 is a perfect cube.

c) For 356, you might use a calculator.

C 356 $\sqrt{x}$ 18.867962

C 356 2nd $\sqrt[x]{y}$ 3 = 7.08734



Key sequences vary among calculators. Check the key sequence for determining square roots and cube roots of numbers on your calculator. Record the correct sequence for your calculator.

Since the square root is not a whole number, 356 is not a perfect square. Since the cube root is not an integer, 356 is not a perfect cube. The number 356 is neither a perfect square nor a perfect cube.

Your Turn

State whether each number is a perfect square, a perfect cube, both, or neither. Use a variety of methods.

a) 125 **b)** 196 **c)** 4096

prime factorization

- the process of writing a number written as a product of its prime factors.
- the prime factorization of 24 is $2 \times 2 \times 2 \times 3$.

WWW Web Link

To learn more about prime factorization and to use a prime factorization tool, go to www.mhrmath10.ca and follow the links.

Did You Know?

Between 1850 and 1750 B.C.E., the Babylonians were applying the Pythagorean relationship. They recorded tables of square roots and cube roots on clay tablets. This was long before Pythagoras was born.

Did You Know?

Canada is the largest producer of uranium in the world. It provides about one third of the world's supply. Uranium is mined mainly in Northern Ontario and Saskatchewan. The mines in Saskatchewan provide the highest grade uranium.

Example 2 Solve Problems Involving Square Roots and Cube Roots

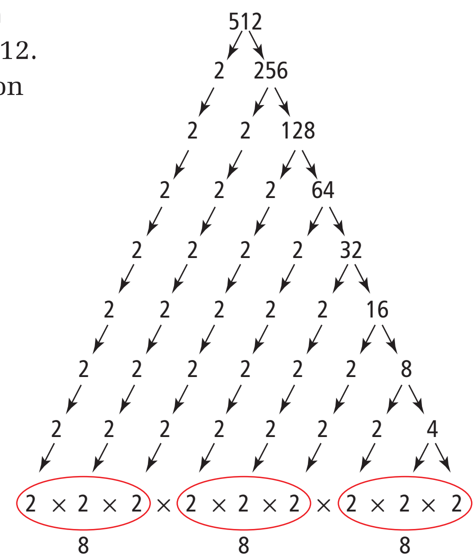
The uranium that Saskatchewan produces in a year has a volume of about 512 m^3. If this volume were made into a single cube, what would be the dimensions of the cube?

Solution

The volume of a cube of length x is given by $V = x^3$. Determine the dimensions of the cube, x, by calculating the cube root of the volume, or $x = \sqrt[3]{V}$.

Method 1: Use Prime Factorization

Determine the cube root of 512. Record the prime factorization for 512. Then, identify the factors that can be cubed to form 512.



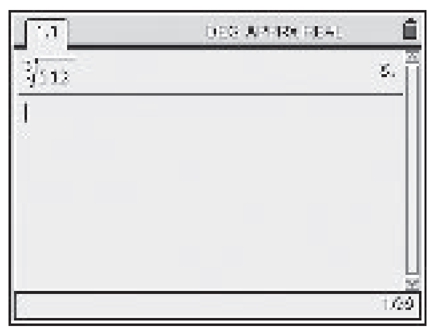
Since there are three equal groups, you know that 512 is a perfect cube.

How do you know that 512 is not a perfect square?

The cube root of 512 is 8.
The cube would be 8 m in length, height, and width.

Method 2: Use a Calculator

C 512 2nd $\sqrt[x]{y}$ 3 = 8.



The cube would be 8 m in length, height, and width.

Your Turn

a) A floor mat for gymnastics is a square with an area of 196 m^2. What is its side length?

b) The volume of a cubic box is 27 000 in.^{3} Use two methods to determine its dimensions.

Key Ideas

- A perfect square is the product of two equal factors. One of these factors is called the square root.

  36 is a perfect square: $\sqrt{36} = 6$ because $6^2 = 36$

- A perfect cube is the product of three equal factors, One of these factors is called the cube root.

  -125 is a perfect cube: $\sqrt[3]{-125} = -5$ because $(-5)^3 = -125$

- Numbers can be both perfect squares and perfect cubes.

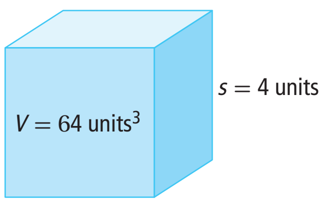
  15 625 is a perfect square: $125^2 = 15\ 625$

  15 625 is a perfect cube: $25^3 = 15\ 625$

- You can use diagrams or manipulatives, factor trees, or a calculator to solve problems involving square roots and cube roots.

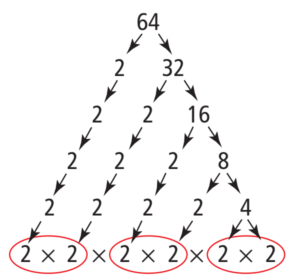
  Determine the cube root of 64.

  - Use a diagram.



The edge lengths represent the cube root:
(4)(4)(4) = 64.

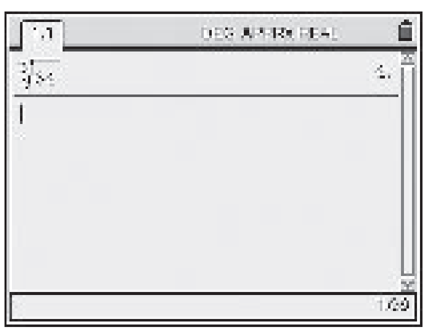
  - Use prime factorization.



There are three equal groups of 4. Therefore, the cube root of 64 is 4.

  - Use a calculator.

C 64 2nd $\sqrt[x]{y}$ 3 = 4.



Check Your Understanding

Practise

1. What is the value of each expression? Express your answers as integers or fractions.

a) 7^2 **b)** -50^2 **c)** $(-3)^2$

d) $\frac{4^2}{5}$ **e)** $\frac{3}{2^2}$ **f)** $\left(\frac{3}{4}\right)^2$

2. Evaluate. Give your answers as integers or fractions.

a) 2^3 **b)** -4^3 **c)** $(-5)^3$

d) $\frac{2^3}{4}$ **e)** $\frac{3}{6^3}$ **f)** $\left(\frac{2}{3}\right)^3$

3. What is the value of each expression?

a) $\sqrt{49}$ **b)** $\sqrt{169}$ **c)** $\sqrt{(25)(4)}$

d) $\frac{16}{\sqrt{64}}$ **e)** $\frac{\sqrt{36}}{3}$ **f)** $\sqrt{9x^2}$

4. Evaluate.

a) $\sqrt[3]{1}$ **b)** $\sqrt[3]{(8)(27)}$ **c)** $\sqrt[3]{8000}$

d) $\frac{\sqrt[3]{64}}{2}$ **e)** $\sqrt[3]{\frac{27}{125}}$ **f)** $\sqrt[3]{64a^3}$

5. Identify each number as a perfect square, a perfect cube, or both. Support your answers using a diagram or a factor tree.

a) 1 **b)** 1000 **c)** 81

d) 169 **e)** 216 **f)** 1024

6. State whether each of the following numbers is a perfect square, a perfect cube, both, or neither.

a) 144 **b)** 2197 **c)** 16

d) 225 **e)** 15 625 **f)** 117 649

7. Evaluate using prime factorization. Explain the process.

a) $\sqrt{100}$ **b)** $\sqrt[3]{8}$ **c)** $\sqrt{81}$

d) $\sqrt[3]{27}$ **e)** $\sqrt{144}$ **f)** $\sqrt{576}$

8. Calculate.

a) $\sqrt{196}$ **b)** $\sqrt[3]{4096}$ **c)** $\sqrt[3]{9261}$

d) $\sqrt[3]{3375}$ **e)** $\sqrt{961}$ **f)** $\sqrt[3]{4913}$

9. Connor needs to replace the edging on a square rug. If the rug has an area of 25 m^2, what length of edging does he need?

10. Serena collected all the garbage she created in one year. The volume of the cube it formed was 343 ft^3. What was the edge length of the cube?

Apply

11. A square wrestling mat has an area of 1444 ft^2.
   a) Before calculating the side length of the mat, estimate two whole numbers between which the answer falls. Which number do you think the answer is closer to?
   b) Calculate the side length.
   c) How does your estimate compare to the calculated answer?

12. Star quilts are squares with a minimum area of 1 m^2 and a maximum area of 9 m^2. What are the possible whole number dimensions of such a quilt?

Did You Know?

The star quilt is a pattern used by many cultures including the Lakota, Dakota, other Sioux nations, and Europeans. It was inspired from the design for buffalo robes. When buffalo were no longer available, the star quilt replaced the buffalo robe in Aboriginal traditions.

13. **Unit Project** The mural shown below was originally created to celebrate Alberta's Centennial in 2005. It was installed at the Centre d'arts visuels de l'Alberta in Edmonton, AB. The mural symbolizes the unity of the francophone communities throughout Alberta. Your art class decides to create a mural mosaic. Your mosaic will highlight the regions of the province or territory where you live.
   a) The class mosaic will be composed of 15-cm by 15-cm squares. How many squares will be needed to create a mural that covers an area of 2.7 m^2?
   b) Design a mural to show a geometric representation of square roots.
   c) How is the mural a geometric representation of square roots?

Les régions se racontent (The regions tell their story)

14. A recycling depot compresses cardboard into cubic bales. If each bale has a volume of 46 656 in.^{3}, what are its edge lengths?

15. **Unit Project** The cubic sculpture shown here is made of steel with copper leaf. It was created by Tony Bloom, an artist from Canmore, AB.

a) If it has a volume of 4913 in.^{3}, what is the length of one edge of the cube?

b) Explain how the sculpture is a geometric representation of a cube root.

16. The surface area of a die is 600 mm^2. What is the volume of the die?

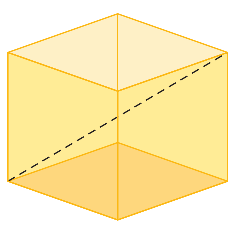
Extend

17. Meteorologists use the formula $D^3 = 684t^2$ to describe violent storms, such as tornadoes and hurricanes. D is the diameter of the storm, in kilometres, and t is the number of hours it will last.

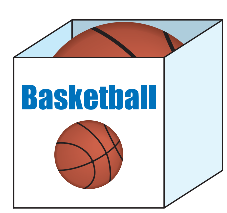
a) If a storm lasts for 4 h, what is its diameter?

b) If the diameter of a hurricane is 30 km, how long will it last?

18. A cube has a volume of 3375 cm^3. What is the diagonal distance through the cube from one corner to the opposite corner?



19. A manufacturer is designing an open, cube-shaped box to hold a basketball. The basketball has a volume of 2304π cm^3.

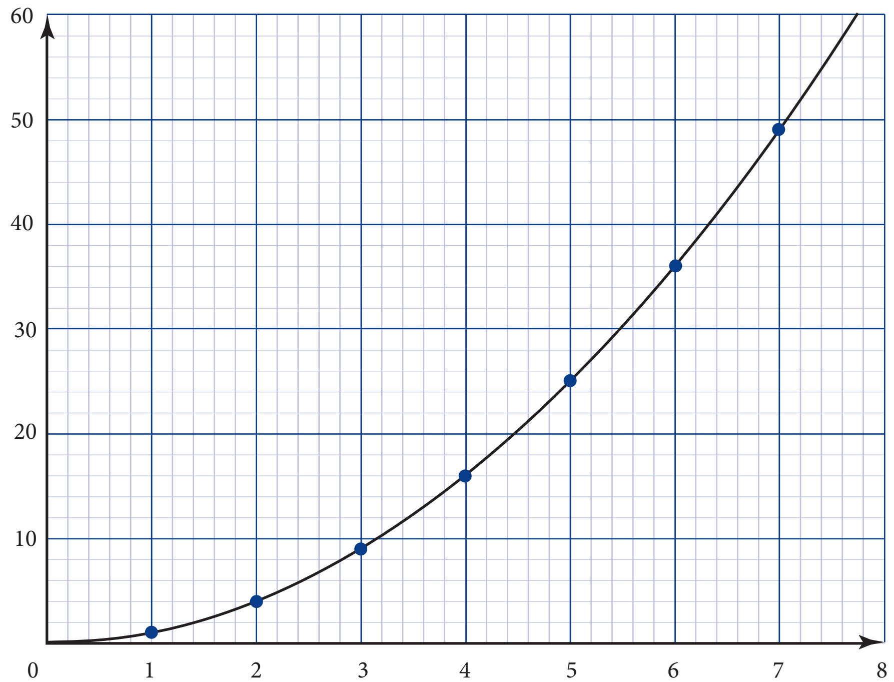


a) How much cardboard is needed to create the smallest box possible using the least amount of material? Do not include seam overlap in your calculations.

b) What is the volume of the box? What are its dimensions?

Create Connections

20. The following graph can be used to determine squares and square roots.



a) Use the graph to complete the following table of values.

Number	0	1	2	3			6	7	
Number Squared	0	1			16	25			64

b) Based on the table, how would you label the axes on the graph?

c) What does each small unit represent on the horizontal axis? vertical axis?

d) Explain how you could use the graph to find the value for 5^2.

e) How could you use the graph to evaluate $\sqrt{49}$?

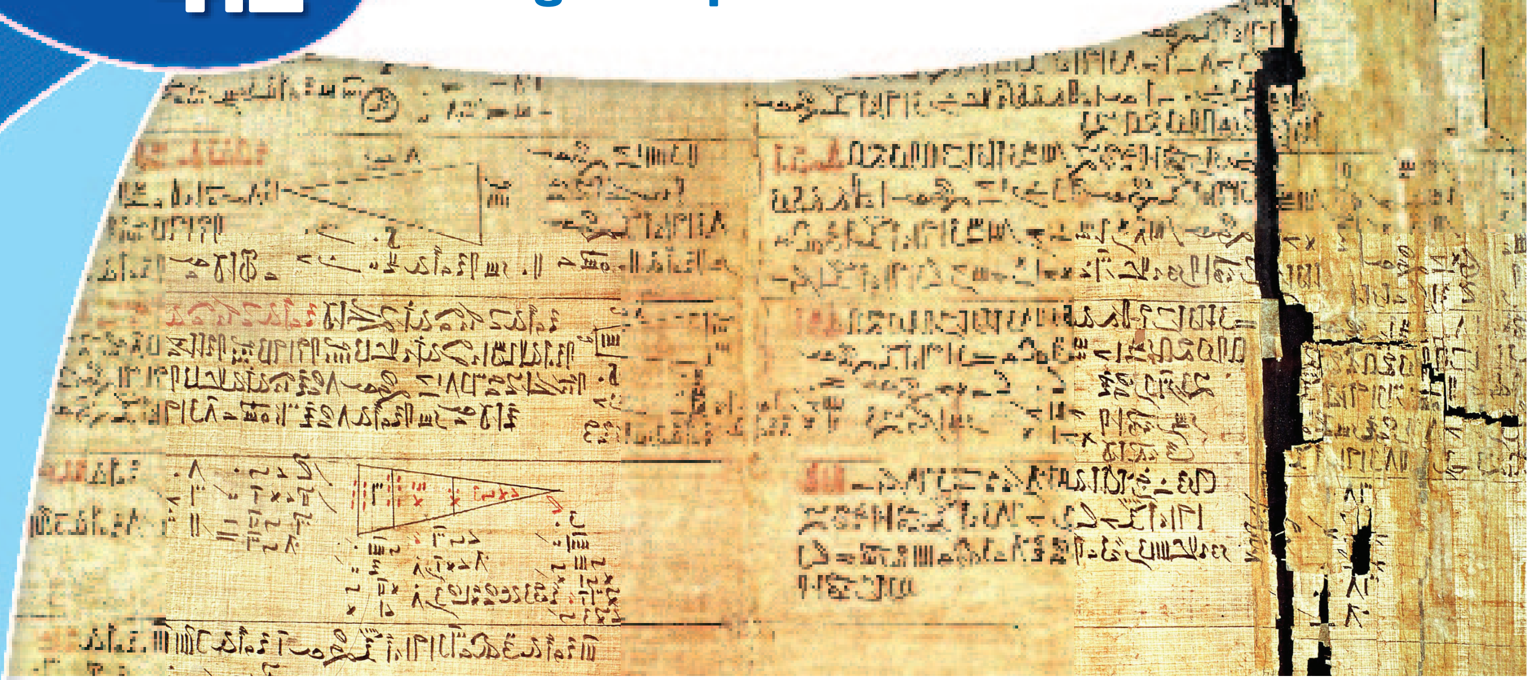
f) Show how you could use the graph to determine the approximate value for $\sqrt{18}$. Multiply your answer by itself. How close is your product to 18?

g) What is an approximation for $(6.2)^2$?

21. a) Make an arithmetic question, involving a square root, that has a value of $\frac{2}{3}$.

b) Make an arithmetic question, involving a cube root, that has a value of $\frac{2}{3}$.

4.2 Integral Exponents



Focus on ...

- applying the exponent laws to expressions using rational numbers or variables as bases and integers as exponents
- converting a power with a negative exponent to an equivalent power with a positive exponent
- solving problems that involve powers with integral exponents

The Rhind mathematical papyrus (RMP) is a valuable source of information about ancient Egyptian mathematics. This practical handbook includes problems that illustrate how Egyptians solved problems related to surveying, building, and accounting. The RMP was written in approximately 1650 B.C.E. How do archaeologists know this?

One way to assess the age of organic matter is by using carbon-14 dating. While they are living, all living things absorb radioactive carbon-14. Papyrus is made from papyrus plants. As soon as the papyrus dies, it stops taking in new carbon. The carbon-14 decays at a constant, known rate and is not replaced. Scientists can measure the amount of carbon-14 remaining. They use a formula involving exponents to accurately assess the age of the papyrus.

Is the quantity of carbon-14 increasing or decreasing? Do you think the exponent in the formula would be positive or negative? Why?

Did You Know?

Carbon-14 dating is accurate for dating artifacts up to about 60 000 years old.

Investigate Negative Exponents

Materials
- ruler

1. On a sheet of paper, draw a line 16 cm long and mark it as shown.

2. Mark a point halfway between 0 and 16. Label the point with its value and its equivalent value in exponential form (2^x). Repeat this procedure until you reach a value of 1 cm.
   a) How many times did you halve the line segment to reach 1 cm?
   b) What do you notice about the exponents as you keep reducing the line segment by half?

3. a) Mark the halfway point between 0 and 1. What fraction does this represent?
   b) Using the pattern established in step 2, what is the exponential form of the fraction?
   c) Halve the remaining line segment two more times.

4. Use a table to summarize the line segment lengths and the equivalent exponential form in base 2.

5. **Reflect and Respond**
   a) Describe the pattern you observe in the exponents as the distance is halved.
   b) Is there a way to rewrite each fraction so that it is expressed as a power with a positive exponent? Try it. Compare this form to the equivalent power with a negative exponent. What is the pattern?
   c) Create a general form for writing any power with a negative exponent as an equivalent power with a positive exponent.

6. a) Carbon-14 has a half-life of 5700 years. This means the rate of decay is $\frac{1}{2}$ or 2^{-1} every 5700 years. What fraction of carbon-14 would be present in organic material that is 11 400 years old? 17 100 years old? Express each answer as a power with a negative exponent. Explain how you arrived at your answers.
   b) Suggest types of situations when a negative exponent might be used.

Did You Know?

The *half-life* of a radioactive element is the amount of time it takes for half of the atoms in a sample to decay. The half-life of a radioactive element is constant. It does not depend upon the quantity or the amount of time that has gone by.

Link the Ideas

You can use the exponent laws to help simplify expressions with integral exponents.

Exponent Law	
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers	$(a^m)(a^n) = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = (a^m)(b^m)$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
Zero Exponent	$a^0 = 1, a \neq 0$

To simplify expressions with integral exponents, you can use the following principle as well as the exponent laws.

A power with a negative exponent can be written as a power with a positive exponent.

- $a^{-n} = \frac{1}{a^n}, a \neq 0$ $\quad 2^{-3} = \frac{1}{2^3}$
- $\frac{1}{a^{-n}} = a^n, a \neq 0$ $\quad \frac{1}{2^{-3}} = 2^3$

Example 1 Multiply or Divide Powers With the Same Base

Write each product or quotient as a power with a single exponent.

a) $(5^8)(5^{-3})$

b) $(0.8^{-2})(0.8^{-4})$

c) $\frac{x^5}{x^{-3}}$

d) $\frac{(2x)^3}{(2x)^{-2}}$

Solution

Use the exponent laws for multiplying or dividing powers with the same base and integral exponents.

a) Method 1: Add the Exponents

$$(5^8)(5^{-3}) = 5^{8 + (-3)}$$
$$= 5^5$$

How do you know that you can add the exponents?

Method 2: Use Positive Exponents

Convert the power with a negative exponent to one with a positive exponent. Rewrite as a division statement. Then, since the bases are the same, you can subtract the exponents.

$$\begin{aligned}(5^8)(5^{-3}) &= (5^8)\left(\frac{1}{5^3}\right)\\ &= \frac{5^8}{5^3}\\ &= 5^{8-3}\\ &= 5^5\end{aligned}$$

How do you know that you can subtract the exponents?

b) **Method 1: Add the Exponents**

$$\begin{aligned}(0.8^{-2})(0.8^{-4}) &= 0.8^{-2+(-4)}\\ &= 0.8^{-6}\end{aligned}$$

Method 2: Use Positive Exponents

$$\begin{aligned}(0.8^{-2})(0.8^{-4}) &= \left(\frac{1}{0.8^2}\right)\left(\frac{1}{0.8^4}\right)\\ &= \frac{1}{(0.8^2)(0.8^4)}\\ &= \frac{1}{0.8^{(2+4)}}\\ &= \frac{1}{0.8^6}\end{aligned}$$

Which method do you prefer? Why?

c) $$\begin{aligned}\frac{x^5}{x^{-3}} &= x^{5-(-3)}\\ &= x^{5+3}\\ &= x^8\end{aligned}$$

What strategy was used?

d) **Method 1: Subtract the Exponents**

$$\begin{aligned}\frac{(2x)^3}{(2x)^{-2}} &= (2x)^{3-(-2)}\\ &= (2x)^5\end{aligned}$$

Method 2: Use Positive Exponents

$$\begin{aligned}\frac{(2x)^3}{(2x)^{-2}} &= (2x)^3\,(2x)^2\\ &= (2x)^{3+2}\\ &= (2x)^5\end{aligned}$$

Your Turn

Simplify each product or quotient.

a) $(2^{-3})(2^5)$

b) $\dfrac{7^{-5}}{7^3}$

c) $\dfrac{(-3.5)^4}{(-3.5)^{-3}}$

d) $\dfrac{(3y)^2}{(3y)^{-6}}$

Example 2 Powers of Powers

Write each expression as a power with a single, positive exponent. Then, evaluate where possible.

a) $(4^3)^{-2}$ **b)** $[(a^{-2})(a^0)]^{-1}$ **c)** $\left(\dfrac{2^4}{2^6}\right)^{-3}$ **d)** $\left[\left(\dfrac{3}{4}\right)^{-2}\left(\dfrac{3}{4}\right)^{-2}\right]^{-2}$

Solution

a) Multiply the exponents. Then, rewrite as a positive exponent.

$$\begin{aligned}(4^3)^{-2} &= 4^{(3)(-2)}\\ &= 4^{-6}\\ &= \frac{1}{4^6}\\ &= \frac{1}{4096}\end{aligned}$$

b) Since the bases are the same, you can multiply the powers by adding the exponents. Raise the result to the exponent -1. Then, multiply.

$$\begin{aligned}[(a^{-2})(a^0)]^{-1} &= (a^{-2+0})^{-1}\\ &= (a^{-2})^{-1}\\ &= a^{(-2)(-1)}\\ &= a^2\end{aligned}$$

How could you use your knowledge of the exponent laws for zero exponents to help simplify the original expression?

c) **Method 1: Simplify Within the Brackets**

Since the bases are the same, you can subtract the exponents. Raise the result to the exponent -3. Then, multiply.

$$\begin{aligned}\left(\frac{2^4}{2^6}\right)^{-3} &= [2^{(4-6)}]^{-3}\\ &= (2^{-2})^{-3}\\ &= 2^{(-2)(-3)}\\ &= 2^6\\ &= 64\end{aligned}$$

Method 2: Raise Each Power to an Exponent

Raise each power to the exponent -3. Then, divide the resulting powers by subtracting the exponents, since they have the same base.

$$\begin{aligned}\left(\frac{2^4}{2^6}\right)^{-3} &= \frac{(2^4)^{-3}}{(2^6)^{-3}}\\ &= \frac{2^{(4)(-3)}}{2^{(6)(-3)}}\\ &= \frac{2^{-12}}{2^{-18}}\\ &= 2^{-12-(-18)}\\ &= 2^6\\ &= 64\end{aligned}$$

Which method do you prefer? Why?

Did You Know?

John Wallis was a professor of geometry at Oxford University in England in 1655. He was the first to explain the significance of zero and negative exponents. He also introduced the current symbol for infinity, ∞.

d) Add the exponents. Raise the resulting power to the exponent -2.

$$\left[\left(\frac{3}{4}\right)^{-2}\left(\frac{3}{4}\right)^{4}\right]^{-2} = \left[\left(\frac{3}{4}\right)^{-2+4}\right]^{-2}$$
$$= \left[\left(\frac{3}{4}\right)^{2}\right]^{-2}$$
$$= \left(\frac{3}{4}\right)^{(2)(-2)}$$
$$= \left(\frac{3}{4}\right)^{-4}$$
$$= \frac{1}{\left(\frac{3}{4}\right)^{4}}$$
$$= \left(\frac{4}{3}\right)^{4}$$
$$= \frac{256}{81}$$

How do you know that you can add the exponents?

Why is base now $\frac{4}{3}$ instead of the original base of $\frac{3}{4}$?

Is it true in all cases that you can express a rational number with a negative exponent as its reciprocal with a positive exponent? Try it out.

Your Turn

Simplify and evaluate where possible.

a) $[(0.6^3)(0.6^{-3})]^{-5}$ **b)** $[(t^{-4})(t^3)]^{-3}$ **c)** $\left(\frac{x^6}{x^4}\right)^{-2}$ **d)** $\left[\frac{(y^2)^0}{(y)^3}\right]^{-3}$

Example 3 Apply Powers With Integral Exponents

It is estimated that there are 117 billion grasshoppers in an area of 39 000 km² of Saskatchewan. Approximately how many grasshoppers are there per square kilometre?

Solution

Method 1: Use Arithmetic

Divide the number of grasshoppers by the total area.

$$\text{grasshoppers per square kilometre} = \frac{117\,000\,000\,000}{39\,000}$$
$$= 3\,000\,000$$

There are approximately 3 000 000 grasshoppers per square kilometre.

Method 2: Use Exponent Rules

Since you cannot enter numbers as large as 117 billion directly into most calculators, rewrite them using exponential form. Then, use the exponent rules to calculate the power of 10.

$$\text{grasshoppers per square kilometre} = \frac{(117)(10^9)}{(39)(10^3)}$$
$$= (3)(10^{(9-3)})$$
$$= (3)(10^6)$$

There are approximately 3 000 000 grasshoppers per square kilometre.

Is it possible to enter numbers expressed using exponential form directly into your calculator? How would doing this help calculate the answer?

Did You Know?

The clear-winged grasshopper is a pest of grasses and cereal grain crops. These insects can completely destroy barley and wheat fields early in the season. Agricultural field workers conduct grasshopper surveys and produce forecasts to help assess the need for control measures to protect crops.

Your Turn

Manitoba Agriculture, Food and Rural Initiatives staff conducted a grasshopper count. In one 25-km^2 area, there were 401 000 000 grasshoppers. Use the following table to assess the degree of grasshopper infestation in this area.

Grasshopper Density
0–4 per square metre = very light
4–8 per square metre = light
8–12 per square metre = moderate
12–24 per square metre = severe
24 per square metre = very severe

Key Ideas

- A power with a negative exponent can be written as a power with a positive exponent.

$$3^{-4} = \frac{1}{3^4} \qquad \left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \left(\frac{3}{2}\right)^2 \qquad \frac{1}{2^{-5}} = 2^5$$

- You can apply the above principle to the exponent laws.

Exponent Law	Example
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	$(3^{-2})(3^4) = 3^{-2+4}$ $= 3^2$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{x^3}{x^{-5}} = x^{3-(-5)}$ $= x^8$
Power of a Power $(a^m)^n = a^{mn}$	$(0.75^4)^{-2} = 0.75^{(4)(-2)}$ $= 0.75^{-8}$ or $\frac{1}{0.75^8}$
Power of a Product $(ab)^m = (a^m)(b^m)$	$(4z)^{-3} = \frac{1}{(4z)^3}$ or $\frac{1}{64z^3}$
Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$	$\left(\frac{t}{3}\right)^{-2} = \left(\frac{3}{t}\right)^2$ $= \frac{3^2}{t^2}$ $= \frac{9}{t^2}$
Zero Exponent $a^0 = 1, a \neq 0$	$(4y^2)^0 = 1$ $-(4y^2)^0 = -1$

Check Your Understanding

Practise

1. For each situation, identify when a positive and a negative exponent would be used.

a) calculating the population growth of a city since 2005 using the expression $150\,000(1.005)^n$

b) calculating the amount of a radioactive substance remaining from a known sample amount using the expression $25\left(\frac{1}{2}\right)^n$

c) determining how many bacteria are present in a culture after h hours using the expression $500(2)^h$

2. Write each expression with positive exponents.

a) b^{-3} **b)** xy^{-4} **c)** $2x^{-2}$

d) $2x^2y^{-1}$ **e)** $-4x^{-5}$ **f)** $-2x^{-3}y^{-4}$

3. Daniel was rewriting the expression $\frac{2x^{-3}}{y^5}$ with positive exponents. He quickly recorded $\frac{2}{x^3y^5}$. Is his answer correct? Justify your answer.

4. Simplify each expression. State the answer using positive exponents.

a) $(4^3)(4^{-5})$ **b)** $\frac{3^{-4}}{3^{-2}}$ **c)** $\frac{12^3}{12^7}$

d) $\left(\frac{8^{-1}}{8^0}\right)^3$ **e)** $(5^4)^{-2}$ **f)** $[(3^2)(2^{-5})]^3$

g) $\left(\frac{5^2}{4^2}\right)^{-1}$ **h)** $(3.2^{-2})^{-3}$ **i)** $4[(2)^{-1}(2)^{-2}]^{-1}$

5. Simplify each expression by restating it using positive exponents only.

a) $\frac{1}{s^2t^{-6}}$ **b)** $[(h)^7(h)^{-2}]^{-2}$ **c)** $\frac{8t}{t^{-3}}$

d) $(2x^{-4})^3$ **e)** $\left(\frac{n^4}{n^{-4}}\right)^{-3}$ **f)** $[(xy^4)^{-3}]^{-2}$

6. Simplify, then evaluate. Express your answers to four decimal places, where necessary.

a) $(0.5^2)^{-3}$ **b)** $\left[\left(\frac{2}{3}\right)^3\right]^{-3}$ **c)** $[(5)(5^3)]^{-1}$

d) $\left(\frac{6^4}{6^4}\right)^{-3}$ **e)** $\left(\frac{8}{8^3}\right)^{-4}$ **f)** $\left[\left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{4}\right)^2\right]^{-1}$

7. A mountain pine beetle population can double every year if conditions are ideal. Assume the forest in Jasper National Park, AB, has a population of 20 000 beetles. The formula $P = 20\,000(2)^n$ can model the population, P, after n years.

   a) How many beetles were there in the forest four years ago? eight years ago?

   b) If the conditions remain ideal, how many beetles will there be two years from now?

8. French-language publishing sales in Canada increased by a growth rate of 1.05 per year from 1996 to 2000. There were sales of \$300 000 in 1996. The formula $S = 300\,000(1.05)^n$ models the sales, S, after n years. Assume that the growth rate stays constant. What would be the projected sales for 2010?

Apply

9. The bacterium *Escherichia coli* is commonly found in the human intestine. A single bacterium has a width of 10^{-3} mm. The head of a pin has a diameter of 1 mm. How many *Escherichia coli* bacteria can fit across the diameter of a pin?

10. A culture of bacteria in a lab contains 2000 bacterium cells. The number of cells doubles every day. This relationship can be modelled by the equation $N = 2000(2)^t$, where N is the estimated number of bacteria cells and t is the time in days.

    a) How many cells were present for each amount of time?

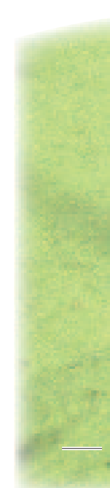
    i) after two days

    ii) after one week

    iii) two days ago

    b) What does $t = 0$ indicate?

11. The Great Galaxy in Andromeda is about 2 200 000 light years from Earth. Light travels 5 900 000 000 000 miles in a year. How many miles is the Great Galaxy in Andromeda from Earth?



12. A red blood cell is about 0.0025 mm in diameter. How large would it appear if it were magnified 10^8 times?

13. Wildlife biologists are tracking the whooping crane population growth at Wood Buffalo National Park, AB. The crane population increased by a growth rate of 7.3% per year from 2002 to 2008. There were 174 whooping cranes in 2002. The rate of growth can be modelled using the formula $P = 174(1.073)^n$, where P is the estimated population and n is the number of years. If conditions remain constant, what is the projected crane population
  a) in 2014?
  b) in 2011?

Did You Know?

The whooping crane is an endangered species. One of the three wild populations of whooping cranes in North America summers at Wood Buffalo National Park in Alberta. It winters in Texas. Projects are ongoing to help the wild population recover.

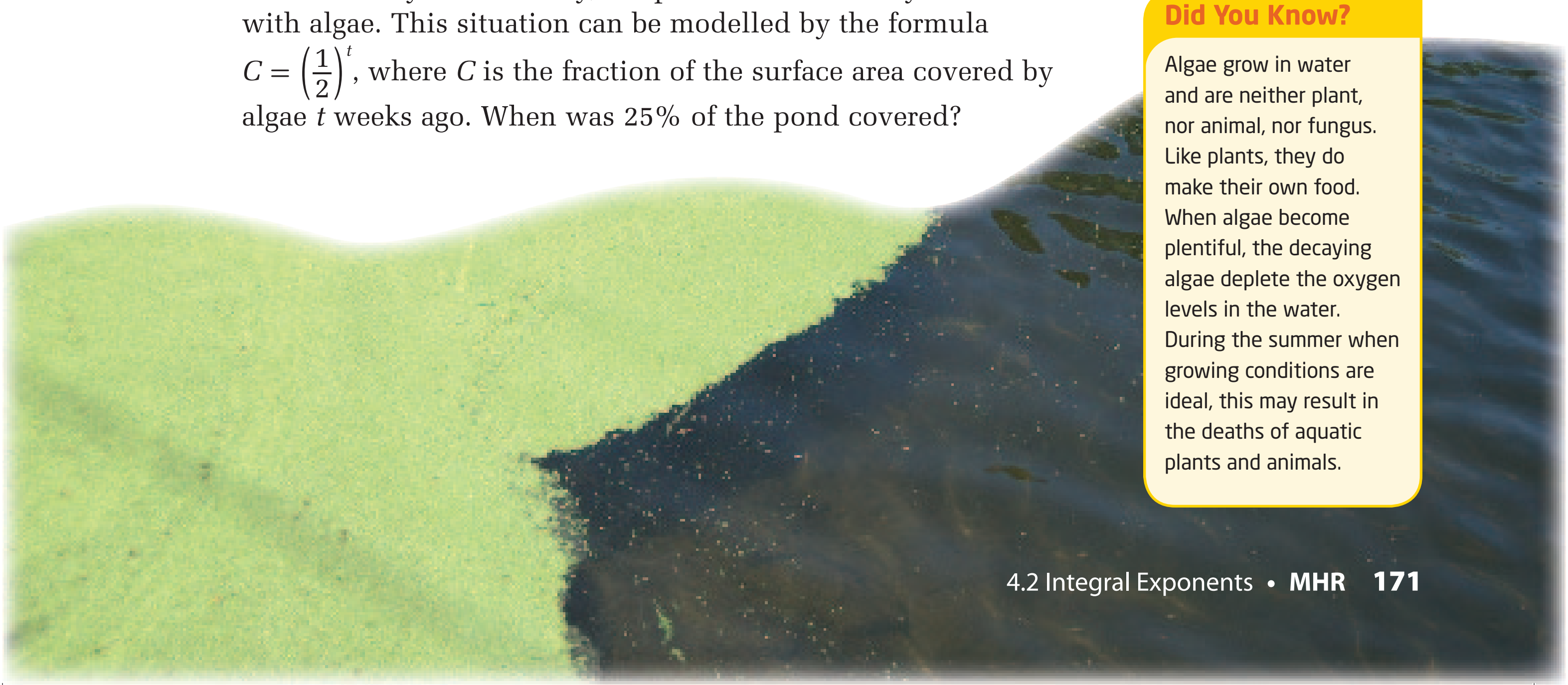
14. There are approximately $(3.2)(10^{18})$ atoms in 1 mg of lead. How many atoms are there in a kilogram of lead? Hint: 1 kg $= 10^6$ mg.

15. Over time, all rechargeable batteries lose their charge, even when not in use. A 12-volt nickel-metal hydride (NiMH) battery, commonly used in power tools, will lose approximately 30% of its charge every month if not recharged. This situation can be modelled by the formula $V = 12(0.70^m)$, where V is the estimated voltage of the battery in volts, and m is the number of months the battery is not used. What is the estimated voltage of an unused battery after 3 months? Assume the battery was initially fully charged.

16. The fraction of the surface area of a pond covered by algae cells doubles every week. Today, the pond surface is fully covered with algae. This situation can be modelled by the formula $C = \left(\frac{1}{2}\right)^t$, where C is the fraction of the surface area covered by algae t weeks ago. When was 25% of the pond covered?

Did You Know?

Algae grow in water and are neither plant, nor animal, nor fungus. Like plants, they do make their own food. When algae become plentiful, the decaying algae deplete the oxygen levels in the water. During the summer when growing conditions are ideal, this may result in the deaths of aquatic plants and animals.



17. Abby, Kevin, and Caleigh are organizing a 12-hour Famine to raise money to help children in developing countries. Participants are to collect pledges in one of two ways. They can ask for a flat rate of \$30 per pledge or they can ask for a pledge of \$0.01 for the first hour and then, every hour after that, double the pledge from the previous hour. Each hourly pledge using the doubling approach can be modelled by the formula $P = 0.01(2)^h$, where P is the hourly pledge amount and h is the number of hours of participation in the famine.

a) Which pledge approach do you think would raise more money?

b) If the maximum number of hours students can participate in the famine is 12, what is the pledge amount for the last hour?

Did You Know?

A team of Canadian scientists learned previously that crude oil spilled on beaches in Nova Scotia did not break down, due to poor soil conditions. The low nutrient concentrations in the soil limited the growth of natural bacteria. Adding fertilizer to the soil increased the rate of bacterial growth. The scientists applied what they learned to the oil spill in the Arctic, and it worked.

18. Following the 1989 Exxon Valdez oil spill, 100 km of Arctic shoreline was contaminated. Crude oil is made up of thousands of compounds. It takes many different kinds of naturally occurring bacteria to break the oil down. Lab technicians identified and counted the bacteria. They monitored how well the oil was degrading. More bacteria and less oil were signs that the shoreline was recovering. The number of bacteria needed to effectively break down an oil spill is 1 000 000 per millilitre of oil. The bacteria double in number every two days. The starting concentration of bacteria is 1000 bacteria per millimetre. This situation can be modelled by the equation $C = 1000(2^d)$, where C is the estimated concentration of bacteria and d is the number of 2-day periods the bacteria grow. Approximately how long would it take for the bacteria to reach the required concentration?

19. From 2001 to 2006 the population of Lloydminster increased at an average annual rate of 2.52%. This can be modelled using the formula $P = 13\ 145(1.0252)^n$, where P is the estimated population and n is the number of years since 2001.

a) What was the population of Lloydminster in 2004?

b) If this rate of increase stays the same, what will the population be in 2012?

Extend

20. Calculate the value of x that makes each statement true.

a) $x^{-4} = \dfrac{81}{16}$

b) $\left(\dfrac{1}{3}\right)^x = 81$

c) $\left(\dfrac{3}{4}\right)^x = \dfrac{64}{27}$

d) $(-5)^x = \dfrac{1}{25}$

21. Use your knowledge of equivalent powers to evaluate $\left(\dfrac{2^5}{8^2}\right)^{-2}$.

22. A fundraising committee plans to donate \$64 000 to six community agencies as follows. The first agency will receive $\frac{1}{2}$. The second agency will receive $\frac{1}{2}$ of what is left. The third agency will receive $\frac{1}{2}$ of what is left, and so on down the line.

a) What fraction of the money will each agency get?

b) How much money will each agency get?

c) Will there be any remaining money after the committee makes these donations? If so, how much?

d) According to the pattern, how many agencies could be supported if no agency is to receive less than \$125?

23. The intensity of light from a stage light decreases exponentially with the thickness of the coloured gels covering it. The intensity, I, in watts per square centimetre, can be calculated using the formula $I = 1200\left(\frac{4}{5}\right)^n$, where n is the number of coloured gels used. What is the intensity of light with

a) no gels? **b)** 2 gels? **c)** 4 gels?

24. The number, N, of radium atoms remaining in a sample that started at 400 atoms can be represented by the equation $N = 400(2)^{\frac{-t}{1600}}$, where t is time, in years.

a) How many atoms are left after 3200 years?

b) What does $t = 0$ represent?

c) What do negative values of t represent?

Create Connections

25. Use a pattern of your choice. Describe the relationship between a negative exponent and its equivalent form with a positive exponent.

26. Describe a problem where negative exponents are used to model a real-life situation. What does the negative exponent represent in your problem?

27. a) Which power is larger: 3^5 or 3^4? Explain how you know. What can you conclude about comparing powers with the same base?

b) Develop an example that shows how to compare powers with the same exponents and different bases. What can you conclude about comparing powers with the same exponents?

c) Which of the following powers is the greatest? How do you know? Arrange the powers from least to greatest.

2^{666} 3^{555} 4^{444} 5^{333} 6^{222}

4.3 Rational Exponents

Focus on ...

- applying the exponent laws to expressions using rational numbers or variables as bases and rational exponents
- solving problems that involve powers with rational exponents

On a piano keyboard, the pitches of any two adjacent keys are related by a ratio equal to $\sqrt[12]{2}$. This is defined as the number that, when multiplied by itself 12 times, results in 2. You may have noticed that there is no $\sqrt[12]{}$ key on your calculator. How can a piano technician evaluate this number?

Roots other than the square root often occur in science, technology, music, art, and other disciplines. How can you represent such roots in a way that makes them easy to work with?

Investigate Rational Exponents

1. According to the product rule for powers

$$\begin{aligned}\left(9^{\frac{1}{2}}\right)\left(9^{\frac{1}{2}}\right) &= 9^{\frac{1}{2}+\frac{1}{2}}\\ &= 9^1\\ &= 9\end{aligned}$$

You can reverse these statements to get

$$\begin{aligned}9^1 &= 9^{\frac{1}{2}+\frac{1}{2}}\\ &= \left(9^{\frac{1}{2}}\right)\left(9^{\frac{1}{2}}\right)\end{aligned}$$

What is the value of $9^{\frac{1}{2}}$? Check your answer with a calculator.

2. Predict values for $4^{\frac{1}{2}}$, $16^{\frac{1}{2}}$, $36^{\frac{1}{2}}$, and $49^{\frac{1}{2}}$. Use a calculator to check your predictions. Were you correct?

3. Predict the value of $8^{\frac{1}{3}}$. Explain your thinking. Check your prediction.

4. Reflect and Respond

a) Explain how determining $49^{\frac{1}{2}}$ and your definition for square root are related.

b) Express the 12th root of 2 as a power. Evaluate using your calculator. Express the answer to six decimal places.

c) Use your calculator to determine the 12th power of your answer to part b). Explain why the answer is not 2.

Link the Ideas

You can use the exponent laws to help simplify expressions with rational exponents.

Exponent Law	
Note that a and b are rational or variable bases and m and n are rational exponents.	
Product of Powers	$(a^m)(a^n) = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = (a^m)(b^m)$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
Zero Exponent	$a^0 = 1, a \neq 0$

To simplify expressions with rational exponents, you can use the following principle as well as the exponent laws.

- $a^{-n} = \frac{1}{a^n}, a \neq 0$ $\quad 3^{-0.2} = \frac{1}{3^{0.2}}$
- $\frac{1}{a^{-n}} = a^n, a \neq 0$ $\quad \frac{1}{3^{-0.2}} = 3^{0.2}$

Example 1 Multiply or Divide Powers With the Same Base

Write each product or quotient as a power with a single exponent.

a) $\left(5^{\frac{1}{3}}\right)\left(5^{\frac{5}{3}}\right)$ **b)** $(x^5)\left(x^{-\frac{1}{2}}\right)$ **c)** $\frac{3^{-\frac{3}{4}}}{3^{0.25}}$ **d)** $\frac{8^{1.8}}{16^{0.3}}$

Solution

Use the exponent laws for multiplying or dividing powers with the same base and rational exponents.

a) Since the bases are the same, you can add the exponents.

$$\begin{aligned}\left(5^{\frac{1}{3}}\right)\left(5^{\frac{5}{3}}\right) &= 5^{\left(\frac{1}{3}+\frac{5}{3}\right)}\\ &= 5^{\frac{6}{3}}\\ &= 5^2\end{aligned}$$

b) Since the bases are the same, you can add the rational exponents.

$$\left(x^5\right)\left(x^{-\frac{1}{2}}\right) = x^{\left[5 + \left(-\frac{1}{2}\right)\right]}$$
$$= x^{\frac{9}{2}}$$

Convert to $x^{\left[\frac{10}{2} + \left(-\frac{1}{2}\right)\right]}$.

c) Convert the rational exponents so both are fractions or decimal numbers. Then, since the bases are the same, you can subtract the exponents.

$$\frac{3^{-\frac{3}{4}}}{3^{0.25}} = \frac{3^{-0.75}}{3^{0.25}}$$
$$= 3^{-0.75 - 0.25}$$
$$= 3^{-1} \text{ or } \frac{1}{3}$$

d) Convert to the same base. Then, subtract the exponents.

$$\frac{8^{1.8}}{16^{0.3}} = \frac{\left(2^3\right)^{1.8}}{\left(2^4\right)^{0.3}}$$
$$= \frac{2^{5.4}}{2^{1.2}}$$
$$= 2^{4.2}$$

How can you use powers of 2 to convert to the same base?

Your Turn

Write each expression as a power with a single exponent.

a) $\left(x^{1.5}\right)\left(x^{3.5}\right)$ **b)** $\left(p^{\frac{-5}{4}}\right)\left(p^{\frac{1}{2}}\right)$

c) $\dfrac{4^{\frac{1}{2}}}{4^{0.5}}$ **d)** $\dfrac{1.5^{\frac{4}{3}}}{1.5^{\frac{1}{6}}}$

Example 2 Simplify Powers With Rational Exponents

Write each expression as a power with a single, positive exponent. Then, evaluate where possible.

a) $\left(4x^3\right)^{0.5}$ **b)** $\left[\left(x^3\right)\left(x^{\frac{3}{2}}\right)\right]^{\frac{1}{2}}$ **c)** $\left(\dfrac{3^4}{16}\right)^{-0.75}$

Solution

a) Raise each term to the exponent, then multiply the exponents.

$$\left(4x^3\right)^{0.5} = \left(4^{0.5}\right)\left(x^3\right)^{0.5}$$
$$= 2x^{(3)(0.5)}$$
$$= 2x^{1.5} \text{ or } 2x^{\frac{3}{2}}$$

What is the value of $4^{0.5}$?

b) Method 1: Add the Exponents

Since the bases are the same, you can add the exponents.
Raise the result to the exponent $\frac{1}{2}$. Then, multiply.

$$\begin{aligned}\left[(x^3)\left(x^{\frac{3}{2}}\right)\right]^{\frac{1}{2}} &= \left(x^{\left(3+\frac{3}{2}\right)}\right)^{\frac{1}{2}}\\ &= \left(x^{\frac{9}{2}}\right)^{\frac{1}{2}}\\ &= x^{\left(\frac{9}{2}\right)\left(\frac{1}{2}\right)}\\ &= x^{\frac{9}{4}}\end{aligned}$$

$$\begin{aligned}3 + \frac{3}{2} &= \frac{6}{2} + \frac{3}{2}\\ &= \frac{9}{2}\end{aligned}$$

Method 2: Apply Power of a Power

Raise each power to the exponent $\frac{1}{2}$. Then, add the exponents of the resulting powers.

$$\begin{aligned}\left[(x^3)\left(x^{\frac{3}{2}}\right)\right]^{\frac{1}{2}} &= \left[(x^3)^{\frac{1}{2}}\right]\left[\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}}\right]\\ &= \left[x^{(3)\left(\frac{1}{2}\right)}\right]\left[x^{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}\right]\\ &= \left(x^{\frac{3}{2}}\right)\left(x^{\frac{3}{4}}\right)\\ &= x^{\left(\frac{6}{4}+\frac{3}{4}\right)}\\ &= x^{\frac{9}{4}}\end{aligned}$$

c) Convert the base to a single fraction with the same exponent.
Then, raise the result to the exponent $-\frac{3}{4}$.

$$\begin{aligned}\left(\frac{3^4}{16}\right)^{-0.75} &= \left(\frac{3^4}{2^4}\right)^{-\frac{3}{4}}\\ &= \left[\left(\frac{3}{2}\right)^4\right]^{-\frac{3}{4}}\\ &= \left(\frac{3}{2}\right)^{(4)\left(-\frac{3}{4}\right)}\\ &= \left(\frac{3}{2}\right)^{-3}\\ &= \left(\frac{2}{3}\right)^{3}\\ &= \frac{8}{27}\end{aligned}$$

Why is $\left(\frac{3}{2}\right)^{-3}$ the same as $\left(\frac{2}{3}\right)^{3}$?

Your Turn

Simplify and evaluate where possible.

a) $(27x^6)^{\frac{2}{3}}$ **b)** $\left[\left(t^{\frac{4}{3}}\right)\left(t^{\frac{1}{3}}\right)\right]^9$ **c)** $\left(\frac{x^3}{64}\right)^{-\frac{2}{3}}$

Example 3 Apply Powers With Rational Exponents

Food manufacturers use a beneficial bacterium called *Lactobacillus bulgaricus* to make yoghurt and cheese. The growth of 10 000 bacteria can be modelled using the formula $N = 10\ 000(2)^{\frac{h}{42}}$, where N is the number of bacteria after h hours.

a) What does the value 2 in the formula tell you?

b) How many bacteria are present after 42 h?

c) How many more bacteria are present after 2 h?

d) How many bacteria are present after 105 h?

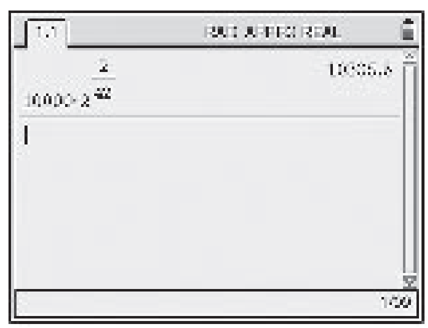
Solution

a) The value 2 indicates that the number of bacteria doubles every 42 h.

b) Substitute the value $h = 42$ into the formula and evaluate.

$N = 10\ 000(2)^{\frac{42}{42}}$
$N = 10\ 000(2)^1$
$N = 20\ 000$
There are 20 000 bacteria after 42 h.

c) Substitute the value $h = 2$ into the formula and evaluate.



$N = 10\ 000(2)^{\frac{2}{42}}$
$N = 10\ 000(1.033\ 558\ldots)$
$N = 10335.58\ldots$
$10\ 335.58\ldots - 10\ 000 = 335.58\ldots$

Why do you subtract 10 000?

There are approximately 336 more bacteria after 2 h.

d) Substitute the value $h = 105$ into the formula and evaluate.

$N = 10\ 000(2)^{\frac{105}{42}}$
$N = 10\ 000(5.656\ 854\ldots)$
$N = 56\ 568.54\ldots$
There are approximately 56 569 bacteria after 105 h.

Your Turn

Cody invests \$5000 in a fund that increases in value at the rate of 12.6% per year. The bank provides a quarterly update on the value of the investment using the formula $A = 5000(1.126)^{\frac{q}{4}}$, where q represents the number of quarterly periods and A represents the final amount of the investment.

a) What is the relationship between the interest rate of 12.6% and the value 1.126 in the formula?

b) What is the value of the investment after the 3rd quarter?

c) What is the value of the investment after 3 years?

Key Ideas

- You can write a power with a negative exponent as a power with a positive exponent.

  $(-9)^{-1.3} = \frac{1}{(-9)^{1.3}}$ $\qquad \frac{1}{2^{-3.2}} = 2^{3.2}$

- You can apply the above principle to the exponent laws for rational exponents.

Exponent Law	Example
Note that a and b are rational or variable bases and m and n are rational exponents	
Product of Powers $(a^m)(a^n) = a^{m+n}$	$\left(x^{\frac{3}{5}}\right)\left(x^{\frac{6}{5}}\right) = x^{\frac{3}{5}+\frac{6}{5}}$ $= x^{\frac{9}{5}}$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4s^{2.5}}{12s^{0.5}} = \frac{1}{3}s^{(2.5-0.5)}$ $= \frac{1}{3}s^2$ or $\frac{s^2}{3}$
Power of a Power $(a^m)^n = a^{mn}$	$(t^{3.3})^{\frac{1}{3}} = t^{(3.3)\left(\frac{1}{3}\right)}$ $= t^{1.1}$
Power of a Product $(ab)^m = (a^m)(b^m)$	$\left(8x^{\frac{1}{2}}\right)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}\left(x^{\frac{1}{2}}\right)^{\frac{2}{3}}$ $= 4x^{\frac{2}{6}}$ or $4x^{\frac{1}{3}}$ How does expressing 8 as 2^3 help simplify?
Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$	$\left(\frac{x^3}{y^6}\right)^{\frac{1}{3}} = \frac{(x^3)^{\frac{1}{3}}}{(y^6)^{\frac{1}{3}}}$ $= \frac{x}{y^2}$
Zero Exponent $a^0 = 1, a \neq 0$	$(-2)^0 = 1$ $-2^0 = -1$

- A power with a rational exponent can be written with the exponent in decimal or fractional form.

  $x^{\frac{3}{5}} = x^{0.6}$

Check Your Understanding

Practise

1. Use the exponent laws to simplify each expression. Where possible, compute numerical values.

a) $(x^3)\left(x^{\frac{7}{3}}\right)$ **b)** $\left(b^{\frac{1}{5}}\right)\left(b^{\frac{9}{5}}\right)$ **c)** $(a^2)^{\frac{3}{2}}$

d) $(k^{4.8})(k^3)$ **e)** $(16)^{0.25}$ **f)** $\left(\frac{-8a^6}{27}\right)^{\frac{1}{3}}$

g) $\left(2x^{\frac{1}{3}}\right)\left(-4x^{\frac{5}{3}}\right)$ **h)** $(9x^2)^{\frac{3}{2}}$ **i)** $(25x^2)^{0.5}$

2. Use the exponent laws to simplify each expression. Leave your answers with positive exponents.

a) $(x^3)\left(x^{\frac{-2}{3}}\right)$ **b)** $(81^{-0.25})^3$ **c)** $\frac{(m^{-2})^{\frac{2}{3}}}{\left(m^{\frac{1}{2}}\right)^4}$

d) $(9p^2)^{-\frac{1}{2}}\left(p^{-\frac{3}{2}}\right)$ **e)** $\left[\frac{x^{-2}}{(xy)^4}\right]^{1.5}$ **f)** $\left[\frac{4x^{-2}}{9y^{-4}}\right]^{-\frac{5}{2}}$

3. For each of the following, use the exponent laws to help identify a value for p that satisfies the equation.

a) $(x^p)^{\frac{1}{3}} = x^{\frac{2}{3}}$ **b)** $(x^p)\left(x^{\frac{3}{4}}\right) = x^2$

c) $\frac{x^p}{x^{-2}} = x^{\frac{5}{2}}$ **d)** $\left(-3x^{\frac{5}{2}}\right)\left(px^{-\frac{1}{2}}\right) = \frac{-3}{4}x^2$

e) $\left(\frac{9a^{-4}}{25}\right)^p = \frac{3}{5a^2}$ **f)** $(2^{-p})(3^p) = \frac{27}{8}$

4. Evaluate without using a calculator. Leave your answers as rational numbers.

a) $8^{\frac{2}{3}}$ **b)** $16^{\frac{1}{4}}$ **c)** $-27^{\frac{4}{3}}$

d) $\left(3^{\frac{1}{6}}\right)\left(3^{\frac{5}{6}}\right)$ **e)** $\left(\frac{36x^0}{25}\right)^{1.5}$ **f)** $\frac{6^{-2}}{36^{-\frac{1}{2}}}$

5. Evaluate using a calculator. Express your answers to four decimal places, if necessary.

a) $(81^{-0.25})^3$ **b)** $(8^3)(8^{1.2})$ **c)** $\left(\frac{2^5}{5^2}\right)^{-\frac{3}{2}}$

d) $\left(\frac{2^3}{8^2}\right)^{\frac{2}{3}}$ **e)** $\left(\frac{-64}{6^{\frac{1}{2}}}\right)^{\frac{4}{3}}$ **f)** $\frac{\left(2^{\frac{1}{2}}\right)^3}{16}$

6. Whonnock Lake, BC is stocked with rainbow trout. The population grows at a rate of 10% per month. The number of trout stocked is given by the expression $250(1.1)^n$, where n is the number of months since the start of the trout season.

   Calculate the number of trout

   a) 5 months after the season opens
   b) $4\frac{1}{2}$ months after the season opens
   c) 2 months before the season opens
   d) $3\frac{1}{2}$ months before the season opens

Did You Know?

In BC, nearly 900 lakes and streams are stocked annually with trout, char, and kokanee.

Apply

7. For each solution, find the step where an error was made. What is the correct answer? Compare your corrections with those of a classmate.

   a) $\frac{t^{1.2}}{t^{-0.5}} = t^{(1.2 - 0.5)}$
   $= t^{0.7}$

   b) $(16x^2)^{0.5} = (16^{0.5})(x^2)^{0.5}$
   $= 8x^{(2)(0.5)}$
   $= 8x$

8. Kelly has been saving money she earned from her paper route for the past two years. She has saved \$1000 to put towards the purchase of a car when she graduates high school. Kelly has two options for investing the money. If she deposits the money into a 3-year term deposit, it earns 1.5% interest per year, but if she deposits the money into a 2-year term deposit, it will earn 2% interest per year. The formula for calculating the value of her investment is $A = 1000(1 + i)^n$, where A is the amount of money at the end of the term, i is the interest rate as a decimal number, and n is the number of years the money is in the term deposit.

   a) Which term deposit will give her the most interest?
   b) How much more interest does this option pay?

9. From the beginning of 2003 to the beginning of 2007, the population of Manitoba increased at an average annual rate of 0.5%. This situation can be modelled with the equation $P = 1.1619(1.005)^n$, where P is the population, in millions, and n is the number of years since the beginning of 2003.

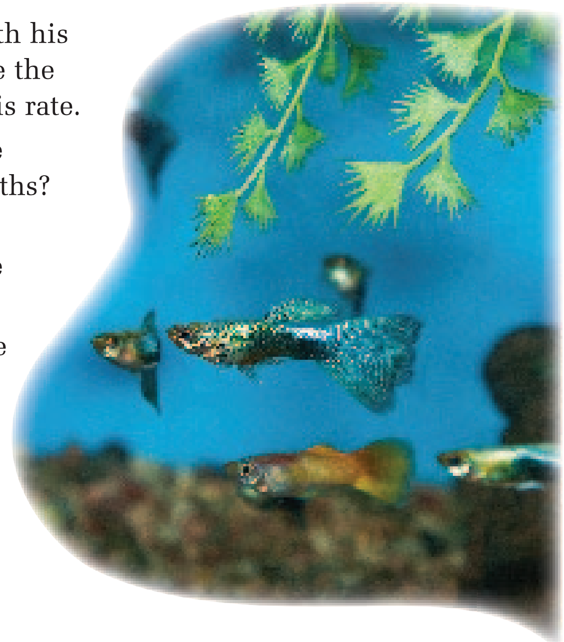
   a) What do you think the number 1.1619 represents?
   b) Assuming that the growth rate continues, what will be the population of Manitoba after 15.5 years?
   c) Assuming that the growth rate was the same prior to 2003, what was the population of Manitoba at the beginning of 1999?

10. Chris buys six guppies. Every month his guppy population doubles. Assume the population continues to grow at this rate.

a) How many guppies will there be after 1 month? 2 months? 3 months? n months?

b) How many guppies will there be after 6.5 months?

c) Can the fish population continue to grow at this rate? Explain.



11. A mutual fund with an initial value of \$10 000 is decreasing in value at a rate of 12% per year. This situation can be represented by the equation $V = 10\ 000(0.88)^n$, where V represents the value of the fund and n the number of years.

a) At this rate, what will be the value of the mutual fund in 5 years and 3 months?

b) If the rate of loss was the same for previous years, what was the value of the fund 3.5 years ago?

12. Martine uses a photographic enlarger that can enlarge a picture to 150% of its previous size. This situation can be modelled by the formula $S = 1.5^t$, where S is the percent increase in the picture size as a decimal number and t is the number of times the enlarger is used.

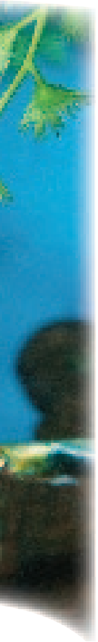
150% means 1.5 times.

a) By how many times is a picture enlarged if the enlarger is used 5 times?

b) How many times would the enlarger need to be used to make a picture at least 25 times as large as the original?

13. Water blocks out sunlight in proportion to its depth. In Qamani'tuaq Lake, NU, $\frac{9}{10}$ of the sunlight reaching the surface of the water can still be seen at a depth of 1 m. This situation can be modelled by the formula $S = 0.9^d$, where S is the fraction of sunlight seen at a depth of d metres. How much sunlight can be seen at a depth of

a) 7.8 m? **b)** 2.75 m?



14. Under certain conditions, the temperature, T, in degrees Celsius, of a cooling object can be modelled using the formula $T = 40(10^{-0.1t})$. In this formula, t is time, in minutes. What is the temperature

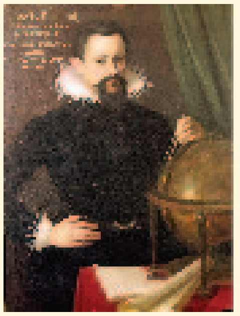
a) after 10 min?

b) after 0.25 h?

15. For any planetary system, the orbital radius of a planet, R, in metres, can be predicted using the formula $R = (KT^2)^{\frac{1}{3}}$. In this formula, K is a constant for the system, and T is the orbital period of the planet. The value of K for objects orbiting the sun is $(3.37)(10^{18})$. If it takes Mercury $(7.60)(10^6)$ s to orbit the sun, what is the orbital radius of Mercury, in metres?

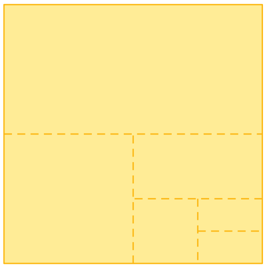
Did You Know?

Johannes Kepler (1571–1630) was a German mathematician and astronomer. He was the first to correctly explain planetary motion.



Extend

16. When a sheet of paper is folded in half, the area of the paper is reduced by half. This situation can be represented by the equation $A = A_0\left(\frac{1}{2}\right)^f$. In this equation, A_0 represents the starting area of the piece of paper and f the number of consecutive folds. How many folds are needed before the area of the folded paper is less than 1% of the original area? Is this possible? Try it.



17. Julia is a veterinarian who needs to determine the remaining concentration of a particular drug in a horse's bloodstream. She can model the concentration using the formula $C = C_0\left(\frac{1}{2}\right)^{\frac{t}{4}}$, where C is an estimate of the remaining concentration of drug in the bloodstream in milligrams per millilitre of blood, C_0 is the initial concentration, and t is the time in hours that the drug is in the bloodstream. At 10:15 a.m. the concentration of drug in the horse's bloodstream was 40 mg/mL.

a) If only a single dose of the drug is given, what will the approximate concentration of the drug be 6 h later?

b) Julia needs to administer a second dose of the drug when the concentration in the horse's bloodstream is down to 30 mg/mL. Estimate after how many hours this would occur.

Create Connections

18. Describe a problem where rational exponents are used to model a real-life situation. Discuss with a classmate what the rational exponent represents in your problem.

19. Describe at least one common error made when simplifying expressions that include powers with rational exponents. Think of at least one strategy you can use to avoid making the error.

4.4 Irrational Numbers

Focus on ...

- representing, identifying, and simplifying irrational numbers
- converting between powers with rational exponents and radicals
- converting between mixed radicals and entire radicals
- solving problems involving radicals

A golden rectangle has sides that are in a ratio that is pleasing to the eye. The ratio of the length to the width in a golden rectangle is called the golden ratio. Many artists use the golden ratio when composing their paintings. This painting titled *Coming Rain* by Ayla Bouvette appears to have several golden rectangles.

The golden ratio is an **irrational number**. Irrational numbers are often called artistic numbers, since they appear in art, architecture, nature, and geometry.

irrational number

- a number that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$
- cannot be expressed as a terminating or repeating decimal
- $\pi = 3.1415...$ $\sqrt{5} = 2.236...$

Did You Know?

Ayla Bouvette is an artist with Okanagan, Anishnaabe, and Red River Métis heritage. Her work uses the themes and designs of her heritage. She sells prints of her works to support local powwow fundraising.

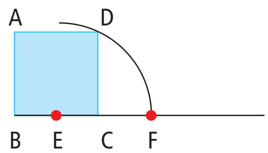
Investigate the Golden Rectangle

Unit Project

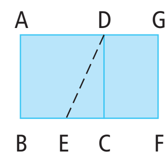
Materials
- ruler
- compasses

1. Draw a square on a blank sheet of paper. Label the vertices of the square ABCD. Measure and record the side length of the square.

2. Complete the following steps.
- Mark the midpoint of side BC as E.
- Extend line BC so that it is about double in length.
- Use compasses to draw an arc with radius DE so that it intersects line BC at point F.



3. Complete the golden rectangle by drawing DG and FG.



4. a) Calculate the length of DE to four decimal places.

b) Measure the length of DE. How does the actual measurement compare to your calculated value?

c) Calculate the length of BF to four decimal places. Hint: DE is the same length as EF.

d) Measure the length of BF. How does the actual measurement compare to your calculated value?

5. Reflect and Respond

a) The ratio of the length to the width in a golden rectangle is called the golden ratio. Write an exact expression for the golden ratio.

b) What is the approximate value of the golden ratio, to two decimal places?

c) In the painting on page 184 describe the golden rectangles you see. Discuss your ideas with a classmate.

6. a) Look for three rectangular shapes in the classroom that you think may be in the golden ratio. Use a table to organize your findings.
- Measure the length and width of each shape.
- Calculate the ratios of the sides as you did for the rectangle you drew.

b) How do the ratios compare? How close were the rectangles you chose to golden rectangles?

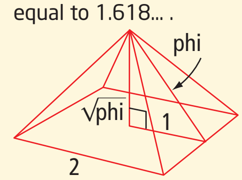
c) Compare your results with those of a classmate.

Web Link

For more information about the golden ratio and the golden rectangle, go to www.mhrmath10.ca and follow the links.

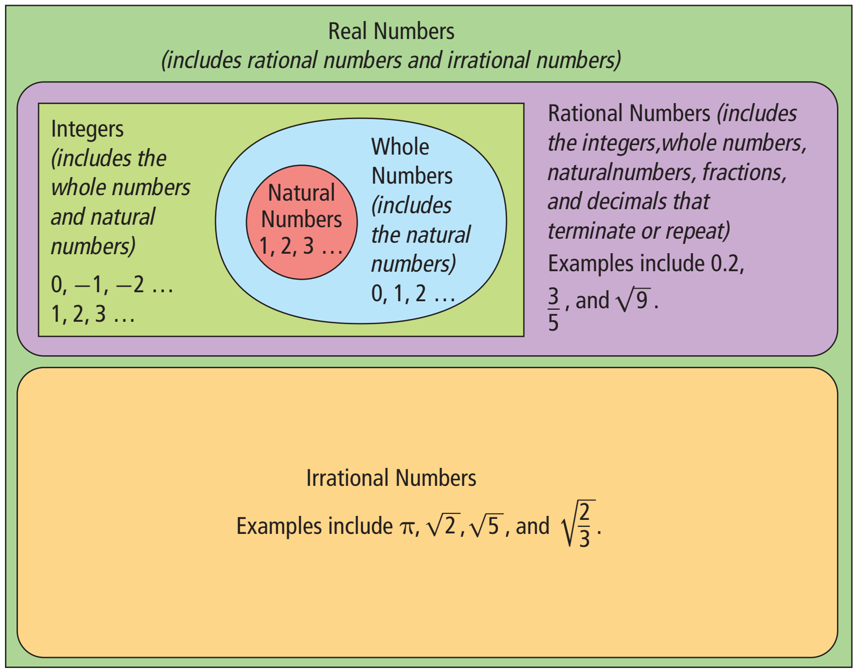
Did You Know?

The Rhind papyrus of Egypt records the building of the Great Pyramid of Giza in 4700 B.C.E. The pyramid has proportions according to the golden ratio. The golden ratio is often represented as *phi,* or ϕ. This is approximately equal to 1.618... .



Link the Ideas

The rational numbers include the natural numbers, whole numbers, and integers. These numbers and the irrational numbers form a set called the real numbers.



What subsets do integers belong to? whole numbers? natural numbers?

Powers with fractional exponents can be written as **radicals** in the form $x^{\frac{1}{n}} = \sqrt[n]{x}$, where $n \neq 0$. When n is even, x cannot be negative, since the product of an even number of equal factors is always positive.

For example, $\left(3^{\frac{1}{2}}\right)\left(3^{\frac{1}{2}}\right) = 3^{\left(\frac{1}{2} + \frac{1}{2}\right)}$
$= 3$

You know that $(\sqrt{3})(\sqrt{3}) = 3$.

Therefore, $3^{\frac{1}{2}} = \sqrt{3}$.

A power can be expressed as a radical in the form

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = (\sqrt[n]{x})^m$$

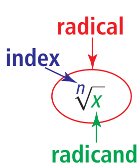
or

$$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

where m and n are integers.

For example, $2^{\frac{3}{4}}$ can be written as $\sqrt[4]{2^3}$ or $\left(\sqrt[4]{2}\right)^3$.

radical
- consists of a root symbol, an index, and a radicand



- can be rational, $\sqrt{4}$, or irrational, $\sqrt{2}$

radicand
- the quantity under the radical sign

index
- indicates what root to take

A fractional exponent can be written in decimal form.

For example, $\sqrt[5]{6^3} = 6^{\frac{3}{5}}$
$= 6^{0.6}$

If the radicand is a number, you can evaluate a power with a fractional or decimal exponent.

For example, $\sqrt{5^3} = 5^{\frac{3}{2}}$
$= 11.1803\ldots$

When the index is 2, it is commonly not written.

Did You Know?

The most famous irrational number is likely pi. The Babylonians (about 2000 to 1600 B.C.E.) were the first to approximate pi to a value of 3. Since that time, pi has been calculated to 1 241 100 000 000 decimal places.

The value of pi is 3.14159265368979323846264338327 95...

Example 1 Convert From a Power to a Radical

Express each power as an equivalent radical.

a) $64^{\frac{1}{2}}$

b) $16^{\frac{3}{4}}$

c) $(8x^2)^{\frac{1}{3}}$

Solution

Write each power as a radical.

Use the denominator of the exponent as the index of the radical.

a) $64^{\frac{1}{2}} = \sqrt{64}$

b) $16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3$

c) $(8x^2)^{\frac{1}{3}} = \sqrt[3]{8x^2}$

Your Turn

Express each power as a radical.

a) $10^{\frac{1}{4}}$

b) $1024^{\frac{1}{3}}$

c) $(x^4)^{\frac{3}{8}}$

Example 2 Convert From a Radical to a Power

Express each radical as a power with a rational exponent.

a) $\sqrt[4]{4^3}$ **b)** $\sqrt[5]{3^4}$ **c)** $\sqrt{s^3}$

Solution

Write each radical as a power.
Use the index as the denominator of the exponent.

a) $\sqrt[4]{4^3} = (4^3)^{\frac{1}{4}}$
$= 4^{\frac{3}{4}}$
$= 4^{0.75}$

b) $\sqrt[5]{3^4} = 3^{\frac{4}{5}}$

c) $\sqrt{s^3} = (s^3)^{\frac{1}{2}}$
$= s^{\frac{3}{2}}$

Your Turn

Express each radical as a power.

a) $\sqrt{125}$ **b)** $\sqrt[3]{y^5}$ **c)** $\sqrt[n]{27^2}$

Example 3 Convert Mixed Radicals to Entire Radicals

Express each **mixed radical** as an equivalent **entire radical**.

a) $5\sqrt{11}$

b) $2\sqrt[3]{5}$

c) $1.5\sqrt[3]{6}$

mixed radical
- the product of a rational number and a radical
- for example, $3\sqrt{2}$, and $\frac{1}{2}\sqrt[3]{6}$

entire radical
- the product of 1 and a radical
- for example, $\sqrt{32}$, and $\sqrt[3]{2^5}$

Solution

a) $5\sqrt{11} = \sqrt{(5^2)}\sqrt{(11)}$
$= \sqrt{(5^2)(11)}$
$= \sqrt{(25)(11)}$
$= \sqrt{275}$

What is the index?
How does it help you convert to an entire radical?

b) $2\sqrt[3]{5} = \sqrt[3]{(2^3)}\sqrt[3]{(5)}$
$= \sqrt[3]{(2^3)(5)}$
$= \sqrt[3]{(8)(5)}$
$= \sqrt[3]{40}$

c) $1.5\sqrt[3]{6} = \sqrt[3]{(1.5^3)}\,\sqrt[3]{(6)}$ or $\frac{3}{2}\sqrt[3]{6} = \sqrt[3]{\left(\frac{3}{2}\right)^3}\,\sqrt[3]{(6)}$

$$\begin{aligned} 1.5\sqrt[3]{6} &= \sqrt[3]{(1.5^3)}\,\sqrt[3]{(6)} \\ &= \sqrt[3]{(1.5^3)(6)} \\ &= \sqrt[3]{(3.375)(6)} \\ &= \sqrt[3]{20.25} \end{aligned}$$

or

$$\begin{aligned} \frac{3}{2}\sqrt[3]{6} &= \sqrt[3]{\left(\frac{3}{2}\right)^3}\,\sqrt[3]{(6)} \\ &= \sqrt[3]{\left(\frac{3}{2}\right)^3(6)} \\ &= \sqrt[3]{\left(\frac{27}{8}\right)(6)} \\ &= \sqrt[3]{\frac{81}{4}} \end{aligned}$$

What could you do to express the radical in lowest terms?

Your Turn

Convert each mixed radical to an equivalent entire radical.

a) $9\sqrt[3]{4}$ **b)** $4.2\sqrt{18}$ **c)** $\frac{1}{2}\sqrt{10}$

Example 4 Convert Entire Radicals to Mixed Radicals

Express each entire radical as an equivalent mixed radical.

a) $\sqrt{27}$ **b)** $\sqrt{50}$ **c)** $\sqrt{48}$ **d)** $\sqrt[4]{80}$

Solution

a)
$$\begin{aligned} \sqrt{27} &= \sqrt{(9)(3)} \\ &= \sqrt{9}\,\sqrt{3} \\ &= 3\sqrt{3} \end{aligned}$$

What value is a perfect square? How does this help you?

b)
$$\begin{aligned} \sqrt{50} &= \sqrt{(25)(2)} \\ &= \sqrt{25}\,\sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

c)
$$\begin{aligned} \sqrt{48} &= \sqrt{(16)(3)} \\ &= \sqrt{16}\,\sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

or

$$\begin{aligned} \sqrt{48} &= \sqrt{(4)(12)} \\ &= \sqrt{4}\,\sqrt{12} \\ &= 2\sqrt{(4)(3)} \\ &= 2\sqrt{4}\,\sqrt{3} \\ &= 2(2)\sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

d)
$$\begin{aligned} \sqrt[4]{80} &= \sqrt[4]{(2)(2)(2)(2)(5)} \\ &= \sqrt[4]{(2^4)(5)} \\ &= 2\sqrt[4]{5} \end{aligned}$$

How does prime factorization help you?

Your Turn

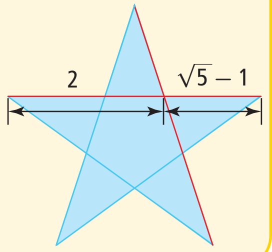
Convert each entire radical to an equivalent mixed radical.

a) $\sqrt{40}$ **b)** $\sqrt{108}$ **c)** $\sqrt[3]{32}$

Did You Know?

The pentagram is also called the star polygon. Inside a pentagram is a pentagon.

Each length of a pentagram intersects two other lengths of the pentagram. The intersection points divide each length according to the golden ratio.



Web Link

For more information about pentagrams and how to draw one, go to www.mhrmath10.ca and follow the links.

Example 5 Order Irrational Numbers

Order these irrational numbers from least to greatest.
$2\sqrt{18}$ $\sqrt{8}$ $3\sqrt{2}$ $\sqrt{32}$

Solution

Method 1: Express Each Irrational Number as an Entire Radical

$$\begin{aligned}2\sqrt{18} &= \sqrt{(2^2)(18)}\\ &= \sqrt{72}\end{aligned}$$

$$\sqrt{8} = \sqrt{8}$$

$$\begin{aligned}3\sqrt{2} &= \sqrt{(3^2)(2)}\\ &= \sqrt{18}\end{aligned}$$

$$\sqrt{32} = \sqrt{32}$$

The radicals in order from least to greatest are
$\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, and $\sqrt{72}$
or
$\sqrt{8}$, $3\sqrt{2}$, $\sqrt{32}$, and $2\sqrt{18}$.

In this case, how could you have used mixed radicals to order the irrational numbers?

Method 2: Estimate the Approximate Values and Plot on a Number Line

Estimate the value of each radical.

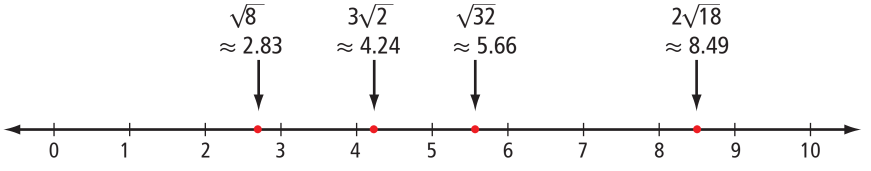
$$\begin{aligned}2\sqrt{18} &= (2)(4.243\ldots)\\ &= 8.485\ldots\\ &\approx 8.49\end{aligned}$$

$$\begin{aligned}\sqrt{8} &= 2.828\ldots\\ &\approx 2.83\end{aligned}$$

$$\begin{aligned}3\sqrt{2} &= (3)(1.414\ldots)\\ &= 4.243\ldots\\ &\approx 4.24\end{aligned}$$

$$\begin{aligned}\sqrt{32} &= 5.657\ldots\\ &\approx 5.66\end{aligned}$$

Plot the approximations on a number line.



Using the approximations on the number line, the radicals in order from least to greatest are $\sqrt{8}$, $3\sqrt{2}$, $\sqrt{32}$, and $2\sqrt{18}$.

Your Turn

Use two different methods to order the following irrational numbers from greatest to least: $2\sqrt{54}$, $\sqrt{192}$, $5\sqrt{10}$.

Example 6 Solve Problems Involving Irrational Numbers

The Seabee Mine is located at Laonil Lake, SK. In 2007, the mine produced a daily average of gold great enough to fill a cube with a volume of 180 cm³. If five days of gold production is cast into a cube, what is its edge length?

Solution

The volume of gold produced in five days is (5)(180) = 900 cm³. The formula for the volume, V, of a cube is $V = s^3$, where s is the length of one side. Substitute 900 for the volume. Solve for s by taking the cube root of both sides of the equation.

Alternatively, you can raise both sides to the exponent $\frac{1}{3}$.

$$\begin{aligned} 900 &= s^3 \\ \sqrt[3]{900} &= \sqrt[3]{s^3} \\ 9.654\ 89\ldots &= s \end{aligned} \quad \text{or} \quad \begin{aligned} 900 &= s^3 \\ 900^{\frac{1}{3}} &= (s^3)^{\frac{1}{3}} \\ 9.654\ 89\ldots &= s \end{aligned}$$

The edge of the cube would be approximately 9.7 cm long.

Your Turn

Assume the Seabee Mine doubles its daily gold production to 360 cm³. What is the edge length of a cube of gold produced in a five-day period?

Did You Know?

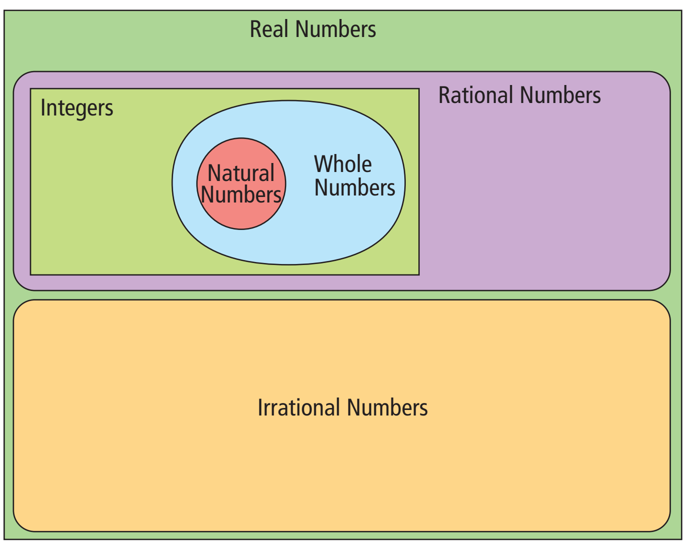
To date, the total amount of gold that has been extracted from Earth would fill only two Olympic sized swimming pools.

Did You Know?

It can be profitable to extract gold from ore grades as low as 0.5 g per 1000 kg of ore. This grade of ore is so low in gold that the gold is not visible. Ore grades of about 30 g per 1000 kg are needed before you can see gold.

Key Ideas

- Rational numbers and irrational numbers form the set of real numbers.



- Radicals can be expressed as powers with fractional exponents.

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

The index of the radical has the same value as the denominator of the fractional exponent.

$\sqrt[3]{10} = 10^{\frac{1}{3}}$ $\quad$ $\sqrt[5]{7^3} = 7^{\frac{3}{5}}$

- Radicals can be entire radicals such as $\sqrt{72}$, $\sqrt[5]{96}$, and $\sqrt[3]{\frac{54}{8}}$. They can also be mixed radicals such as $6\sqrt{2}$, $2\sqrt[5]{3}$, and $\frac{3\sqrt{2}}{2}$. You can convert between entire radicals and mixed radicals.
- You can order radicals that are irrational numbers using different methods:
  - Use a calculator to produce approximate values.
  - Express each irrational number as an entire radical.

Check Your Understanding

Practise

1. Express each power as an equivalent radical.

a) $4^{\frac{3}{2}}$ **b)** $32^{\frac{1}{5}}$ **c)** $64^{0.5}$

d) $\left(\frac{1}{100}\right)^{\frac{1}{4}}$ **e)** $\left(\frac{y^4}{x^3}\right)^{\frac{1}{3}}$ **f)** $(m^n)^{\frac{3}{2}}$

2. Express each radical as a power.

a) $\sqrt{(12p)^3}$ **b)** $\sqrt[5]{5^3}$ **c)** $\sqrt[4]{x^3}$

d) $\sqrt[3]{\frac{s^3}{t^5}}$ **e)** $\sqrt{y^{\frac{5}{3}}}$ **f)** $\sqrt[n]{8}$

3. Evaluate each expression. State the result to four decimal places, if necessary.

a) $\sqrt{0.36}$ **b)** $(27)^{\frac{1}{3}}$ **c)** $4\sqrt{17}$

d) $(65)^{\frac{2}{3}}$ **e)** $0.3(22)^{\frac{1}{2}}$ **f)** $\frac{\sqrt{36}}{\sqrt{7}}$

4. Express each mixed radical as an equivalent entire radical.

a) $3\sqrt{11}$ **b)** $7\sqrt{2}$ **c)** $3\sqrt{5}$

d) $2\sqrt{7}$ **e)** $3\sqrt{3}$ **f)** $10\sqrt{6}$

5. Express each mixed radical as an equivalent entire radical.

a) $2\sqrt[3]{7}$ **b)** $3\sqrt[3]{3}$ **c)** $10\sqrt[3]{5}$

d) $4\sqrt[3]{2}$ **e)** $3\sqrt[4]{2}$ **f)** $2\sqrt[4]{5}$

6. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{12}$ **b)** $\sqrt{50}$ **c)** $\sqrt{48}$

d) $\sqrt{72}$ **e)** $\sqrt{45}$ **f)** $\sqrt{500}$

7. Express each entire radical as an equivalent mixed radical.

a) $\sqrt[3]{24}$ **b)** $\sqrt[3]{54}$ **c)** $\sqrt[3]{243}$

d) $\sqrt[3]{40}$ **e)** $\sqrt[4]{32}$ **f)** $\sqrt[4]{243}$

8. Order each set of numbers from least to greatest. Then, identify the irrational numbers.

a) $\frac{5}{8}$ $\quad 0.\overline{6}$ $\quad \sqrt{0.25}$ $\quad \sqrt[3]{0.84}$

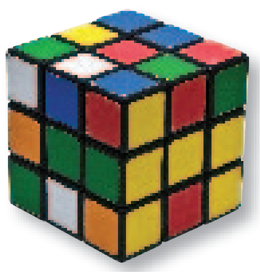
b) $3\sqrt{28}$ $\quad \sqrt{225}$ $\quad 15\frac{4}{5}$ $\quad \sqrt[4]{625}$

9. Plot each set of numbers on a number line. Which of the numbers in each set is irrational?

a) $3\sqrt{4}$ $\quad 6.\overline{6}$ $\quad \sqrt{39}$ $\quad \sqrt[3]{515}$

b) $4\frac{3}{11}$ $\quad \sqrt[3]{125}$ $\quad \frac{4\sqrt{125}}{5}$ $\quad 3\sqrt{8}$

10. The Rubik's Cube is a mechanical puzzle. Calculate the edge length of a Rubik's Cube with a volume of 38.44 cm^3, to three decimal places.



11. Pacific halibut are the largest of all flatfish. The relationship between the length and mass of Pacific halibut can be approximated using the equation $l = 0.46\sqrt[3]{m}$. In this equation, l is the length, in metres, and m is the mass, in kilograms. Use the equation to predict the length of a 25-kg Pacific halibut.

Did You Know?

The range of the Pacific halibut extends along the Pacific coast to the Bering Sea. These fish are important to coastal First Nations who harvest them for food and ceremonial purposes.

Apply

12. Police can estimate the speed of a car by the length of the skid marks made when the driver braked. The formula is $v = \sqrt{30df}$. In this formula, v is the speed, in miles per hour, d is the length of the skid marks, in feet, and f is the coefficient of friction. What was the speed of a vehicle if the skid marks were 75 ft long and the coefficient of friction was 0.7?

13. **Unit Project** Christina is a weaver in Pangnirtung, NU. The dimensions of the tapestry that she is working on represent the golden ratio.

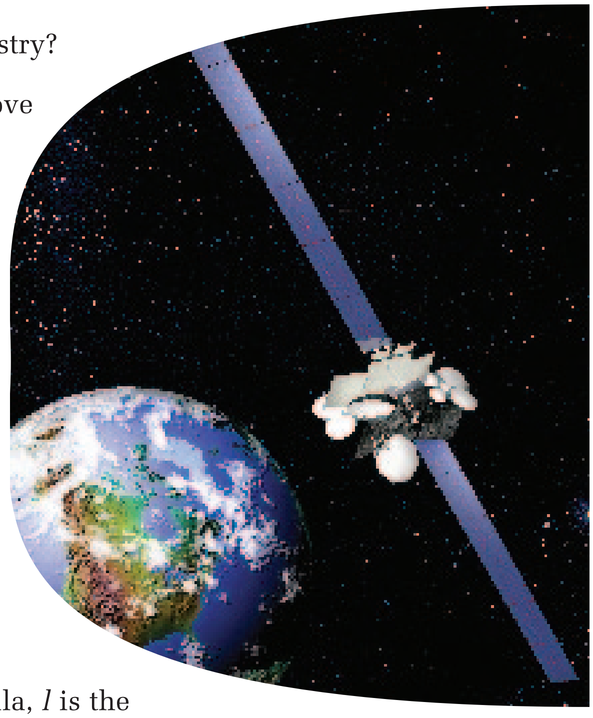
a) If the longer dimension of the tapestry is 60 cm, what is the shorter dimension? Express the answer to the nearest hundredth of a centimetre.

b) What is the total area of the tapestry?

Did You Know?

The velocity of a satellite in *geosynchronous orbit* around Earth matches the rotation of Earth. Since the orbital velocity matches Earth's rotation, the satellite appears to stay in one spot over Earth. However, the satellite is actually travelling at more than 11 000 km/h.

14. When a satellite is h kilometres above Earth, the time, t, in minutes, to complete one orbit is given by the formula $t = \dfrac{\sqrt{(6370 + h)^3}}{6024}$.



a) A telecommunications satellite is placed 30 km above Earth. How long does it take the satellite to make one orbit?

b) A satellite is placed in geosynchronous orbit about Earth. What must its altitude be?

Web Link

For information about Canada's satellites, go to www.mhrmath10.ca and follow the links.

15. The formula $l = \dfrac{8t^2}{\pi^2}$ represents the swing of a pendulum. In this formula, l is the length of the pendulum, in feet, and t is the time, in seconds, it takes to swing back and forth once. What is the length of a pendulum that makes one swing in 2 s?

16. An electronics store owner researched the number of customers who would attend a limited-time sale. She modelled the relationship between the sales discount and the length of the sale using the formula $N = 580\sqrt[3]{Pt}$. In this formula, N is the number of customers expected, P is the percent of the sales discount, and t is the number of hours of the sale. What sales discount should the store offer in order to attract 500 shoppers in 8 h?

Kira made a start to the solution. Complete her work.

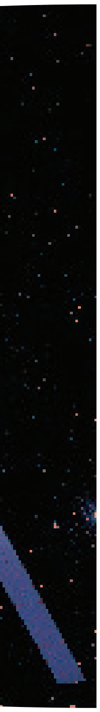
$$N = 580\sqrt[3]{Pt}$$
$$500 = 580\sqrt[3]{P(8)}$$
$$\frac{500}{580} = \frac{580}{580}\sqrt[3]{8P}$$

17. The amount of current, I, in amperes, that an appliance uses can be calculated by the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where P is the power, in watts, and R is the resistance, in ohms. How much current does an appliance use if $P = 120$ W and $R = 3\ \Omega$? Express your answer to one decimal place.

18. **Unit Project** Many aspects of nature, such as the spiral patterns of leaves and seeds, can be described using the Fibonacci sequence. The sequence is 1, 1, 2, 3, 5, 8, 13, … . The expression for the nth term of the Fibonacci sequence is called Binet's formula. The formula is $F_n = \frac{1}{\sqrt{5}}\left(\frac{1 + \sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1 - \sqrt{5}}{2}\right)^n$. Use Binet's formula to find F_3.

WWW Web Link

Pine cones, which grow in spirals, show the Fibonacci spirals. Try to locate spiral patterns in the pine cone shown. To learn more about Fibonnaci spirals, go to www.mhrmath10.ca and follow the links.



Extend

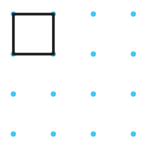
19. Express as a power with a rational exponent.

a) $\sqrt{\sqrt{2^{\frac{4}{5}}}}$ **b)** $\sqrt[4]{\sqrt{256}}$

20. Is the statement $\sqrt[4]{(-x)^4} = x$ sometimes, always, or never true? Explain your reasoning.

Create Connections

21. A 1-by-1 square can be drawn on 5-by-5 dot paper as shown.



a) Draw as many different sized squares as possible on a piece of 5-by-5 dot paper. How many did you find?

b) What is the side length of each square? Which side lengths are rational and which are irrational?

c) What is the area of each square?

22. Describe the relationship between a radical and its equivalent power with a rational exponent.

23. Research the history of up to three algebraic or mathematical symbols of your choice. You might consider the radical sign, pi, phi, or zero. For each symbol, explain who developed it and why they created it.

24. **Unit Project** Use what you have learned about radicals to analyse the golden ratio. Use the following methods as a guide.
- Make a timeline about the history of the golden ratio.
- Explain the exact relationship between the dimensions of the golden rectangle and the golden ratio.
- Use a visual to help describe one other example of the golden ratio.

4 Review

4.1 Square Roots and Cube Roots, pages 152–161

1. Which of the following numbers are perfect squares, perfect cubes, or both?

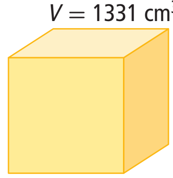
a) 81 **b)** 64 **c)** 125
d) 121 **e)** 729 **f)** 196

2. Use prime factorization to evaluate $\sqrt{144}$.

3. Calculate.

a) $\sqrt{121}$ **b)** $\sqrt[3]{216}$ **c)** $\sqrt[3]{8000}$

4. What are the dimensions of the cube?



5. Jan wants to build a fence around her garden to prevent deer from eating the vegetables. The deer fencing costs \$15 per metre. How much will it cost to enclose a square garden with an area of 81 m^2?

4.2 Integral Exponents, pages 162–173

6. Write as a power with a positive exponent.

a) $(x^{-4})^2$ **b)** $\dfrac{s^3}{s^{-3}}$

c) $\dfrac{(-2.6)^4}{(-2.6)^{-2}}$ **d)** $\dfrac{(4k)^2}{(4k)^{-3}}$

7. Evaluate each expression. Express your answers to four decimal places, if necessary.

a) $(3^{-2})^{-2}$ **b)** $\left[\dfrac{4.5}{(3^2)(1.5)}\right]^3$

c) $[(1.2^3)(1.2^{-2})]^{-4}$ **d)** $\left[\dfrac{(4x^{-2})^{-2}}{(4x)^3}\right]^2$

8. A ball is dropped from a height of 3 m and allowed to bounce freely. The height, h, in metres, that it rebounds can be modelled using the formula $h = 3(0.7)^n$. In this formula, n is the number of bounces.

a) How high does the ball reach on the third bounce? Express the answer to two decimal places.

b) After how many bounces does the ball reach a maximum height of 0.5 m?

9. A radioactive element has a half-life of one week. The formula for the amount of the element remaining is $A = 500\left(\frac{1}{2}\right)^n$, where n is the number of weeks. How much of a 500-g sample of the element

a) remains after five weeks?

b) was there four weeks ago?

10. In a national park, the caribou population increased by a growth factor of 1.05 per year over a 15-year period from 1993 to 2008. There were 1400 caribou at the beginning of 1993 This situation can be modelled by the formula $P = 1400(1.05)^n$, where P is the estimated caribou population and n is the number of years since 1993.

a) If the growth rate remained constant, how many caribou were there after 1 year? 2 years? 3 years? n years?

b) How many caribou will there be at the beginning of 2011?

c) Assume that the growth rate was the same before 1993. How many caribou were there at the beginning of 1990?

4.3 Rational Exponents, pages 174–183

11. Simplify each expression. Express the answers with positive exponents.

a) $\left(x^{\frac{-4}{3}}\right)^{\frac{1}{4}}$ b) $\frac{4^{\frac{2}{5}}}{4^{-0.6}}$ c) $(16g^8)^{\frac{-3}{4}}$ d) $\left(\frac{t^2}{0.5t^{\frac{-1}{3}}}\right)^3$

12. a) Use mental math to simplify $\left(\frac{1}{3}\right)^{-1}(27)^{\frac{1}{3}}$. Show your thinking.

b) What shortcuts have you developed for working with negative exponents? Discuss your strategies with a classmate.

13. Evaluate each expression. Express your answers to four decimal places, if necessary.

a) $\left(12^{\frac{2}{5}}\right)(12^{0.4})$ b) $\left(3^{\frac{-2}{3}}\right)^3$ c) $\left(\frac{0.5^{0.5}}{0.5^{-2}}\right)^4$ d) $\left(\frac{8^4}{2}\right)^{\frac{1}{2}}$

14. Barb incorrectly simplified $(27x^4)^{\frac{2}{3}}$ as $18x^{\frac{8}{3}}$. What error did she make? What is the correct answer?

15. Jason invested \$6500 into a fund that increases in value at a rate of 6.2% per year. Jason receives a statement on the value of his investment based on the formula $A = 6500(1.062)^t$, where A is the total value of the investment and t is the number of annual periods. What is the value of Jason's fund after three annual periods?

Did You Know?

A tsunami can travel at speeds up to 800 km/h. In 1964, the strongest earthquake of the century in North America resulted in a tsunami travelling at more than 700 km/h toward the BC coast. The 4.3-m wave caused extensive flooding and property damage in Port Alberni, BC.

16. The speed that a tsunami can travel is modelled by the equation $s = 356(d)^{\frac{1}{2}}$. In the equation, s is the speed, in kilometres per hour, and d is the average depth of the water, in kilometres. What is the depth of a tsunami travelling at a speed of 145 km/h?

17. The cost, C, to make computer chips can be calculated using the formula $C = 1000n^{\frac{2}{3}} + 1500$, where n is the number of chips. What is the cost of producing 10 000 computer chips?

4.4 Irrational Numbers, pages 184–195

18. Write each power as an equivalent radical.
    a) $x^{\frac{3}{5}}$ **b)** $(27t^2)^{\frac{2}{3}}$ **c)** $\left(\frac{g^3}{18}\right)^{0.5}$

19. Express each radical as a power.
    a) $\sqrt{(xp)^5}$ **b)** $\sqrt[3]{2^5}$ **c)** $3\sqrt[5]{x^4}$

20. Convert each mixed radical to an equivalent entire radical.
    a) $3\sqrt{12}$ **b)** $2\sqrt{10}$ **c)** $4\sqrt[3]{5}$ **d)** $-2\sqrt[3]{2}$

21. Express each entire radical as an equivalent mixed radical.
    a) $\sqrt{180}$ **b)** $\sqrt{192}$ **c)** $\sqrt[3]{128}$ **d)** $\sqrt[4]{48}$

22. Identify the irrational numbers in each set. Then, order all the numbers from greatest to least.
    a) $0.\overline{24}$ $\frac{\pi}{3}$ $\sqrt{0.9}$ $\sqrt[5]{96}$ **b)** $18^{\frac{1}{2}}$ $\sqrt{36}$ $6.\overline{2}$ $2\sqrt[3]{27}$

23. Use a graphic organizer of your choice to represent the similarities and differences between rational and irrational numbers. Explain the key ideas on your organizer.

Traditional Cree hand drum, from Winnipeg, MB showing feathers, the moon and sun, and animal footprints. Traditional Cree hand drums are used in ceremony, cultural events such as round dances, and as artwork. Traditional drums should always be handled with respect following appropriate protocol.

24. **a)** A circular drum head has an area of 300 cm². What is the approximate radius of the drum? Express your answer to one decimal place.
    b) What is the surface area of a drum head with a radius of 12 cm?

4 Practice Test

Multiple Choice

For #1 to #7, choose the best answer.

1. Which of the following numbers is both a perfect square and a perfect cube?

A 8 **B** 16 **C** 32 **D** 64

2. Which expression is not equivalent to $(2x^3)^{-4}$?

A $\frac{16}{x^{12}}$ **B** $\frac{1}{16x^{12}}$

C $\frac{2^{-4}}{x^{12}}$ **D** $\frac{x^{-12}}{16}$

3. Oxalic acid is $(3.0)(10^3)$ times as acidic as acetic acid. Acetic acid is $(2.8)(10^4)$ as strong as boric acid. How many times as acidic is oxalic acid as boric acid?

A $(9.3)(10^1)$ **B** $(3.1)(10^4)$

C $(5.8)(10^7)$ **D** $(8.4)(10^7)$

Did You Know?

Oxalic acid is found in rhubarb. Acetic acid is found in vinegar. Boric acid is an antiseptic used to treat minor burns.

4. Certain bacteria in soil can double in number every 4 h. If the initial population is 2000, what will the approximate population of the bacteria be after 16 h?

A 131 070 000 **B** 128 000

C 32 000 **D** 2064

5. Which of the following is equivalent to $\sqrt[4]{405}$?

A $3\sqrt[4]{5}$ **B** $5\sqrt[4]{3}$

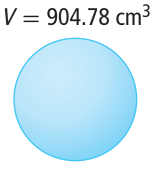
C $5\sqrt[4]{81}$ **D** $81\sqrt[4]{3}$

6. Which power is equivalent to $\sqrt{2^3(3)^{-2}}$?

A $\frac{2^{\frac{3}{2}}}{3}$ **B** $2^{\frac{3}{2}}(3)$

C $\frac{3}{2^{\frac{3}{2}}}$ **D** $2^{-\frac{3}{2}}(3)$

7. The radius, r, of a sphere is given by the equation $r = \sqrt[3]{\frac{3V}{4\pi}}$, where V is the volume of the sphere. What is the approximate radius of this sphere?



A 2.3 cm **B** 6.0 cm

C 12.87 cm **D** 72.0 cm

Short Answer

8. A cube-shaped storage container has a volume of $10.648\ \text{m}^3$.

a) Before calculating, estimate two whole numbers between which the edge length of the container lies. Which number do you think the actual value is closer to?

b) What are the dimensions of the container?

9. Explain why any irrational number cannot be a rational number.

10. a) The whole grid of a Sudoku puzzle has an area of $169\ \text{cm}^2$. What are the dimensions of the grid?

b) What is the side length of each of the nine sections? Express your answer to the nearest tenth of a centimetre.

Did You Know?

A Sudoku puzzle is a Japanese logic puzzle on a grid that has nine 3-by-3 sections.

5	8				7			
		6	4	5		8		
		7		3				1
1					9		2	
8								6
	6		2					4
7				9		2		
		1		2	3	9		
			8				1	3

11. Amaiya in Kugaaruk, NU is video conferencing with Ed in Muenster, SK. She sends the following homework solution to him. What error did she make? Find the step and correct the answer.

$$\begin{aligned}\frac{2^{-\frac{1}{2}}}{2^{0.5}} &= \frac{2^{-0.5}}{2^{0.5}}\\ &= 2^{-0.5+0.5}\\ &= 2^{0}\\ &= 1\end{aligned}$$

12. The remaining mass, M, of radioactive iodine-131 is given by the equation $M = M_0\left(2^{-\frac{t}{8}}\right)$, where M_0 is the original mass, in grams, and t is time, in days. If the original mass is 250 g, how much iodine-131 remains after 24 days? Express the answer to two decimal places.

13. As you go higher above the surface of Earth, the distance to the horizon becomes greater. The distance, d, in kilometres, to the horizon is given by the formula $d = 3.572\sqrt{h}$, where h is the height of the object, in metres. What is the distance to the horizon from the top of Mount Robson, which has a height of 3959 m? State the result to one decimal place.

Did You Know?

Mount Robson, BC, is the highest point in the Canadian Rockies.

14. A town's rabbit population is growing. The expected population can be modelled using the formula $P = 1580(1.018)^n$, where P is the estimated population and n is the number of years.

a) What is the current rabbit population?

b) What is the expected population in 6 years?

Extended Response

15. Ming and Brian were ordering the irrational numbers $3\sqrt{27}$, $3\sqrt{48}$, $4\sqrt{12}$, and $6\sqrt{3}$ from least to greatest.

a) Ming plans to draw a number line. Describe the rest of her method.

b) Brian converted $4\sqrt{12}$ to $\sqrt{192}$. Describe the rest of his method.

c) Use the method of your choice to order the irrational numbers.

CHAPTER

5 Polynomials

Historically, mathematics has involved simplifying complex concepts and calculations so that they can be done more quickly and easily. Take, for example, the Chinese work, *Arithmetic in Nine Sections*, written around 200 B.C.E. It begins, "three sheafs of good crop, two sheafs of mediocre crop, and one sheaf of bad crop are sold for 29 dou." Today, we would represent this situation with the polynomial equation $3g + 2m + b = 29$, where the variables g, m, and b represent the prices for the *good*, *mediocre*, and *bad* crops. Problem situations that involve factoring polynomials were originally represented by the ancient Greeks using geometric constructions. Today, those problems are still represented using diagrams. They can also be represented using algebraic expressions.

Big Ideas

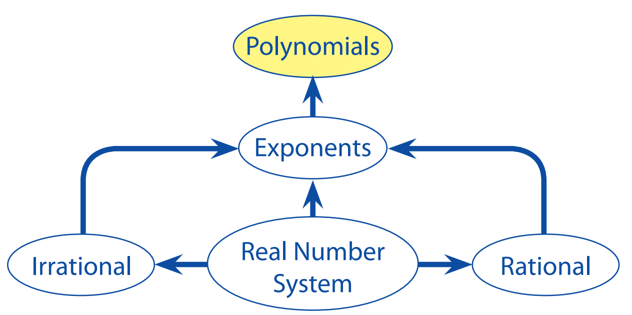
When you have completed this chapter, you will be able to ...

- identify the factors of whole numbers and algebraic expressions
- multiply polynomials concretely, pictorially, and algebraically
- factor polynomials concretely, pictorially, and algebraically

Key Terms

polynomial
binomial
distributive property
trinomial
greatest common factor
least common multiple
difference of squares

Your Algebra and Number Organizer



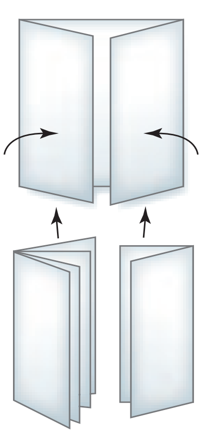
Recreation Facilities Manager

Recreation Facilities Managers are involved in their communities through maintaining and often improving recreation facilities such as pools, arenas, and fitness and dance studios. They schedule the use of the facilities by individuals and community groups. They also oversee the financial aspect of the facilities, as well as the staff. They may also make suggestions for any renovations or facility improvements.

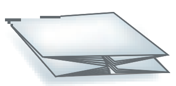
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 5.

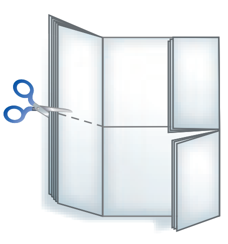
1. Fold a sheet of 11 × 17 paper as shown. Then, fold three sheets of 8.5 × 11 paper. Attach two pages inside the left flap and one page inside the right flap.



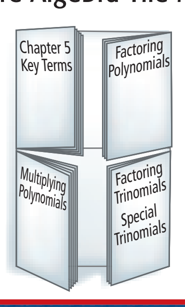
2. Fold in half.



3. Cut tabs along the fold lines.



4. Label as shown. Label the back What I Need to Work On, and Project Ideas and Questions. Label the inside centre Algebra Tile Models.



5.1 Multiplying Polynomials

Focus on …
- multiplying polynomials
- explaining how multiplication of binomials is related to area and to the multiplication of two-digit numbers

Geometric abstraction is a form of abstract art based on the use of geometric shapes. Piet Mondrian is one of many artists who used this style of painting. Mondrian's art has influenced designers of everything from cups to buildings.

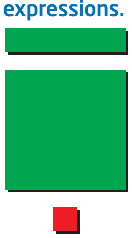
In what ways can you relate **polynomial** multiplication to Mondrian's painting?

polynomial
- a sum of monomials
- for example, $x + 5$, $2a^2 - 6ab + 18b^2$

Investigate Multiplying Polynomials

Materials
- algebra tiles

You can use algebra tiles to model algebraic expressions.



positive *x*-tile

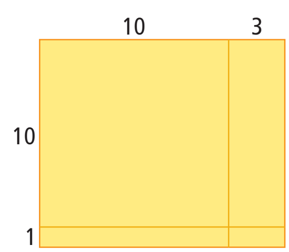
positive x^2-tile

positive 1-tile

The same tiles in white represent negative quantities.

1. Complete the multiplication (13)(11).

2. a) You can express 13 as the sum 10 + 3, and 11 as the sum 10 + 1. Complete an area model with the dimensions 10 + 3 and 10 + 1.

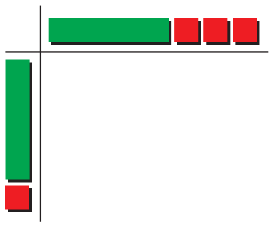


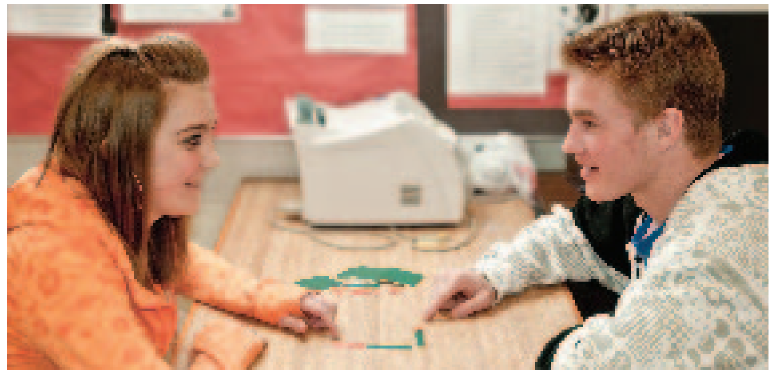
b) How does your model show the factors 10 + 3 and 10 + 1?

c) What is the product?

3. Compare the methods in step 1 and step 2. How are they similar? How are they different?

4. a) Use algebra tiles to determine the product $(x + 3)(x + 1)$. Sketch your model.





b) How does your model show the factors $x + 3$ and $x + 1$?

c) What is the product?

d) How is your model similar to and different from the model you used in step 2?

5. Use algebra tiles to determine each product.

a) $(x + 5)(x - 2)$

b) $(p + 4)(p + 4)$

c) $(2x + 4)(3x - 1)$

6. **Reflect and Respond** Discuss the following questions with a partner.

a) Use your answers from step 5 to look for patterns that relate the factors to the products.

b) Use the patterns you found to explain how you could multiply **binomials** without using algebra tiles.

binomial

- a polynomial with two terms
- for example, $x + 3$, $2x - 5y$

Link the Ideas

You can make connections between multiplying whole numbers and multiplying polynomials. Multiply 42 by 26 using the **distributive property**.

$$\begin{aligned}(42)(26) &= (40 + 2)(20 + 6)\\ &= 40(20 + 6) + 2(20 + 6)\\ &= (40)(20) + (40)(6) + (2)(20) + (2)(6)\\ &= 800 + 240 + 40 + 12\\ &= 1092\end{aligned}$$

distributive property

- the rule that states $a(b + c) = ab + ac$
- for example, $40(20 + 6) = (40)(20) + (40)(6)$

Example 1 Multiply Binomials

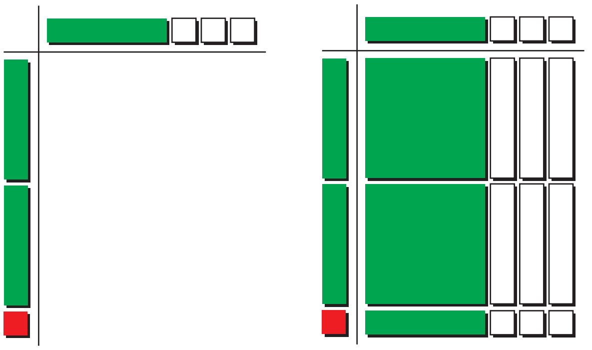
Multiply.

a) $(x - 3)(2x + 1)$ **b)** $(x - 2y)(x - 4y)$

Solution

a) Method 1: Use Algebra Tiles

Use algebra tiles to show the dimensions $x - 3$ and $2x + 1$. Then, complete a rectangle that has these dimensions.



There are two x^2-tiles, six negative x-tiles, one positive x-tile, and three negative 1-tiles in the rectangle.
Therefore, $(x - 3)(2x + 1) = 2x^2 - 5x - 3$.

To create an area model, you place the negative tiles on top of the positive tiles. Explain why.

Method 2: Use the Distributive Property

$$\begin{aligned}(x - 3)(2x + 1) &= x(2x + 1) - 3(2x + 1)\\ &= (x)(2x) + (x)(1) + (-3)(2x) + (-3)(1)\\ &= 2x^2 + 1x - 6x - 3\\ &= 2x^2 - 5x - 3\end{aligned}$$

Multiply each term in the first binomial by each term in the second binomial. Then, combine like terms.

b) $$\begin{aligned}(x - 2y)(x - 4y) &= x(x - 4y) - 2y(x - 4y)\\ &= x^2 - 4xy - 2xy + 8y^2\\ &= x^2 - 6xy + 8y^2\end{aligned}$$

Check:
You can verify your work by substituting values for the variables x and y. For example, substitute $x = 5$ and $y = 1$.

Left Side	Right Side
$(x - 2y)(x - 4y)$	$x^2 - 6xy + 8y^2$
$= [5 - 2(1)][5 - 4(1)]$	$= (5)^2 - 6(5)(1) + 8(1)^2$
$= (5 - 2)(5 - 4)$	$= 25 - 30 + 8$
$= (3)(1)$	$= 25 - 22$
$= 3$	$= 3$

Left Side = Right Side

Your Turn

Determine each product.

a) $(x - 3)(x - 5)$ **b)** $(5m - 1)(2m - 6)$

Example 2 Multiply a Binomial and a Trinomial

Multiply the following binomial and **trinomial**.
$(x + 2)(2x^2 - 5x + 1)$

trinomial
- a polynomial with three terms
- for example, $x^2 + 3x - 1$, $2x^2 - 5xy + 10y^2$

Solution

$$\begin{aligned}
&(x + 2)(2x^2 - 5x + 1)\\
&= x(2x^2 - 5x + 1) + 2(2x^2 - 5x + 1)\\
&= 2x^3 - 5x^2 + x + 4x^2 - 10x + 2\\
&= 2x^3 - x^2 - 9x + 2
\end{aligned}$$

Multiply each term in the binomial by each term in the trinomial. Then, combine like terms.

Your Turn

Determine each product.

a) $(r - 4)(3r^2 + 8r - 6)$

b) $(5x - 3)(2x^2 - 6x + 12)$

Example 3 Perform Operations on Products of Polynomials

Simplify.

a) $(x + 1)(5x + 3) + 3(2x + 4)(6x - 2)$

b) $(3w - 2)(4w + 5) - (w - 7)(2w + 3)$

Solution

a)
$$\begin{aligned}
&(x + 1)(5x + 3) + 3(2x + 4)(6x - 2)\\
&= x(5x + 3) + 1(5x + 3) + 3[2x(6x - 2) + 4(6x - 2)]\\
&= 5x^2 + 3x + 5x + 3 + 3(12x^2 - 4x + 24x - 8)\\
&= 5x^2 + 3x + 5x + 3 + 36x^2 - 12x + 72x - 24\\
&= 41x^2 + 68x - 21
\end{aligned}$$

When you have three factors, you can multiply in any order. What are some other ways you could multiply $3(2x + 4)(6x - 2)$?

b)
$$\begin{aligned}
&(3w - 2)(4w + 5) - (w - 7)(2w + 3)\\
&= (3w)(4w + 5) + (-2)(4w + 5) - [w(2w + 3) - 7(2w + 3)]\\
&= 12w^2 + 15w - 8w - 10 - (2w^2 + 3w - 14w - 21)\\
&= 12w^2 + 15w - 8w - 10 - 2w^2 - 3w + 14w + 21\\
&= 10w^2 + 18w + 11
\end{aligned}$$

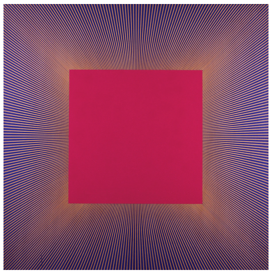
Your Turn

Multiply and then combine like terms.

a) $(x + 3)(5x - 2) + 4(x - 1)(2x + 5)$

b) $2(3x - 2) - (4x + 7)(2x - 5)$

Example 4 Apply Binomial Multiplication

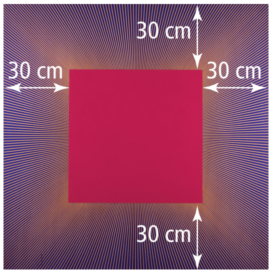


The painting shown is *Deep Magenta Square* by Richard Anuszkiewicz. It can be used to represent binomial multiplication. The length of the red square in the painting is unknown. The width of the border around the square is 30 cm.

a) What polynomial expression represents the total area of the painting?

b) What is the total area of the painting if the red square has an area of 3600 cm²?

Solution



a) Let x represent the length of the red square. The length of the painting can be represented by $x + 30 + 30 = x + 60$. The area of the painting can be represented by the polynomial expression $(x + 60)(x + 60) = x^2 + 120x + 3600$.

b) If the red square has an area of 3600 cm², the side length of the red square is $\sqrt{3600} = 60$. Substitute this value into the expression $(x + 60)(x + 60)$ or the expression $x^2 + 120x + 3600$.

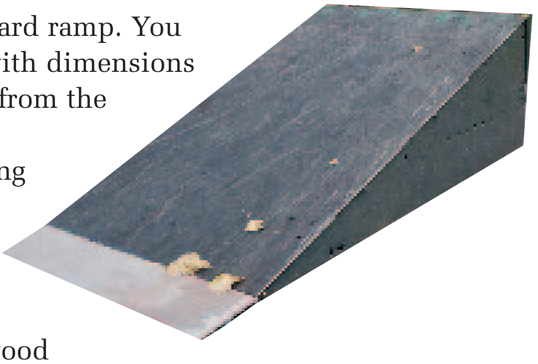
$$\begin{aligned}(60 + 60)(60 + 60) &= (120)(120)\\ &= 14\,400\end{aligned}$$

or

$$\begin{aligned}(60)^2 + 120(60) + 3600 &= 3600 + 7200 + 3600\\ &= 14\,400\end{aligned}$$

The area of the painting is 14 400 cm².

Your Turn



You are building a skateboard ramp. You have a piece of plywood with dimensions of 4 ft by 8 ft. You cut x ft from the length and width.

a) Sketch a diagram showing the cuts made to the piece of plywood. Label the dimensions.

b) What is the area of the remaining piece of plywood that will be used for the ramp?

Key Ideas

- You can use the distributive property to multiply polynomials. Multiply each term in the first polynomial by each term in the second polynomial.

$$\begin{aligned}(3x - 2)(4x + 5) &= (3x)(4x + 5) - 2(4x + 5)\\ &= 12x^2 + 15x - 8x - 10\\ &= 12x^2 + 7x - 10\end{aligned}$$

$$\begin{aligned}(c - 3)(4c^2 - c + 6) &= c(4c^2 - c + 6) - 3(4c^2 - c + 6)\\ &= 4c^3 - c^2 + 6c - 12c^2 + 3c - 18\\ &= 4c^3 - 13c^2 + 9c - 18\end{aligned}$$

Check Your Understanding

Practise

1. Multiply using algebra tiles.

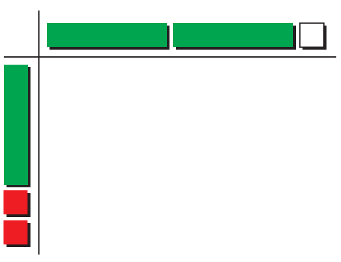
a) $(x - 2)(x + 3)$ **b)** $(3x - 4)(2x - 1)$

c) $(x - 5)(x - 2)$ **d)** $(x + 3)^2$

e) $(x + 4)(x + 7)$ **f)** $(2x - 5)(x - 3)$

$(x + 3)^2$ means $(x + 3)(x + 3)$.

2. a) What product does the algebra tile model show?



b) What are the dimensions of the model?

3. Multiply using the distributive property.

a) $(x + 5)(x - 2)$ **b)** $(x - 3)^2$

c) $(c - d)(c + d)$ **d)** $(4x + y)(x + y)$

e) $(y + 3)^2$ **f)** $(4j + 2k)(6j - 3k)$

4. Use the distributive property to determine each product.

a) $x(3x^2 - 5x + 8)$ **b)** $a(7b^2 + b - 1)$

c) $(x - 3)(6x^2 - 4x - 12)$ **d)** $(2x - 1)(5x^2 + 4x - 5)$

e) $(4s^2 + s)(3s^2 - 2s + 6)$ **f)** $(2y^2 + 3y - 1)(y^2 + 4y + 5)$

5. Match each binomial multiplication on the left with a trinomial on the right.

a) $(x + 1)(x - 2)$
b) $(x - 3)(x - 4)$
c) $(x - 1)^2$
d) $(x + 4)(x - 3)$
e) $(x - 3)(x - 5)$
f) $(x + 3)^2$
g) $(x + 9)(x + 4)$
h) $(x - 6)(x - 3)$

A $x^2 + 13x + 36$
B $x^2 - x - 2$
C $x^2 - 2x - 1$
D $x^2 + x - 12$
E $x^2 + 6x + 9$
F $x^2 - 2x + 1$
G $x^2 - 9x + 18$
H $x^2 - 7x + 12$
I $x^2 - 7x - 12$
J $x^2 - 8x + 15$

6. Multiply. Then, combine like terms.

a) $(4n + 2) + (2n - 3)(3n - 2)$
b) $(f + 7)(2f - 4) - (3f + 1)^2$
c) $(b - 2d)(5b - 3d) + (b + d)(4b + d)$
d) $(4x - 2)(3x - 5) + 2(7x + 5)(2x - 6)$
e) $3(5a + 3c)(2a - 3c) - (4a + c)^2$
f) $(y^2 - 5y - 6)(4y^2 + 6y + 1)$

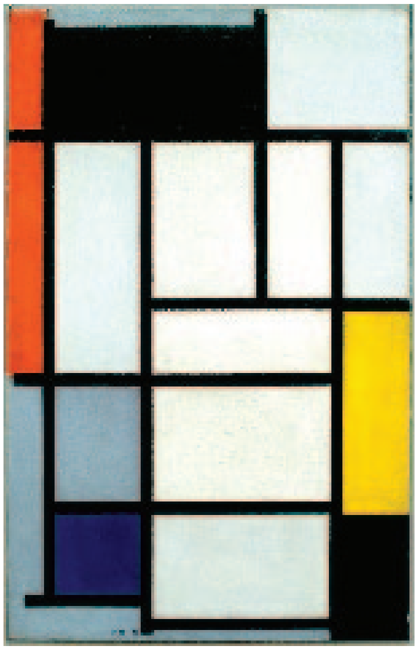
Apply

7. The painting shown is by Métis artist Leah Marie Dorion from Prince Albert, Saskatchewan. It is called *Hawk Woman* (2006). The frame is 2 in. wide on each side of the square painting. Write an expression to represent the dimensions and area of the painting. Multiply, and then combine like terms.

8. **Unit Project** Sketch an area model or an algebraic model to represent each multiplication. Use specific polynomials for each multiplication. Label your diagrams. Then, write the result of each multiplication as an equation.

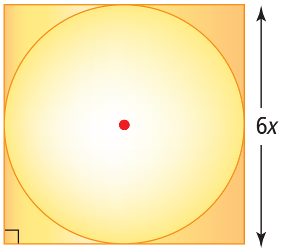
a) (monomial)(binomial)
b) (binomial)(binomial)
c) (binomial)(trinomial)

9. **Unit Project** Use an arrangement of algebra tiles to show combining like terms of polynomials. Arrange them artistically. Use the style of Piet Mondrian's paintings, shown here and on page 204. Write the corresponding algebraic equation that summarizes your result.



10. Darien trimmed a square photo to fit into a rectangular frame. He cut 7 cm from one side and 4 cm from the other. Let x represent the side length of the original square photo. Write an expression for the area of the trimmed photo. Multiply, and then combine like terms.

11. A circle is inset into a square with a side length of $6x + 4$, as shown. Write an expression to represent the area of the circle. Multiply, and then combine like terms.



12. Bryan was asked to multiply two binomials. He completed the following work.

$$\begin{aligned} (p+3)(p+7) &= p^2 + 7p + 3p + 21 && \text{Step 1} \\ &= p^2 + 10p + 21 && \text{Step 2} \\ &= 11p^2 + 21 && \text{Step 3} \end{aligned}$$

a) Is Bryan's work correct? If not, which step is incorrect?

b) Choose any number for p. Determine whether the following equation is true.
$(2p - 3)(p + 4) = 2p^2 - 5p - 12$

13. The Li family has a house with a length of 13 m and a width of 9 m. Due to lot restrictions, they can make an addition of only y metres to the width and x metres to the length.

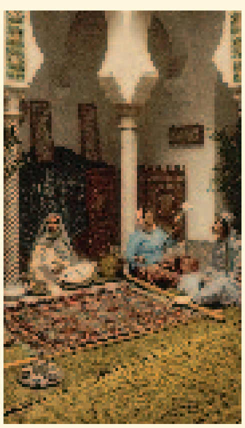
a) Sketch a diagram of the area of the house. Label the dimensions.

b) Write an expression for the area of the house, including the addition.

c) Calculate the area if $x = 1$ m and $y = 2$ m.

Did You Know?

Authentic oriental rugs are hand woven and knotted. They are produced primarily in Asia, the Middle East, and India. The rugs are made for decorative, practical, and sometimes spiritual purposes.



14. **a)** Susan is making a rectangular area rug with a similar design to the square rug she made earlier. What are the dimensions if the new rug is 2 ft longer and 1 ft narrower than the square rug?

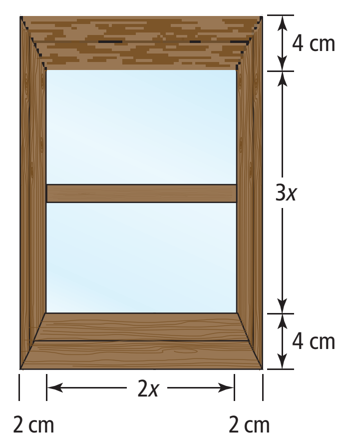
b) Write an expression for the area of the new rug.

c) If the square rug is 3 ft by 3 ft, which rug has the greater area? Show your work.

15. Vera is installing a kitchen window that has a height-to-width ratio of 3 : 2. The window frame adds 4 cm to the width and 8 cm to the height.

a) Write a polynomial expression that represents the total area of the window, including the frame. Multiply and combine like terms.

b) Calculate the area when $x = 12$ cm.



16. André multiplied the expression $(2x - 4)(3x + 5)$. When he checked his answer, he discovered an error.

$$\begin{aligned}(2x - 4)(3x + 5) &= 2x(3x + 5) - 4(3x + 5)\\ &= 6x^2 + 10x - 12x + 20\\ &= 6x^2 - 2x + 20\end{aligned}$$

Check:
Let $x = 4$.

Left Side	Right Side
$(2x - 4)(3x + 5)$	$6x^2 - 2x + 20$
$= [2(4) - 4][3(4) + 5]$	$= 6(4)^2 - 2(4) + 20$
$= (8 - 4)(12 + 5)$	$= 96 - 8 + 20$
$= (4)(17)$	$= 108$
$= 68$	

a) Explain how André knew that he had made an error.

b) Explain the error and how to correct it.

Extend

17. The average number of burgers, b, sold at The Burger Barn daily can be represented by $b = 550 - 100p$, where p is the price of a burger, in dollars.

a) How does the average number of burgers sold change as the price of a burger increases?

b) Solve the equation for p.

c) The revenue from burger sales can be represented by $R = np$, where R is the total revenue, in dollars, and n is the number of burgers sold. Substitute your expression for p from part b). Then, multiply to get an expression for the daily burger revenue.

Create Connections

18. a) Choose four consecutive whole numbers. Multiply the first and last numbers. Multiply the second and third numbers. Repeat this multiplication with several different groups of four consecutive whole numbers. What pattern to you notice?

b) Let n represent your first number. What algebraic expressions represent your second, third, and fourth numbers?

c) Use algebraic multiplication to show that your pattern in part a) is always true.

19. The product of 45 and 34 can be thought of as $(40 + 5)(30 + 4)$. You can represent $40 + 5$ as $4t + 5$, where t represents 10.

a) What expression could represent $30 + 4$?

b) Use binomial multiplication of the algebraic expression. Substitute to find the product of 45 and 34.

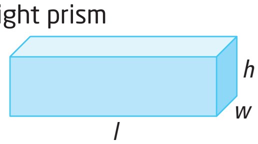
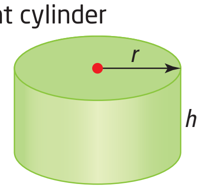
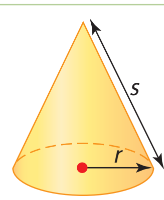
5.2 Common Factors

Focus on …

- determining prime factors, greatest common factors, and least common multiples of whole numbers
- writing polynomials in factored form
- applying your understanding of factors and multiples to solve problems

Cubism is an early 20th-century art style. It was pioneered by artists Pablo Picasso and Georges Braque. In cubist artworks, natural forms are broken up. The pieces are reassembled into simplified 3-D shapes. The idea is to portray an object from multiple points of view at the same time. The painting shown is Picasso's *Factory, Horta de Ebbo* (1909).

When calculating the surface area of a 3-D shape, the same formula can often be used in different ways. For example, the formula for the surface area of a right prism, a right cylinder, or a right cone can be written in two forms:

Shape	Formula #1	Formula #2
Right prism 	$SA = 2lw + 2lh + 2wh$	$SA = 2(lw + lh + wh)$
Right cylinder 	$SA = 2\pi r^2 + 2\pi rh$	$SA = 2\pi r(r + h)$
Right cone 	$SA = \pi r^2 + \pi rs$	$SA = \pi r(r + s)$

Compare the two surface area formulas for each shape. What is similar about the two formulas? What is different about the two formulas? Is there one surface area formula for each shape that you prefer to use? Explain.

Investigate Common Factors

1. a) Write the number 30 as a product of prime factors. How do you know the factors are prime?

b) Can you write the number 1 as a product of prime factors? Explain why or why not.

c) Can you write the number 0 as a product of prime factors? Explain why or why not.

2. Write each of the following pairs of numbers as a product of prime factors. Identify the **greatest common factor (GCF)** of each pair.

a) 60 and 48 **b)** 25 and 40

c) 16, 24, and 36

greatest common factor (GCF)

- the largest factor shared by two or more terms
- for example, the GCF of 12 and 28 is 4

3. Identify the **least common multiple (LCM)** of each pair of numbers.

a) 12 and 15 **b)** 20 and 25

c) 18 and 32

least common multiple (LCM)

- the smallest multiple shared by two or more terms
- Multiples of 6 and 8 are 24, 48, 72, … . The LCM is 24.

4. a) What is the GCF of 72 and 48?

b) Write each number as the product of two factors, where one factor is the GCF.

c) Explain how you determined the second factor.

5. a) Identify the GCF of each pair of terms.

6^2 and 6^3 8^4 and 8^7 x^5 and x^2

b) Compare the methods you used to identify the GCF of whole numbers and the GCF of variable terms. What are the similarities and differences between the methods?

6. a) Identify the GCF of x^5 and x^7.

b) Write each term as the product of two factors, where one factor is the GCF.

c) Explain how you determined the second factor.

7. a) Identify the GCF of the polynomial $12x^4 + 8x^3$.

How would writing each term as a product help?

b) Rewrite the polynomial as the sum of products. Express each term as a product of two factors, where the first factor is the GCF.

c) Explain how you determined the second factor.

8. Reflect and Respond Explain how to factor a polynomial using the GCF.

Link the Ideas

Factor out the GCF from a polynomial by dividing each term by the GCF. Then, the polynomial can be written in a simpler form to solve more complex problems.

$15x^2 + 10x = 5x(3x + 2)$

Example 1 Determine the Greatest Common Factor

Determine the GCF of $16x^2y$ and $24x^2y^3$.

Did You Know?

For *prime factorization*, write a number as the product of prime numbers. For example, the prime factorization of 18 is $2 \times 3 \times 3$.

Solution

Method 1: Use Prime Factorization

List the prime factorization of the numerical coefficients.

$16 = (2)(2)(2)(2)$

$24 = (2)(2)(2)(3)$

$$\begin{aligned}\text{Numerical GCF} &= (2)(2)(2)\\ &= 8\end{aligned}$$

List the prime factorization of the variables.

$x^2y = (x)(x)(y)$

$x^2y^3 = (x)(x)(y)(y)(y)$

$$\begin{aligned}\text{Variable GCF} &= (x)(x)(y)\\ &= x^2y\end{aligned}$$

Therefore, the GCF of $16x^2y$ and $24x^2y^3$ is $8x^2y$.

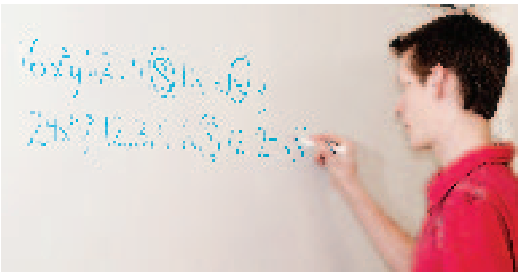
Method 2: List the Factors

Write the factors of each term.

$16x^2y$: 1, 2, 4, 8, 16, x, x^2, y

$24x^2y^3$: 1, 2, 3, 4, 6, 8, 12, 24, x, x^2, y, y^2, y^3

The greatest common factors are 8, x^2, and y.
Therefore, the GCF of $16x^2y$ and $24x^2y^3$ is $8x^2y$.



Your Turn

Determine the GCF of each pair of terms.

a) $5m^2n$ and $15mn^2$

b) $48ab^3c$ and $36a^2b^2c^2$

Example 2 Write a Polynomial in Factored Form

Write $7a^2b - 28ab + 14ab^2$ in factored form.

Solution

Identify the GCF of the numerical coefficients by listing the prime factorization for each coefficient.

$7 = 7$
$28 = (2)(2)(7)$
$14 = (2)(7)$
The GCF is 7.

Identify the GCF of the variables.

$a^2b = (a)(a)(b)$

$ab = (a)(b)$

$ab^2 = (a)(b)(b)$

The GCF is ab.

Therefore, the GCF of $7a^2b - 28ab + 14ab^2$ is $7ab$.

Divide each term by the GCF.
$7a^2b - 28ab + 14ab^2 = 7ab(a - 4 + 2b)$

Check:
Multiply.

$$\begin{aligned} 7ab(a - 4 + 2b) &= (7ab)(a) + (7ab)(-4) + (7ab)(2b) \\ &= 7a^2b - 28ab + 14ab^2 \end{aligned}$$

Multiplying is the reverse of factoring.

Your Turn

Write each polynomial in factored form.

a) $27r^2s^2 - 18r^3s^2 - 36rs^3$
b) $4np^2 + 10n^4p - 12n^3p$

Example 3 Determine Binomial Factors

Write each expression in factored form.

a) $3x(x - 4) + 5(x - 4)$

b) $y^2 + 8xy + 2y + 16x$

Solution

a) The GCF can be a binomial expression. The GCF for the terms $3x(x - 4)$ and $5(x - 4)$ is $(x - 4)$.

$3x(x - 4) + 5(x - 4) = (x - 4)(3x + 5)$

b) A GCF may be found by grouping terms.

$$\begin{aligned} y^2 + 8xy + 2y + 16x &= (y^2 + 8xy) + (2y + 16x) \\ &= y(y + 8x) + 2(y + 8x) \\ &= (y + 2)(y + 8x) \end{aligned}$$

Check:

Multiply.

$$\begin{aligned} (y + 2)(y + 8x) &= y(y + 8x) + 2(y + 8x) \\ &= y^2 + 8xy + 2y + 16x \end{aligned}$$

Your Turn

Write each expression in factored form.

a) $4(x + 5) - 3x(x + 5)$

b) $a^2 + 8ab + 2a + 16b$

Example 4 Using the Greatest Common Factor to Solve a Problem

Paula has 18 toonies, 30 loonies, and 48 quarters. She wants to group her money so that each group has the same number of each coin and there are no coins left over.

a) What is the maximum number of groups she can make?

b) How many of each coin will be in each group?

c) How much money will each group be worth?

Solution

a) To find the maximum number of groups, identify the GCF of 18, 30, and 48.
The factors of 18 are 1, 2, 3, 6, 9, and 18.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.
The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.
The GCF is 6. Therefore, the maximum number of groups is 6.

b) Divide each number of coins by the GCF.
$\frac{18}{6} = 3 \qquad \frac{30}{6} = 5 \qquad \frac{48}{6} = 8$
There will be 3 toonies, 5 loonies, and 8 quarters in each group.

c) Multiply the number of each coin by its value.
Toonies: (3)($2) = $6
Loonies: (5)($1) = $5
Quarters: (8)($0.25) = $2
6 + 5 + 2 = 13
Each group will have a value of $13.

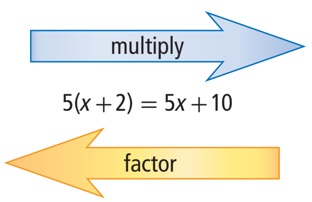
Your Turn

The students in Mr. Noyle's Construction class have decided they want to build dog houses for their class project. The class will be split up into groups. Each group will construct their dog house with the same type and amount of lumber. Mr. Noyle has 24 ten foot 1 by 4s, 32 eight foot 2 by 4s, and 8 sheets of plywood (4′ by 8′) available to use for this project.

a) What is the maximum number of groups of students that can build dog houses?
b) How much of each lumber type will each group have to work with?
c) What is the total length of 2 by 4s and 1 by 4s that each group will have to work with?

Key Ideas

• Factoring is the reverse of multiplying.



- To find the GCF of a polynomial find the GCF of the coefficients and variables.
- To factor a GCF from a polynomial divide each term by the GCF.
- Polynomials can be written as a product of the GCF and the sum or difference of the remaining factors.

  $2m^3n^2 - 8m^2n + 12mn^4 = 2mn(m^2n - 4m + 6n^3)$
- A common factor can be any polynomial, such as a binomial.

  $a(x + 2) - b(x + 2)$ has a common factor of $x + 2$.

Check Your Understanding

Practise

1. Copy the table. List all of the factors of each pair of numbers. Then, identify the greatest common factor (GCF).

a) 20: 30: GCF:	**b)** 28: 40: GCF:
c) 30: 48: GCF:	**d)** 36: 27: GCF:

2. Identify the GCF of the following sets of numbers.

a) 48 and 36
b) 144 and 96
c) 81 and 54
d) 256, 216, and 78
e) 50, 100, and 625

3. Identify the least common multiple (LCM) of the following sets of numbers.

a) 12 and 16
b) 15 and 20
c) 18 and 30
d) 10, 15, and 25
e) 22, 33, and 44

4. Identify the GCF of the following sets of terms.

a) $15a^2b$ and $18ab$
b) $27m^2n^3$ and $81m^3n$
c) $8x^2y^2$ and $24x^3y^3$
d) $12a^3bc^2$, $28a^2c$, and $36a^2b^2c^2$
e) $14p^4q^5$, $-24p^5q^4$, and $7p^3q^3$

5. Factor the following polynomials.

a) $5x + 15$

b) $3y^2 - 5y$

c) $w^2x + w^2y - w^2z$

d) $6a^3b - 18ab^2$

e) $9x^3 - 12x^2 + 6x$

6. State the missing factor.

a) $6a^2bc + 9ab^2 = (\blacksquare)(2ac + 3b)$

b) $3s^2 - 15 = 3(\blacksquare)$

c) $3d^2 - 21d = 3d(\blacksquare)$

d) $16x^2 - 2x = 2x(\blacksquare)$

e) $12x^2y^2 - 16xy = (\blacksquare)(3xy - 4)$

7. Factor the following polynomials.

a) $3y(y - 2) + 4(y - 2)$

b) $5a(a - 4) - 2(a - 4)$

c) $2cx - 8x + 7c - 28$

d) $3x^2 - 9x - 8x + 24$

e) $2y^4 + y^3 - 10y - 5$

Apply

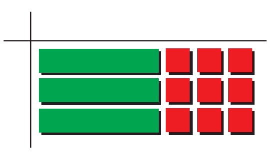
8. Mei is stacking toy blocks that are 12 cm tall next to blocks that are 18 cm tall. What is the shortest height at which the two stacks will be the same height?

9. Explain the difference between listing the factors of a number and listing the multiples of a number.

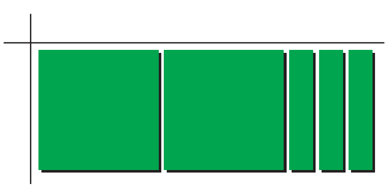
10. The models show rectangles of algebra tiles. Answer the following questions for each rectangle.

- What expression does each model represent?
- What are the possible dimensions that could produce each rectangle?
- Write an expression for each model, using your dimensions.

a)



b)

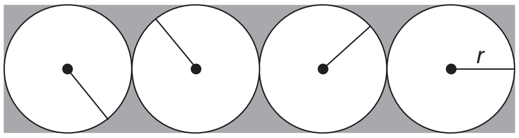


11. a) Write a polynomial with two terms that have a GCF of $6x$.
    b) Write a polynomial with two terms that have a GCF of $4ab$.
    c) Write a polynomial with three terms that have a GCF of $2m^3n^2$.

12. Each of the following factored polynomials has an error or is not fully factored. Describe what needs to be fixed and correct each one.
    a) $15x^2 - 3x = 3x(5x - 0)$
    b) $5x(x - 2) - (x - 2) = (x - 2)(5x)$
    c) $9a^2b^3 - 27a^2b^2 + 81a^3b^3 = 9ab(ab^2 - 3ab + 9a^2b^2)$
    d) $4fx + 16f + 2x + 8 = 2f(2x + 8) + 1(2x + 8)$
    e) $2p^2 - 20p + 6p - 10 = 2p(p - 10 - 3) - 10$
    $= 2p(p - 23)$

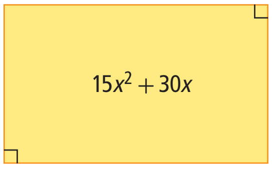
13. Nikolai has 30 pencils, 48 pens, and 36 erasers. He needs to package these items in containers for the participants of a workshop he is giving. He wants to divide them into identical containers, so that each container has the same number of each of the pencils, pens, and erasers. If he wants each container to have the greatest number of items possible, how many plastic containers does he need?

14. Some natural gas meters have four dials to show the gas use. Write a factored expression to represent the area of the metal plate around the dials, shaded in grey.



15. Mario wants to cut two pieces of material into equal-size squares with no material wasted. One piece measures 12 in. by 36 in. The other measures 6 in. by 42 in. What is the largest size square that he can cut?

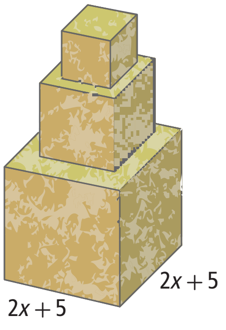
16. A rectangle has an area that can be represented by the expression $15x^2 + 30x$. The length and width can be found by factoring the expression. Write possible expressions for the length and the width.



Extend

17. The greatest common factor of two numbers is 871. Both numbers are even. Neither is divisible by the other. What are the smallest two numbers they could be?

18. The pedestal design is made up of square-based prisms. The base length of each prism is 3 cm less than that of the layer immediately below.



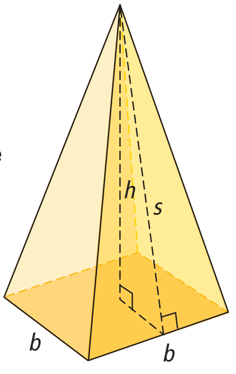
a) Write an algebraic expression for the total top surface area of the three prisms used to make the pedestal.

b) Multiply and then simplify.

c) Factor the expressions from part b).

Create Connections

19. The surface area of a square-based pyramid is $SA = b^2 + 2bs$.



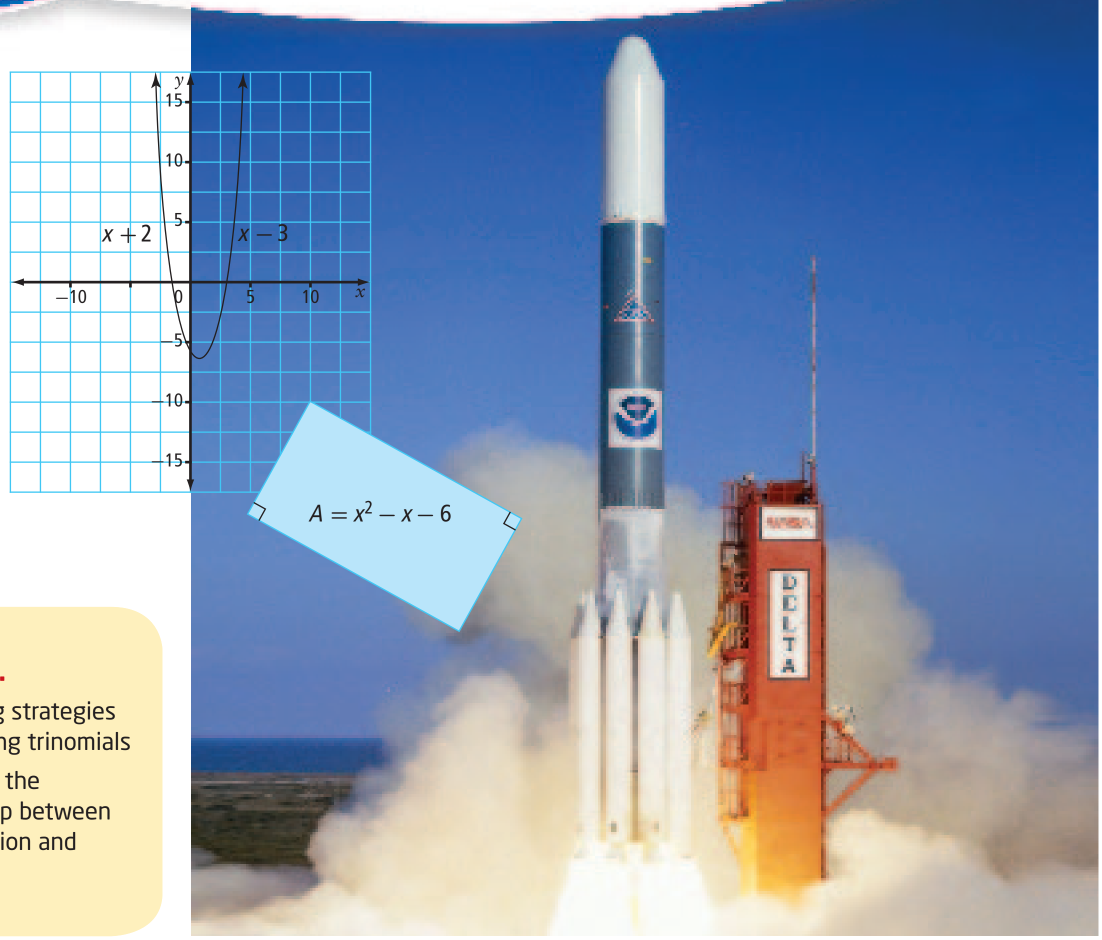
a) Write the formula in factored form.

b) Use both forms of the formula to calculate the surface area when $b = 5$ cm and $s = 4$ cm.

c) What is the same about each surface area you calculated? What is different about each surface area you calculated?

d) Is there one surface area formula you prefer to use? Explain.

5.3 Factoring Trinomials



Focus on ...

- developing strategies for factoring trinomials
- explaining the relationship between multiplication and factoring

The trinomial, rectangle, and graph shown all have something in common. They each represent the same relationship. These kinds of relationships allow us to represent physical situations, such as the dimensions of a field or the height of a rocket, with mathematical expressions. Then, the expressions can be used to solve a variety of real-life problems.

Materials

- algebra tiles

Investigate Factoring Trinomials

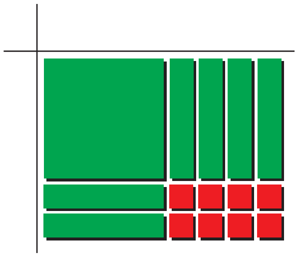
1. a) Use algebra tiles to model $(x + 4)(x + 1)$ as the dimensions of a rectangle.

b) Complete the rectangle. What is the product of $(x + 4)(x + 1)$?

c) How is the product represented by the algebra tiles?

d) How are the factors represented by the algebra tiles?

2. a) Use algebra tiles to factor the trinomial $x^2 + 6x + 8$. Create a rectangle so that the length and width represent the factors of the trinomial.



b) Place tiles along the top and left side of the rectangle to show the length and width of the rectangle. What are the two dimensions?

c) Record the dimensions as a product of binomials. What is this product equivalent to?

d) Multiply the two binomials. Compare the result to the original trinomial. Are they equivalent?

3. Repeat step 2 for each trinomial.

a) $x^2 + 5x + 6$

b) $x^2 + 8x + 12$

c) $x^2 + 3x + 2$

4. Each trinomial in step 2 and step 3 is of the form $x^2 + bx + c$. What do you notice about b and c and the binomial factors for each trinomial? Describe the relationship.

5. Test your observations from step 4 on each of the following trinomials. Use algebra tiles to check your answer.

a) $x^2 + 7x + 6$

b) $x^2 + 8x + 15$

6. **Reflect and Respond** Describe a process for finding the factors of a trinomial of the form $x^2 + bx + c$.

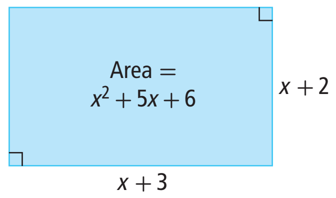
Link the Ideas

A rectangle can have an area that is a trinomial. By finding the dimensions of the rectangle, you are reversing the process of multiplying two binomials. This process is called *factoring*.

You can factor a trinomial of the form $x^2 + bx + c$ and the form $ax^2 + bx + c$ by studying patterns. Observe patterns that result from multiplying two binomials.

Factor Trinomials of the Form $ax^2 + bx + c$, $a = 1$

Multiply $x + 2$ and $x + 3$.



$$\begin{aligned}(x + 2)(x + 3) &= x^2 + 3x + 2x + (2)(3)\\ &= x^2 + 3x + 2x + (2)(3)\\ &= x^2 + (3 + 2)x + (2)(3)\end{aligned}$$

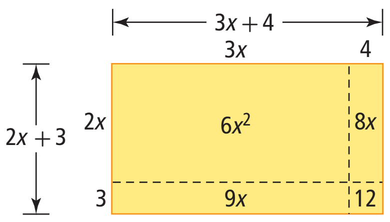
Since $(x + 2)(x + 3) = x^2 + 5x + 6$
and $(x + 2)(x + 3) = x^2 + (3 + 2)x + (2)(3)$,
then $x^2 + 5x + 6 = x^2 + (3 + 2)x + (2)(3)$.

Note that $3 + 2 = 5$ and $(2)(3) = 6$.

To factor trinomials of the form $x^2 + bx + c$, you can use patterns. Replace bx with two terms whose integer coefficients have a sum of b and a product of c.

Factor Trinomials of the Form $ax^2 + bx + c$, $a \neq 1$

Multiply $2x + 3$ and $3x + 4$.



$$\begin{aligned}(2x + 3)(3x + 4) &= 6x^2 + 8x + 9x + 12\\ &= 6x^2 + 17x + 12\end{aligned}$$

You can combine the two middle terms because they are like terms.

Notice the patterns:
- The sum of 8 and 9 is 17.
- The product of 8 and 9 is the same as (6)(12).

To factor trinomials of the form $ax^2 + bx + c$, you can use patterns. Replace bx with two terms whose integer coefficients have a sum of b and a product of $(a)(c)$.

Example 1 Factor Trinomials of the Form $ax^2 + bx + c$, $a = 1$

Factor, if possible.

a) $x^2 + 5x + 4$

b) $x^2 + 4x + 6$

c) $x^2 - 29x + 28$

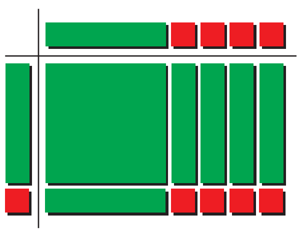
d) $x^2 + 3xy - 18y^2$

Solution

a) Factor $x^2 + 5x + 4$.

Method 1: Use Algebra Tiles

Arrange one x^2-tile, five x-tiles, and four 1-tiles into a rectangle. Then, add tiles to show the dimensions.



The dimensions of the rectangle are $x + 4$ and $x + 1$.
Therefore, the factors are $x + 4$ and $x + 1$.

How do you know that the dimensions are correct?

Method 2: Use a Table

Use a table to find two integers with
- a product of 4
- a sum of 5

In order to have a positive product and a positive sum, what signs do the two integers need to have?

Factors of 4	Product	Sum
1, 4	4	5
2, 2	4	4

Therefore, the factors are $x + 1$ and $x + 4$.

Check:
Multiply.

$$\begin{aligned}(x + 4)(x + 1) &= x(x + 1) + 4(x + 1)\\ &= x^2 + 1x + 4x + 4\\ &= x^2 + 5x + 4\end{aligned}$$

Did You Know?

When a polynomial cannot be factored such that the factors include only integer coefficients, we say that the polynomial cannot be factored *over the integers*.

b) Use a table to find two integers with
- a product of 6
- a sum of 4

In order to have a positive product and a positive sum, what signs do the two integers need to have?

Factors of 6	Product	Sum
1, 6	6	7
2, 3	6	5

No two integers have a product of 6 and sum of 4.
Therefore, you cannot factor $x^2 + 4x + 6$ over the integers.

c) Use a table to find two integers with
- a product of 28
- a sum of -29

In order to have a positive product and a negative sum, what signs do the two integers need to have?

Factors of 28	Product	Sum
$-1, -28$	28	-29
$-2, -14$	28	-16
$-4, -7$	28	-11

Therefore, the factors are $x - 1$ and $x - 28$.

Check:
Multiply.

$$\begin{aligned}(x - 1)(x - 28) &= x(x - 28) - 1(x - 28)\\ &= x^2 - 28x - 1x + 28\\ &= x^2 - 29x + 28\end{aligned}$$

d) Use a table to find two integers with
- a product of -18
- a sum of 3

In order to have a negative product and a positive sum, what signs do the two integers need to have?

Factors of -18	Product	Sum
$1, -18$	-18	-17
$2, -9$	-18	-7
$3, -6$	-18	-3
$6, -3$	-18	3
$9, -2$	-18	7
$18, -1$	-18	17

Therefore, the factors are $x + 6y$ and $x - 3y$.
Check:
Multiply.

$$\begin{aligned}(x + 6y)(x - 3y) &= x(x - 3y) + 6y(x - 3y)\\ &= x^2 - 3xy + 6xy - 18y^2\\ &= x^2 + 3xy - 18y^2\end{aligned}$$

Your Turn

Factor, if possible.

a) $x^2 + 7x + 10$

b) $r^2 - 10rs + 9s^2$

Example 2 Factor Trinomials of the Form $ax^2 + bx + c, a \neq 1$

Factor, if possible.

a) $3x^2 + 8x + 4$

b) $6x^2 - 5xy + y^2$

c) $3x^2 + 2x + 4$

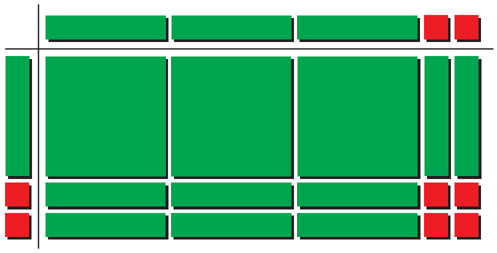
d) $24x^2 - 30x - 9$

Solution

a) First, check for a GCF. The GCF of the polynomial $3x^2 + 8x + 4$ is 1.

Method 1: Use Algebra Tiles

Arrange three x^2-tiles, eight x-tiles, and four 1-tiles into a rectangle. Then, add tiles to show the dimensions.



The dimensions of the resulting rectangle are $3x + 2$ and $x + 2$.

How do you know that the dimensions are correct?

Check:

Multiply.

$$\begin{aligned}(3x + 2)(x + 2) &= 3x(x + 2) + 2(x + 2)\\ &= 3x^2 + 6x + 2x + 4\\ &= 3x^2 + 8x + 4\end{aligned}$$

Method 2: Use a Table

Use a table to find two integers with
- a product of $(3)(4) = 12$
- a sum of 8

What signs do the two integers need to have?

Factors of 12	Product	Sum
1, 12	12	13
2, 6	12	8
3, 4	12	7

Write $8x$ as the sum $2x + 6x$. Then, factor by grouping.

$$\begin{aligned}3x^2 + 8x + 4 &= 3x^2 + (2x + 6x) + 4\\ &= (3x^2 + 2x) + (6x + 4)\\ &= x(3x + 2) + 2(3x + 2)\\ &= (3x + 2)(x + 2)\end{aligned}$$

Therefore, the factors are $3x + 2$ and $x + 2$.

Check:

Multiply.

$$\begin{aligned}(3x + 2)(x + 2) &= 3x(x + 2) + 2(x + 2)\\ &= 3x^2 + 6x + 2x + 4\\ &= 3x^2 + 8x + 4\end{aligned}$$

b) First, check for a GCF. The GCF of the polynomial $6x^2 - 5xy + y^2$ is 1. Use a table to find two integers with

- a product of 6
- a sum of -5

Factors of $6y^2$	Product	Sum
$-1, -6$	6	-7
$-2, -3$	6	-5

Write $-5xy$ as $-2xy - 3xy$. Then, factor by grouping.

$$\begin{aligned} 6x^2 - 5xy + y^2 &= 6x^2 + (-2xy - 3xy) + y^2 \\ &= (6x^2 - 2xy) + (-3xy + y^2) \\ &= 2x(3x - y) - y(3x - y) \\ &= (3x - y)(2x - y) \end{aligned}$$

Therefore, the factors are $3x - y$ and $2x - y$.

Check:

Multiply.

$$\begin{aligned} (3x - y)(2x - y) &= 3x(2x - y) - y(2x - y) \\ &= 6x^2 - 3xy - 2xy + y^2 \\ &= 6x^2 - 5xy + y^2 \end{aligned}$$

What signs do the two integers need to have?

c) First, check for a GCF. The GCF of the polynomial $3x^2 + 2x + 4$ is 1. Use a table to find two integers with

- a product of $(3)(4) = 12$
- a sum of 2

Factors of 12	Product	Sum
1, 12	12	13
2, 6	12	8
3, 4	12	7

No two integers have a product of 12 and sum of 2.
Therefore, you cannot factor $3x^2 + 2x + 4$ over the integers.

What signs do the two integers need to have?

d) First, remove the greatest common factor (GCF). The GCF of the polynomial is 3. Therefore, $24x^2 - 30x - 9 = 3(8x^2 - 10x - 3)$. Use a table to find two integers with

- a product of $(8)(-3) = -24$
- a sum of -10

Factors of -24	Product	Sum
$-1, 24$	-24	23
$-2, 12$	-24	10
$-3, 8$	-24	5
$-4, 6$	-24	2
$-6, 4$	-24	-2
$-8, 3$	-24	-5
$-12, 2$	-24	-10
$-24, 1$	-24	-23

What signs do the two integers need to have?

Write $-10x$ as $-12x + 2x$. Then, factor by grouping.

$$\begin{aligned} 3(8x^2 - 10x - 3) &= 3(8x^2 - 12x + 2x - 3) \\ &= 3[(8x^2 - 12x) + (2x - 3)] \\ &= 3[4x(2x - 3) + 1(2x - 3)] \\ &= 3(4x + 1)(2x - 3) \end{aligned}$$

Therefore, the factors are 3, $4x + 1$, and $2x - 3$.

Check:

Multiply.

$$\begin{aligned} 3(4x + 1)(2x - 3) &= 3[4x(2x - 3) + 1(2x - 3)] \\ &= 3(8x^2 - 12x + 2x - 3) \\ &= 3(8x^2 - 10x - 3) \\ &= 24x^2 - 30x - 9 \end{aligned}$$

Your Turn

Factor, if possible.

a) $2x^2 + 7x - 4$

b) $-3s^2 - 51s - 30$

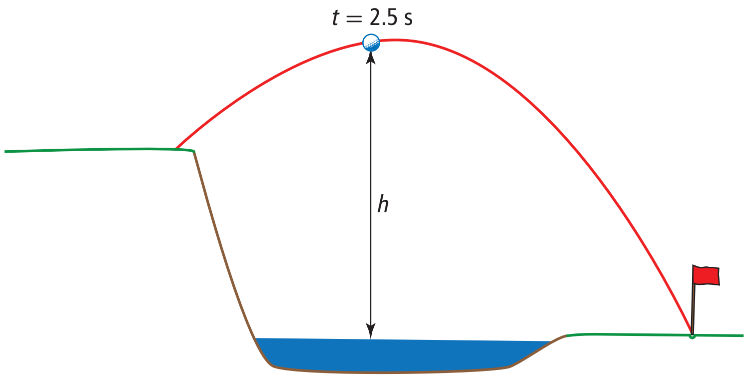
c) $2y^2 + 7xy + 3x^2$

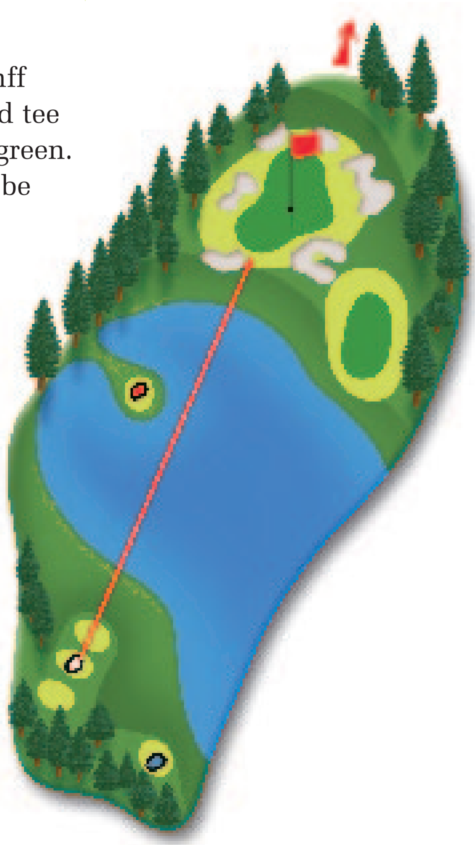
Example 3 Apply Factoring

The world famous *Devil's Cauldron* is the 4th hole at the Banff Springs Golf Course. This is a tough tee shot from an elevated tee that must carry the ball across a glacial lake to a small bowl green. The approximate height of the ball during a typical shot can be represented by the formula $h = -5t^2 + 25t + 30$, where t is the time, in seconds, and h is the height of the ball relative to the green, in metres.

a) Write the formula in factored form.

b) What is the height of the golf ball after 2.5 s?





Why is it easier if you remove a GCF of -5 instead of $+5$?

Solution

a) The expression for the height of the golf ball can be factored by first removing the GCF. The GCF of -5, 25, and 30 is -5.

$-5t^2 + 25t + 30 = -5(t^2 - 5t - 6)$

Use a table to find two integers with

- a product of -6
- a sum of -5

Factors of –6	Product	Sum
1, –6	–6	–5
2, –3	–6	–1
3, –2	–6	1
6, –1	–6	5

Therefore, the factors are $t + 1$ and $t - 6$.
The factored form is $h = -5(t + 1)(t - 6)$.

Check:
Multiply.

$$\begin{aligned}-5(t + 1)(t - 6) &= -5[t(t - 6) + 1(t - 6)]\\ &= -5(t^2 - 6t + t - 6)\\ &= -5(t^2 - 5t - 6)\\ &= -5t^2 + 25t + 30\end{aligned}$$

b) Substitute $t = 2.5$ into $h = -5t^2 + 25t + 30$ or $h = -5(t + 1)(t - 6)$.

$h = -5(2.5)^2 + 25(2.5) + 30$ or $h = -5(2.5 + 1)(2.5 - 6)$
$h = -5(6.25) + 62.5 + 30$ $\quad$ $h = -5(3.5)(-3.5)$
$h = -31.25 + 62.5 + 30$ $\quad$ $h = 61.25$
$h = 61.25$

After 2.5 s, the golf ball is 61.25 m above the green.

Your Turn

A rescue worker launches a signal flare into the air from the side of a mountain. The height of the flare can be represented by the formula $h = -16t^2 + 144t + 160$. In the formula, h is the height, in feet, above ground, and t is the time, in seconds.

a) What is the factored form of the formula?
b) What is the height of the flare after 5.6 s?

Key Ideas

- To factor a trinomial of the form $x^2 + bx + c$, first find two integers with
  - a product of c
  - a sum of b

  For $x^2 + 12x + 27$, find two integers with
  - a product of 27
  - a sum of 12

  The two integers are 3 and 9.
  Therefore, the factors are $x + 3$ and $x + 9$.

- To factor a trinomial of the form $ax^2 + bx + c$, first factor out the GCF, if possible. Then, find two integers with
  - a product of $(a)(c)$
  - a sum of b

  Finally, write the middle term as a sum. Then, factor by grouping.

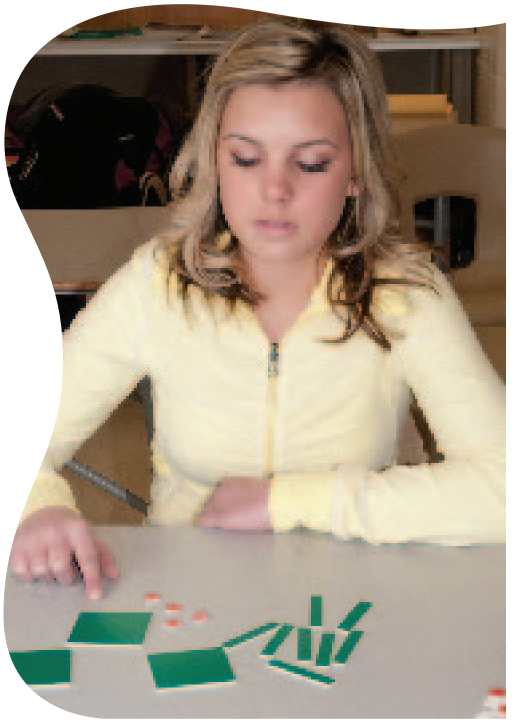
  For $8k^2 - 16k + 6$, the GCF is 2, so $8k^2 - 16k + 6 = 2(4k^2 - 8k + 3)$.

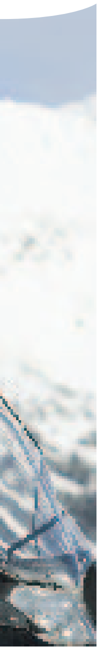
  Identify two integers with
  - a product of $(4)(3) = 12$
  - a sum of -8

  The two integers are -2 and -6. Use these two integers to write the middle term as a sum. Then, factor by grouping.

  $2(4k^2 - 2k - 6k + 3) = 2(2k - 3)(2k - 1)$

- You cannot factor some trinomials, such as $x^2 + 3x + 5$ and $3x^2 + 5x + 4$, over the integers.

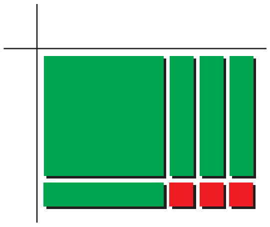


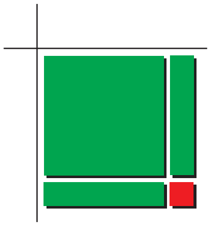


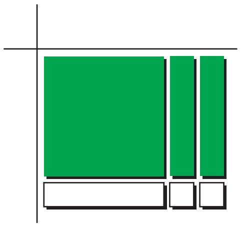
Check Your Understanding

Practise

1. Write the trinomial represented by each rectangle of algebra tiles. Then, determine the dimensions of each rectangle.

   a) 

   b) 

   c) 

   d) 

2. Use algebra tiles or a diagram to factor each trinomial.
   a) $2x^2 + 5x + 3$
   b) $3x^2 + 7x + 4$
   c) $3x^2 + 7x - 6$
   d) $6x^2 + 11x + 4$

3. Identify two integers with the given product and sum.
   a) product $= 45$, sum $= 14$
   b) product $= 6$, sum $= -5$
   c) product $= -10$, sum $= 3$
   d) product $= -20$, sum $= -8$

4. Factor, if possible.
   a) $x^2 + 7x + 10$
   b) $j^2 + 12j + 27$
   c) $k^2 + 5k + 4$
   d) $p^2 + 9p + 12$
   e) $d^2 + 10d + 24$
   f) $c^2 + 4cd + 21d^2$

5. Factor each trinomial.
   a) $m^2 - 7m + 10$
   b) $s^2 + 3s - 10$
   c) $f^2 - 7f + 6$
   d) $g^2 - 5g - 14$
   e) $b^2 - 3b - 4$
   f) $2r^2 - 14rs + 24s^2$

6. Factor, if possible.

a) $2x^2 + 7x + 5$ **b)** $6y^2 + 19y + 8$

c) $3m^2 + 10m + 8$ **d)** $10w^2 + 15w + 3$

e) $12q^2 + 17q + 6$ **f)** $3x^2 + 7xy + 2y^2$

7. Factor, if possible.

a) $4x^2 - 11x + 6$ **b)** $w^2 + 11w + 25$

c) $x^2 - 5x + 6$ **d)** $2m^2 + 3m - 9$

e) $6x^2 - 3xy - 3y^2$ **f)** $12y^2 + y - 1$

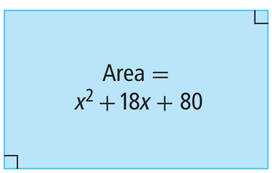
g) $6c^2 + 7cd - 10d^2$ **h)** $4k^2 + 15k + 9$

i) $a^2 + 11ab + 24b^2$ **j)** $6m^2 + 13mn + 2n^2$

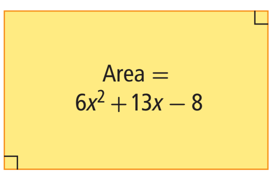
Apply

8. Identify binomials that represent the length and width of each rectangle. Then, calculate the dimensions of the rectangle if $x = 15$ cm.

a)



b)



9. Determine two values of b that allow each expression to be factored.

a) $x^2 + bx + 12$ **b)** $y^2 - by + 4$

c) $x^2 - bx - 8$ **d)** $p^2 + bp - 10$

10. Determine two values of c that allow each expression to be factored.

a) $x^2 + 6x + c$ **b)** $a^2 - 8a - c$

c) $x^2 - x + c$ **d)** $w^2 + 2w - c$

11. Find two values of n that allow each trinomial to be factored over the integers.

a) $x^2 + nx + 16$

b) $3y^2 + ny + 25$

c) $6a^2 + nab + 7b^2$

12. Determine one value of k that allows each trinomial to be factored over the integers.

a) $36m^2 + 18m + k$

b) $18x^2 - 15x + k$

c) $kp^2 - 18pq + 16q^2$

13. a) Make up an example of a trinomial expression that cannot be factored.

b) Explain why it cannot be factored.

14. **Unit Project** Use algebra tiles or area models to show the following relationships. Create a poster displaying your models.

a) the relationship between a monomial multiplied by a binomial and common factoring

b) the relationship between a binomial multiplied by a binomial and factoring a trinomial of the form $ax^2 + bx + c$, where a, b, and c are integers

15. You can estimate the height, h, in metres, of a toy rocket at any time, t, in seconds, during its flight. Use the formula $h = -5t^2 + 23t + 10$. Write the formula in factored form. Then, calculate the height of the rocket 3 s after it is launched.

16. The total revenue from sales of ski jackets can be modelled by the expression $720 + 4x - 2x^2$, where x represents the number of jackets sold above the minimum needed to break even. Revenue is also calculated as the product of the number of jackets sold and the price per jacket. Factor the given expression to determine the number sold and the price per jacket. The minimum price of a jacket is \$18.
Hint: As the price increases, the number sold decreases.

Extend

17. Find three values of k such that the trinomial $3x^2 + kx + 5$ can be factored over the integers.

18. A square has an area of $9x^2 + 30xy + 25y^2$ square centimetres. What is the perimeter of the square? Explain how you determined your answer.

19. You have been asked to factor the expression $30x^2 - 39xy - 9y^2$ Explain how you would factor this expression. What are the factors?

20. The area of a certain shape can be represented by the expression $8x^2 + 10x - 7$.

a) Identify a possible shape.

b) Write expressions for the possible dimensions of the shape you identified in part a).

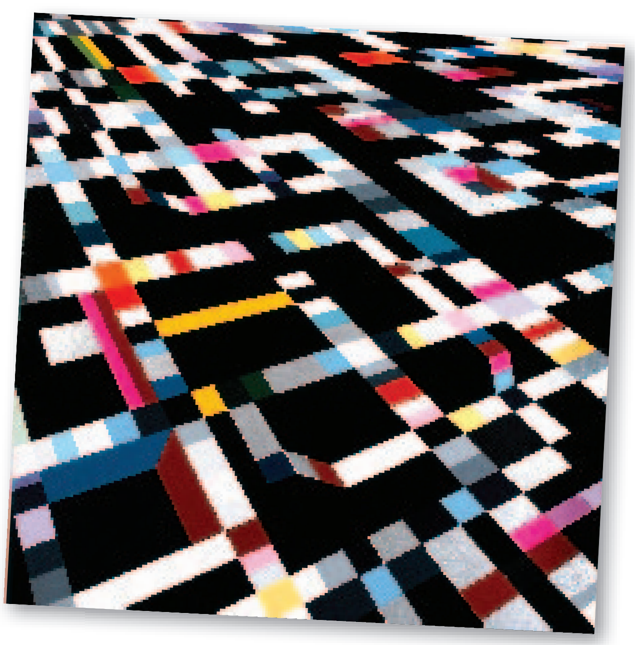
Create Connections

21. Describe, using examples, how multiplying binomials and factoring a trinomial are related.

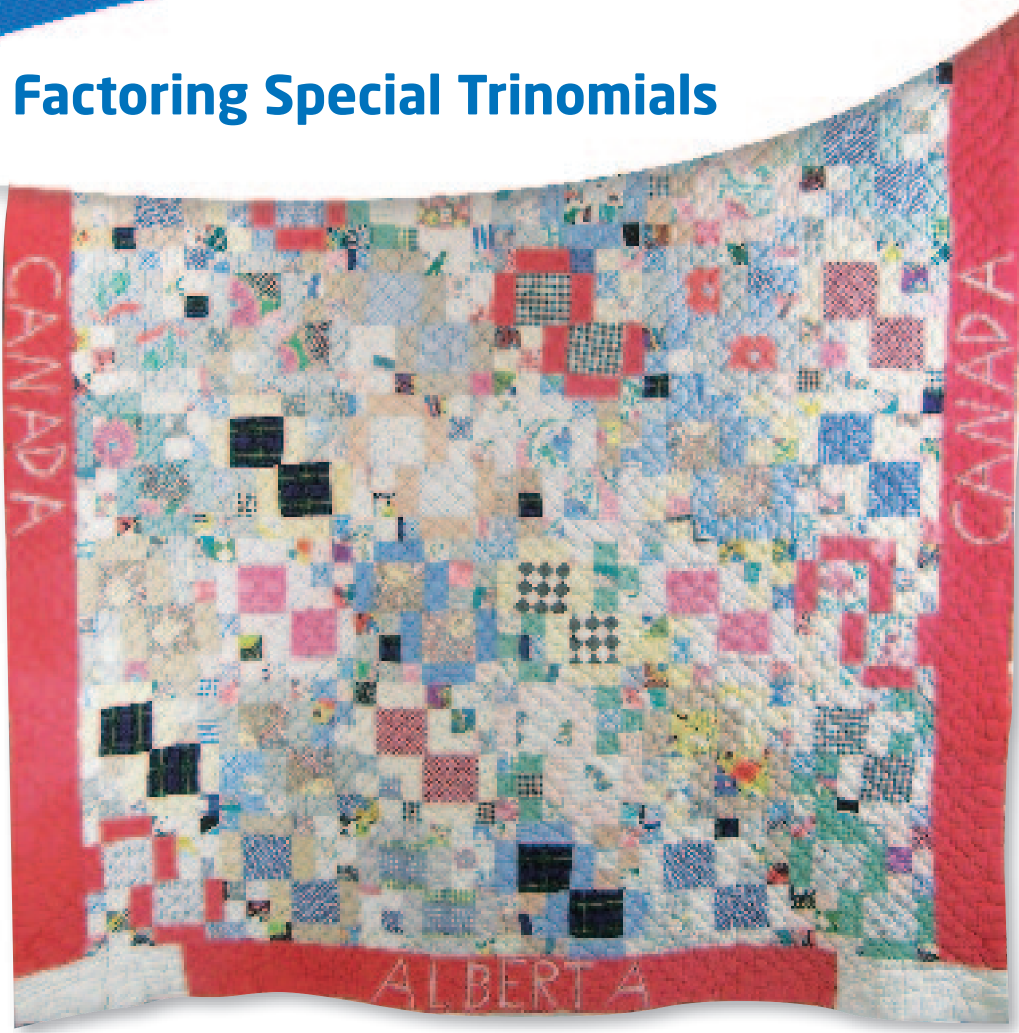
22. **Unit Project**

a) Use algebra tiles to create a model of a polynomial of your choice.

b) Create a piece of art that includes your polynomial in some way. Your artwork may be a drawing, painting, sculpture, or other form of your choice.



5.4 Factoring Special Trinomials



Focus on ...

- factoring the difference of squares
- factoring perfect squares

Did You Know?

Quilting has often been a way to unite people from different countries and cultures. The quilt shown here was part of a collection of quilts made by the Canadian Red Cross during WWII. These quilts were sent to families in Britain who had been displaced because of the war.

difference of squares

- an expression of the form $a^2 - b^2$ that involves the subtraction of two squares
- for example, $x^2 - 4$, $y^2 - 25$

Materials

- centimetre grid paper
- scissors

Patchwork quilts are made of square pieces of fabric sewed together to form interesting patterns. How could you relate these squares to polynomials and their factors?

Some polynomials, like perfect square trinomials and **differences of squares**, follow patterns that allow you to recognize a type of factoring method to use.

Investigate Factoring Differences of Squares

1. Cut a 10-cm by 10-cm square out of a piece of centimetre grid paper.

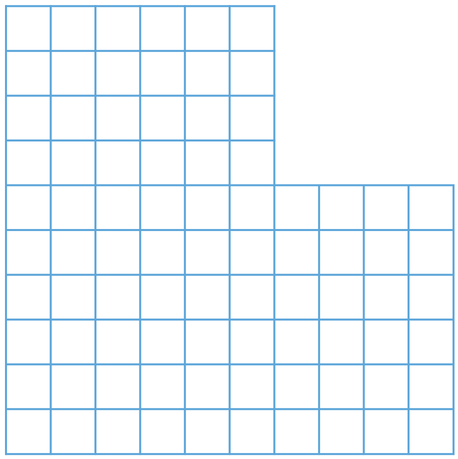
a) What is the area of the square?

b) How did you calculate this area?

2. Cut a 4-cm by 4-cm piece from the corner of the square.

a) What is the area of this cutout piece?

b) How did you calculate this area?



3. a) Calculate the area of the remaining paper.

b) How did you calculate this area?

c) Are there other methods you could use to calculate this area? Explain.

4. Make one cut to the irregular shape that remains, so that you can rearrange it to form a rectangle.

a) What are the dimensions of the rectangle?

b) What is the area of the rectangle?

c) How is the area of the shape in step 3 related to the area of this rectangle?

5. Repeat step 1 to step 4 for two additional squares of different sizes.

a) Can each irregular shape always be rearranged into a rectangle? Compare your answer with a partner's.

b) List the dimensions of each rectangle.

c) Explain how the area of the cutout shape relates to the area of the rectangle.

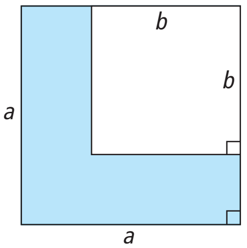
6. a) Write an algebraic expression to represent the area remaining when a square of area 25 cm^2 is removed from a square of area x^2 square centimetres.

b) If the resulting shape is rearranged into a rectangle, what are its dimensions?

c) Explain the relationship between your answers to parts a) and b).

d) Write an equation showing this relationship.

7. **Reflect and Respond** The diagram shows what remains when a square of dimensions b by b is removed from a square of dimensions a by a.



a) Write an expression to represent the area of the remaining shaded shape.

b) The shape is rearranged to form a rectangle. What are the dimensions of the rectangle?

c) Write an expression to represent the area of the rectangle.

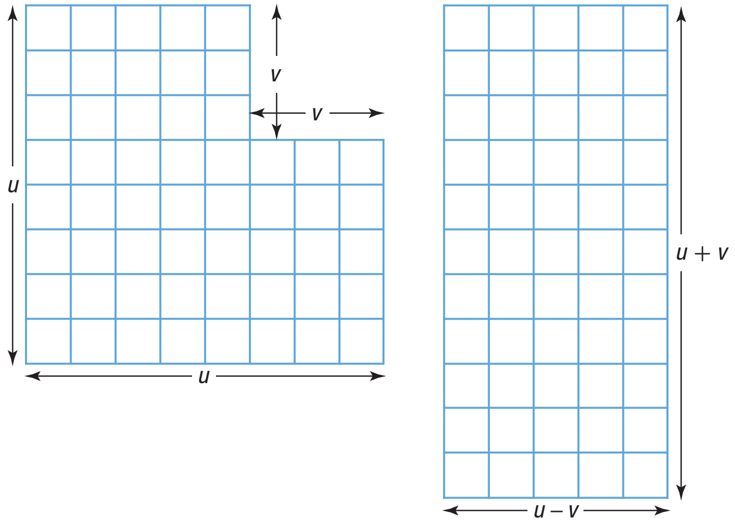
d) Write an equation to show the relationship between the area of the remaining shape and the area of the rectangle.

8. a) What are the patterns you observed from cutting out and rearranging the squares?

b) What conclusions can you make about subtracting the area of a smaller square from the area of a larger square?

Link the Ideas

When you cut a square out of a square, the area of the remaining shape is a difference of two squares. When you cut and rearrange this paper into a rectangle, you can write the area as a product of its dimensions.



You will find patterns helpful in factoring polynomials with special products. These include differences of squares and perfect square trinomials.

Difference of Squares

When you multiply the sum and the difference of two terms, the product will be a difference of squares.

$$\begin{aligned}(u + v)(u - v) &= (u)(u - v) + (v)(u - v)\\ &= (u)(u) - (u)(v) + (v)(u) + (v)(-v)\\ &= u^2 - uv + uv - v^2\\ &= u^2 - v^2\end{aligned}$$

In a difference of squares
- the expression is a binomial
- the first term is a perfect square: u^2
- the last term is a perfect square: v^2
- the operation between the two terms is subtraction

A difference of squares, $u^2 - v^2$, can be factored into $(u + v)(u - v)$.

Perfect Square Trinomial

When you square a binomial, the result is a perfect square trinomial.

$$\begin{aligned}(x + 5)^2 &= (x + 5)(x + 5)\\ &= x(x + 5) + 5(x + 5)\\ &= x^2 + 5x + 5x + 25\\ &= x^2 + 10x + 25\end{aligned}$$

In a perfect square trinomial
- the first term is a perfect square: x^2
- the last term is a perfect square: 5^2
- the middle term is twice the product of the square root of the first term and the square root of the last term: $(2)(x)(5) = 10x$

Example 1 Factor a Difference of Squares

Factor each binomial, if possible.

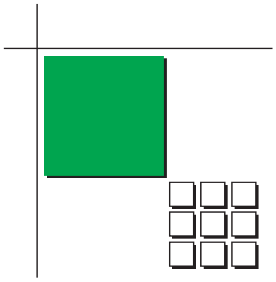
a) $x^2 - 9$ **b)** $16c^2 + 25a^2$

c) $m^2 + 16$ **d)** $7g^3h^2 - 28g^5$

Solution

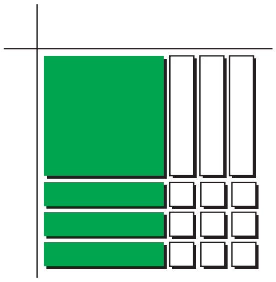
a) Method 1: Use Algebra Tiles

Create an algebra tile model to represent $x^2 - 9$.

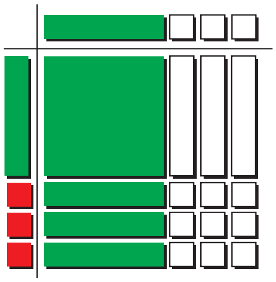


What is the result when you combine a positive x-tile and a negative x-tile?

Add three positive x-tiles and three negative x-tiles to represent the middle term.



The dimensions represent the factors of $x^2 - 9$.



The factors are $x - 3$ and $x + 3$.
Therefore, $x^2 - 9 = (x - 3)(x + 3)$.
Check:
Multiply.

$$\begin{aligned}(x - 3)(x + 3) &= x(x + 3) - 3(x + 3)\\ &= x^2 + 3x - 3x - 9\\ &= x^2 - 9\end{aligned}$$

Method 2: Factor by Grouping

$$\begin{aligned}x^2 - 9 &= x^2 - 3x + 3x - 9\\ &= (x^2 - 3x) + (3x - 9)\\ &= x(x - 3) + 3(x - 3)\\ &= (x - 3)(x + 3)\end{aligned}$$

The middle term must be included. Add in the zero pairs.

Method 3: Factor as a Difference of Squares

The binomial $x^2 - 9$ is a difference of squares.
The first term is a perfect square: x^2
The last term is a perfect square: 3^2
The operation is subtraction.

$$\begin{aligned} x^2 - 9 &= x^2 - 3^2 \\ &= (x - 3)(x + 3) \end{aligned}$$

b) You can write $-16c^2 + 25a^2$ as $25a^2 - 16c^2$.
The binomial $25a^2 - 16c^2$ is a difference of squares.
The first term is a perfect square: $(5a)^2$
The last term is a perfect square: $(4c)^2$
The operation is subtraction.

$$\begin{aligned} 25a^2 - 16c^2 &= (5a)^2 - (4c)^2 \\ &= (5a - 4c)(5a + 4c) \end{aligned}$$

Check:
Multiply.

$$\begin{aligned} (5a - 4c)(5a + 4c) &= (5a)(5a + 4c) + (-4c)(5a + 4c) \\ &= 25a^2 + 20ac - 20ac - 16c^2 \\ &= 25a^2 - 16c^2 \\ &= -16c^2 + 25a^2 \end{aligned}$$

c) The binomial $m^2 + 16$ can be written as a trinomial where the middle term is $0m$.
$m^2 + 0m + 16$
To factor this expression, you need to find two integers with
- a product of 16
- a sum of 0

Since the product is positive, both integers need to be either positive or negative.
If both integers are either positive or negative, a sum of 0 is not possible. Therefore, the binomial $m^2 + 16$ cannot be factored over the integers.

d) First, factor out the GCF from $7g^3h^2 - 28g^5$.
$7g^3h^2 - 28g^5 = 7g^3(h^2 - 4g^2)$
The binomial is a difference of squares.
The first term is a perfect square: h^2
The last term is a perfect square: $(2g)^2$
The operation is subtraction.

$$\begin{aligned} 7g^3h^2 - 28g^5 &= 7g^3(h^2 - 4g^2) \\ &= 7g^3[h^2 - (2g)^2] \\ &= 7g^3(h - 2g)(h + 2g) \end{aligned}$$

Check:
Multiply.

$$\begin{aligned} 7g^3(h - 2g)(h + 2g) &= 7g^3[h(h + 2g) - 2g(h + 2g)] \\ &= 7g^3(h^2 + 2gh - 2gh - 4g^2) \\ &= 7g^3(h^2 - 4g^2) \\ &= 7g^3h^2 - 28g^5 \end{aligned}$$

Your Turn

Factor each binomial, if possible.

a) $49a^2 - 25$

b) $125x^2 - 40y^2$

c) $9p^2q^2 - 25$

Example 2 Factor Perfect Square Trinomials

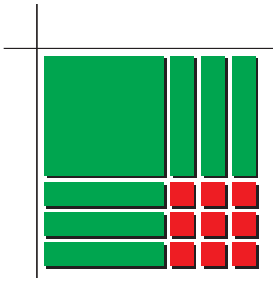
Factor each trinomial, if possible.

a) $x^2 + 6x + 9$ **b)** $2x^2 - 44x + 242$ **c)** $c^2 - 12x - 36$

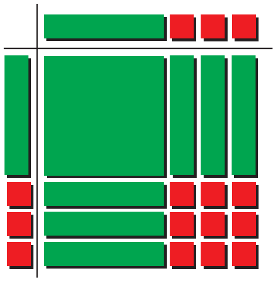
Solution

a) Method 1: Use Algebra Tiles

Create an algebra tile model to represent $x^2 + 6x + 9$.



The dimensions represent the factors of $x^2 + 6x + 9$.



The factors are $x + 3$ and $x + 3$.

Therefore, $x^2 + 6x + 9 = (x + 3)(x + 3)$

$$= (x + 3)^2$$

Check:

Multiply.

$$\begin{aligned}(x + 3)(x + 3) &= x(x + 3) + 3(x + 3)\\ &= x^2 + 3x + 3x + 9\\ &= x^2 + 6x + 9\end{aligned}$$

Method 2: Factor by Grouping

$$\begin{aligned}x^2 + 6x + 9 &= (x^2 + 3x) + (3x + 9)\\ &= x(x + 3) + 3(x + 3)\\ &= (x + 3)(x + 3)\\ &= (x + 3)^2\end{aligned}$$

Method 3: Factor as a Perfect Square Trinomial

The trinomial $x^2 + 6x + 9$ is a perfect square.
The first term is a perfect square: x^2
The last term is a perfect square: 3^2
The middle term is twice the product of the square root of the first term and the square root of the last term: $(2)(x)(3) = 6x$
The trinomial is of the form $(ax)^2 + 2abx + b^2$.

$$\begin{aligned} x^2 + 6x + 9 &= (x + 3)(x + 3) \\ &= (x + 3)^2 \end{aligned}$$

b) First, factor out the GCF from $2x^2 - 44x + 242$.
$2x^2 - 44x + 242 = 2(x^2 - 22x + 121)$
The first term in the brackets is a perfect square: x^2
The last term in the brackets is a perfect square: 11^2
The middle term is twice the product of the square root of the first term and the square root of the last term: $(2)(x)(11) = 22x$
The trinomial is of the form $(ax)^2 - 2abx + b^2$.

$$\begin{aligned} 2x^2 - 44x + 242 &= 2(x^2 - 22x + 121) \\ &= 2(x - 11)(x - 11) \\ &= 2(x - 11)^2 \end{aligned}$$

Check:
Multiply.

$$\begin{aligned} 2(x - 11)(x - 11) &= 2[x(x - 11) - 11(x - 11)] \\ &= 2(x^2 - 11x - 11x + 121) \\ &= 2(x^2 - 22x + 121) \\ &= 2x^2 - 44x + 242 \end{aligned}$$

c) The trinomial $c^2 - 12x - 36$ is not a perfect square.
The first and last terms are perfect squares.
The middle term is twice the product of the square root of the first term and the square root of the last term.
However, the trinomial is not of the form $(ax)^2 + 2abx + b^2$ or $(ax)^2 - 2abx + b^2$.
Therefore, the trinomial cannot be factored over the integers.

Your Turn

Factor each trinomial, if possible.

a) $x^2 - 24x + 144$
b) $y^2 - 18y - 81$
c) $3b^2 + 24b + 48$

Key Ideas

- Some polynomials are the result of special products. When factoring, you can use the pattern that formed these products.

  Difference of Squares:

  The expression is a binomial.
  The first term is a perfect square.
  The last term is a perfect square.
  The operation between the terms is subtraction.

  $$\begin{aligned} x^2 - 25 &= x^2 - 5^2 \\ &= (x - 5)(x + 5) \end{aligned}$$

  Perfect Square Trinomial:

  The first term is a perfect square.
  The last term is a perfect square.
  The middle term is twice the product of the square root of the first term and the square root of the last term.
  The trinomial is of the form $(ax)^2 + 2abx + b^2$ or $(ax)^2 - 2abx + b^2$.

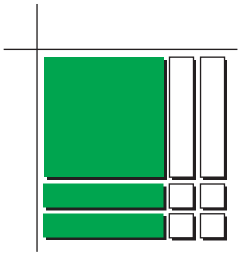
  $$\begin{aligned} x^2 + 16x + 64 &= x^2 + 8x + 8x + 64 \\ &= x(x + 8) + 8(x + 8) \\ &= (x + 8)(x + 8) \end{aligned}$$

Check Your Understanding

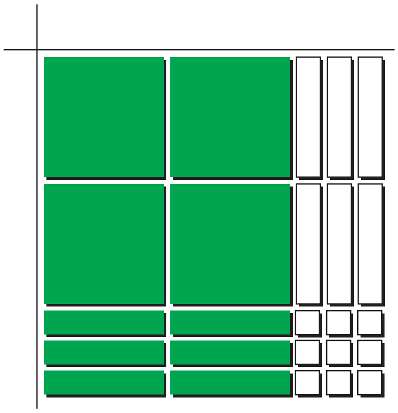
Practise

1. Identify the factors of the polynomial shown by each algebra tile model.

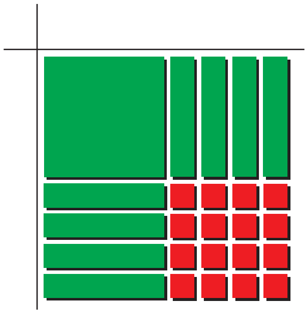
a)



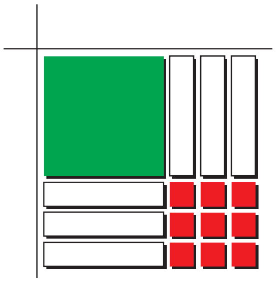
b)



c)



d)



2. Determine each product.

a) $(x - 8)(x + 8)$ **b)** $(2x + 5)(2x - 5)$

c) $(3a - 2b)(3a + 2b)$ **d)** $3(t - 5)(t + 5)$

3. What is each product?

a) $(x + 3)^2$ **b)** $(3b - 5a)^2$

c) $(2h + 3)^2$ **d)** $5(x - 2y)^2$

4. Identify the missing values for a difference of squares or a perfect square trinomial.

a) $\blacksquare - y^2 = (\blacksquare - y)(m + \blacksquare)$

b) $16r^6 - \blacksquare = (\blacksquare - \blacksquare)(\blacksquare + 9)$

c) $x^2 - 12x + \blacksquare = (\blacksquare - 6)^2$

d) $4x^2 + \blacksquare + \blacksquare = (\blacksquare + 5)^2$

e) $\blacksquare + \blacksquare + 49 = (5x + \blacksquare)(\blacksquare + \blacksquare)$

5. Factor each binomial, if possible.

a) $x^2 - 16$ **b)** $b^2 - 121$

c) $w^2 + 169$ **d)** $9a^2 - 16b^2$

e) $36c^2 - 49d^2$ **f)** $h^2 + 36f^2$

g) $121a^2 - 124b^2$ **h)** $100 - 9t^2$

6. Factor each trinomial, if possible.

a) $x^2 + 12x + 36$ **b)** $x^2 + 10x + 25$

c) $a^2 - 24a - 144$ **d)** $m^2 - 26m + 169$

e) $16k^2 - 8k + 1$ **f)** $49 - 14m + m^2$

g) $81u^2 + 34u + 4$ **h)** $36a^2 + 84a + 49$

7. Factor completely.

a) $5t^2 - 100$ **b)** $10x^3y - 90xy$

c) $4x^2 - 48x + 36$ **d)** $18x^3 + 24x^2 + 8x$

e) $x^4 - 16$ **f)** $x^4 - 18x^2 + 81$

Apply

8. Determine two values of n that allow each polynomial to be a perfect square trinomial. Then, factor.

a) $x^2 + nx + 25$

b) $a^2 + na + 100$

c) $25b^2 + nb + 49$

d) $36t^2 + nt + 121$

9. Each of the following polynomials cannot be factored over the integers. Why not?

a) $25a^2 - 16b$ **b)** $x^2 - 7x - 12$

c) $4r^2 - 12r - 9$ **d)** $49t^2 + 100$

10. **Unit Project** Use models or diagrams to show what happens to the middle terms when you multiply two factors that result in a difference of squares. Include at least two specific examples.

11. Many number tricks can be explained using factoring. Use $a^2 - b^2 = (a - b)(a + b)$ to make the following calculations possible using mental math.

a) $19^2 - 9^2$ **b)** $28^2 - 18^2$

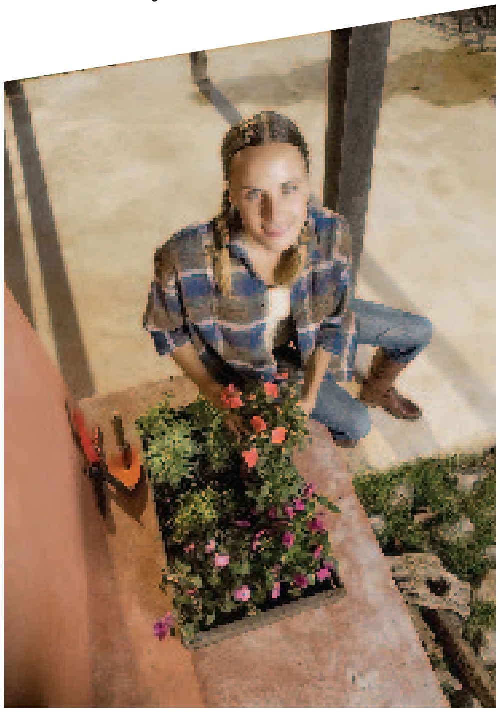
c) $35^2 - 25^2$ **d)** $5^2 - 25^2$

12. **Unit Project**

a) Use models or diagrams to show the squaring of a binomial. Include at least two specific examples.

b) Create a rule for squaring any binomial. Show how your rule relates to your models or diagrams.

13. Zoë wants to construct a patio in the corner of her property. The area of her square property has a side length represented by x metres. The patio will take up a square area with a side length represented by y metres. Write an expression, in factored form, to represent the remaining area of the property.

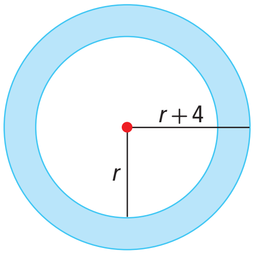


14. The diagram shows two concentric circles with radii r and $r + 4$.

a) Write an expression for the area of the shaded region.

b) Factor this expression completely.

c) If $r = 6$ cm, calculate the area of the shaded region. Give your answer to the nearest tenth of a square centimetre.

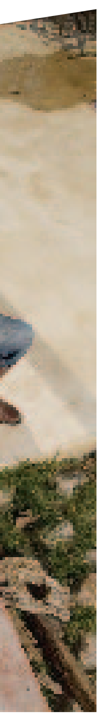


15. An object is reduced or enlarged uniformly in all dimensions. The print shown is a watercolour painting called *August Chinook* by Gena LaCoste of Medicine Hat, Alberta. This print is going to be enlarged by a factor of 3. The side length of the original can be represented by $(2x - 3)$ cm.

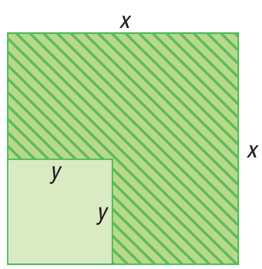
a) Use your understanding of differences of squares to write an expression that represents the difference in the areas of the original print and the enlargement.

b) Multiply this expression to write it in the form $ax^2 + bx + c$.

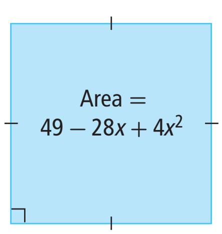
c) Verify that your expressions in parts a) and b) are correct by substituting a value for x.



16. Explain how the diagram shows a difference of squares.



17. The area of a square can be given by $49 - 28x + 4x^2$, where x represents a positive integer. Write a possible expression for the perimeter of the square.



Did You Know?

The painted butterfly drum, made by Odin Lonning, is a circular drum made from rawhide over a cedar frame.

18. The circular area of the painted butterfly drum can be represented by the expression $(9x^2 + 30x + 25)\pi$. Determine an expression for the smallest diameter the drum could have.

Traditional Tlingit hand drums are used in ceremony, cultural and social events, and as artwork. Traditional drums should always be handled with respect following appropriate protocol.

19. State whether the following equations are *sometimes*, *always*, or *never* true. Explain your reasoning.

a) $a^2 - 2ab - b^2 = (a - b)^2$, $b \neq 0$

b) $a^2 + b^2 = (a + b)(a + b)$

c) $a^2 - b^2 = a^2 - 2ab + b^2$

d) $(a + b)^2 = a^2 + 2ab + b^2$

20. Rahim and Kate are factoring $16x^2 + 4y^2$. Who is correct? Explain your reasoning.

Rahim

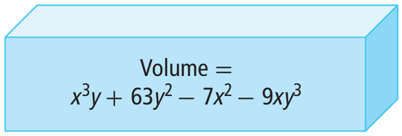
$16x^2 + 4y^2 = 4(4x^2 + y^2)$

Kate

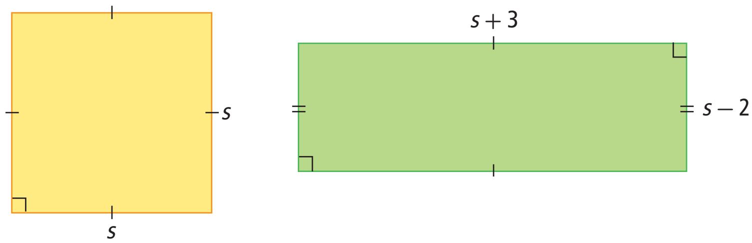
$16x^2 + 4y^2 = 4(4x^2 + y^2)$
$= 4(2x + y)(2x - y)$

Extend

21. The volume of a rectangular prism is $x^3y + 63y^2 - 7x^2 - 9xy^3$. Determine expressions for the dimensions of the prism.



22. The area of the square shown is $16x^2 - 56x + 49$. What is the area of the rectangle in terms of x?



23. **a)** The difference of squares of two numbers is the same as their sum. What integers satisfy this condition? Show how you determined your answer.

b) Based on your observations in part a), identify two integers from 11 to 20 which have a difference of squares that can be expressed as the sum of the integers.

Create Connections

24. **a)** If $x^2 + bx + c$ is a perfect square, how are b and c related?

b) If $ax^2 + bx + c$ is a perfect square, how are a, b, and c related?

25. Use two ways to show that $a^2 - b^2 = (a - b)(a + b)$.

26. What is the difference in factoring $x^2 + 2bx + b^2$ and $x^2 - 2bx + b^2$?

27. To determine the product of two numbers that differ by 2, square their average and then subtract 1. Use this method to find the following products.

$(29)(31) = ■$
$(59)(61) = ■$

a) Explain this method using a difference of squares.

b) Develop a similar method for multiplying two numbers that differ by 6.

c) Explain your method from part b) using a difference of squares.

5 Review

5.1 Multiplying Polynomials, pages 204–213

1. Draw a diagram to model each product.

a) $(x + 5)(x - 3)$ **b)** $(y + 3)^2$

2. Determine the product and then combine like terms.

a) $(x + 7)(x + 3)$ **b)** $(b + 9)(b - 9)$

c) $(y - 11)(y + 11)$ **d)** $(3a + 8b)(5a + 6b)$

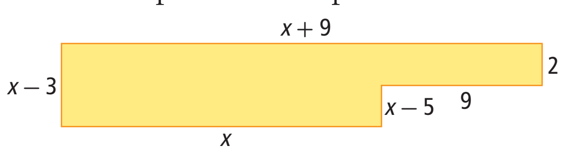
e) $-5(2x + 5b)^2$ **f)** $-(a - 6b)(a + 6b)$

3. Multiply and then combine like terms.

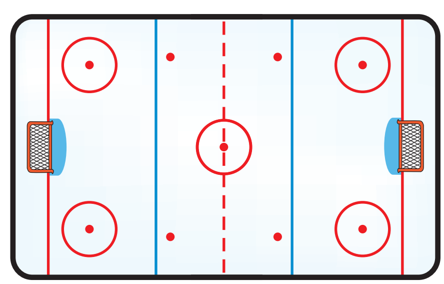
a) $(a^2 + 6a + 2)(a - 3)$

b) $2b(4b - 7)(3b + 2) - b(5b + 2)(b - 6)$

4. Write an expression to represent the area of the figure. Simplify.



5. The length of the ice surface of a hockey rink is represented by $5x + 25$. The width is represented by $2x + 10$. What expression represents the area of the ice surface?



5.2 Common Factors, pages 214–223

6. Identify the GCF of each set of terms.

a) 16 and 64 **b)** 81 and 108

c) 144, 60, and 54 **d)** $2x^2$ and $4x$

e) $10x^2$ and $20y^2$ **f)** $3xy$ and $7xz$

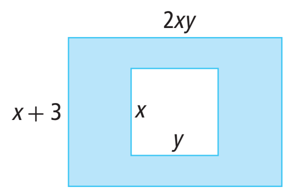
7. Identify the LCM of the following pairs of numbers.

a) 18 and 27 **b)** 125 and 15

8. Use algebra tiles or a diagram to factor each polynomial.

a) $x^2 + 5x$ **b)** $8x^2 + x$

9. Write an expression in fully factored form for the shaded area.

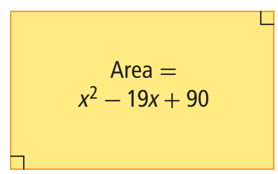


5.3 Factoring Trinomials, pages 224–237

10. Use algebra tiles or a diagram to factor each trinomial.
   - a) $x^2 + 6x + 9$
   - b) $x^2 + 12x + 35$
   - c) $12x^2 - 5x - 3$
   - d) $3x^2 - 13x + 10$

11. Factor, if possible.
   - a) $x^2 - 4x - 12$
   - b) $x^2 - 7x + 12$
   - c) $30x^2 + 9x - 12$
   - d) $-6x^2 - 34x + 12$
   - e) $-2x^2 + 16x - 30$
   - f) $x^3 + 3x^2 - 28x$

12. Identify binomials to represent the length and width of the rectangle. Then, calculate the dimensions of the rectangle if $x = 11$ cm.

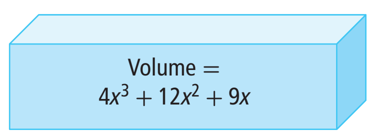


5.4 Factoring Special Trinomials, pages 238–251

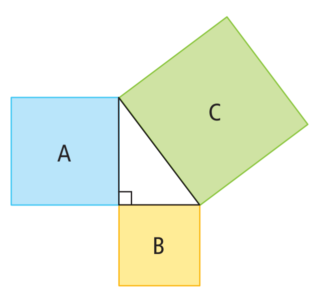
13. Factor fully.
   - a) $x^2 - 100$
   - b) $c^2 - 25$
   - c) $9x^2 - 16$
   - d) $128 - 18x^2$
   - e) $1 - 225y^2$
   - f) $-3x^2 + 27y^2$

14. Verify that each trinomial is a perfect square. Then, factor.
   - a) $y^2 + 16y + 64$
   - b) $x^2 - 20x + 100$
   - c) $225 - 90y + 9y^2$
   - d) $121c^2 + 308cd + 196d^2$

15. a) Write algebraic expressions for the dimensions of the rectangular prism.
   - b) Describe the faces of the prism.
   - c) Calculate the surface area if $x = 3$ cm.



16. The expression x^2 represents the area of square A, y^2 the area of square B, and $x^2 + y^2$ the area of square C. The side lengths of squares A and B are increased by 2 units. Write an expression for the new area of square C.

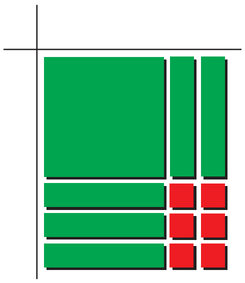


5 Practice Test

Multiple Choice

For #1 to #4, choose the best answer.

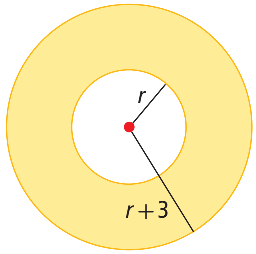
1. What binomial product does the diagram represent?



A $(x + 2)(x + 3)$ **B** $(x + 2)(x - 3)$

C $(x - 2)(x - 3)$ **D** $(x - 2)(x + 3)$

2. What is an algebraic expression for the area of the shaded region?



A $\pi(r^2 - 9)$ **B** $3\pi(2r + 3)$

C $\pi(2r^2 + 6r + 9)$ **D** 9π

3. Fully factored, $8m^2 + 16m - 10$ can be represented as

A $2(2m - 1)(2m + 5)$ **B** $2(4m + 10)(2m - 5)$

C $(4m^2 + 20)(2m - 4)$ **D** $(4m - 2)(2m + 5)$

4. Fully factored, $4y^2 - 64$ can be represented as

A $y(y^2 - 16)$ **B** $4(y - 4)^2$

C $4(y - 4)(y + 4)$ **D** $4(y - 2)(y + 2)$

Short Answer

5. What is the GCF of 56, 20, and 228? What is their LCM?

6. Simplify.

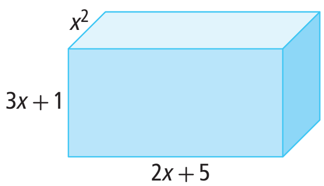
a) $(x - 3)(x - 9)$ **b)** $(2x + 3)(2x - 1)$

c) $-3(x - 4)^2 + 2(x - 3)(x + 3)$ **d)** $(3c + d)^2 + 2c(c - d)$

e) $2(x - 1)(x - 6) - 3(2x - 1)^2$ **f)** $(2c + 3d)^2 - 3(c + 1)^2$

7. a) Write an algebraic expression for the volume of the rectangular prism. Simplify the expression.

b) Calculate the volume if $x = 2$ cm.



8. Factor fully.

a) $x^2 + 10x + 25$ **b)** $25r^2 - 20rs + 4s^2$ **c)** $5x^2 - 5$

d) $1 - 49m^2$ **e)** $5m^2 + 17m + 6$ **f)** $m^2 - 9mn + 14n^2$

9. Factor, if possible.

a) $3y^3 - 7y^2 + 2y$ **b)** $4m^2 + 16$

c) $6y^2 + y - 1$ **d)** $x(m - 2) - 4(m - 2)$

e) $y^2 + 2x + 2y + xy$ **f)** $9t - 4t^3$

10. The face of a Canadian $20 bill has an area that can be represented by the expression $10x^2 + 9x - 40$.

a) Factor $10x^2 + 9x - 40$ to find expressions to represent the dimensions of the bill.

b) If x represents 32 mm, what are the dimensions of the bill? Express your answer in millimetres.

11. Trafalgar Fountain in Regina is a circular fountain with a radius of $2x$ metres. The circular pool surrounding the fountain is an additional 3 m in radius. What is an expression for the area of the base of the pool? Multiply and combine like terms.

Did You Know?

Trafalgar Fountain is one of a pair of fountains made in London, England. The fountain in Regina commemorates the founding of the Northwest Mounted Police headquarters in 1882. The other fountain is located in Ottawa.

Extended Response

12. Brendan factored the trinomial $8y^2 - 12y - 18$ in this way:

$$\begin{aligned} 8y^2 - 12y - 18 &= 2(4y^2 - 6y - 9) \\ &= 2(2y - 3)(2y - 3) \\ &= 2(2y - 3)^2 \end{aligned}$$

Is Brendan's factoring correct? Explain.

13. The volume of a rectangular prism is represented by $12x^3 - 3x$.

a) Factor the expression fully.

b) Sketch the prism and label its dimensions.

c) If x represents 6 cm, what are the dimensions of the prism?

2 Unit Connections

Unit 2 Project

In this unit, you have seen how artists use mathematics in their work. Now, it is your turn to create art from mathematics. Your artwork can be historical or contemporary. It can involve nature, stylized arrangements of models, or another idea of your choice that relates to this unit.

You can present your creation as a video, cartoon, painting, or in another form of your choice. Include a brief report, describing how the mathematics from Unit 2 specifically relates to your work of art.

Unit Review

Chapter 4 Exponents and Radicals

Write the letter of the value that represents each term.

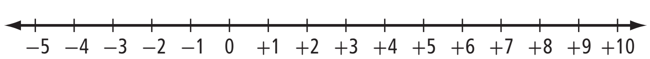
1. radicand	**A** the number 4 in $\sqrt[4]{16}$
2. rational exponent	**B** the number 3 in $2\sqrt{3}$
3. irrational number	**C** $\sqrt{20}$
4. index	**D** $3^{\frac{2}{3}}$

5. Sort the following numbers into perfect squares, perfect cubes, or neither. Identify the value of each perfect root.

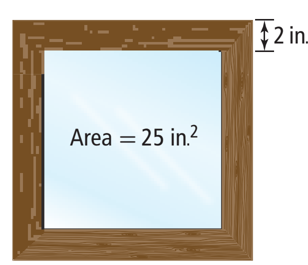
16, 15, 27, -4, 125, 1000, 169, -8, 99

6. Copy the number line shown. Order each of the following numbers on your number line.

$\sqrt{5}$, $\sqrt[3]{-8}$, $\sqrt{12}$, $\sqrt[5]{32}$, $\sqrt{65}$



7. The area of the glass within a square picture frame is 25 in.^{2}. The width of the frame around the glass is 2 in. Determine the area of the entire framed picture.



8. A child's toy cube is made up of 26 small cubes, plus one invisible small cube in the centre of the same dimension. The volume of the large toy cube is 729 cm^3. What is the area of one side of a small cube?

9. Express each radical as a mixed radical in simplest form.

   a) $\sqrt{12}$ **b)** $\sqrt{162}$ **c)** $\sqrt[3]{16}$

10. Express each mixed radical as an entire radical.

   a) $2\sqrt{5}$ **b)** $5\sqrt{3}$ **c)** $2\sqrt[3]{5}$

11. Express each power as an equivalent radical.

   a) $7^{\frac{4}{5}}$ **b)** $\left(\frac{27}{8}\right)^{\frac{1}{3}}$ **c)** $(6x^2)^{\frac{1}{4}}$

12. Express each radical as a power.

   a) $\sqrt{\frac{m^3}{n^2}}$ **b)** $\sqrt[4]{6^3}$ **c)** $\sqrt[3]{8s^4}$

13. Simplify each expression. Write your answers with positive exponents.

   a) $\left(\frac{1}{3}\right)^5\left(\frac{1}{3}\right)^{\frac{1}{2}}$ **b)** $\frac{(x^3y^{-1})}{(x^6y^4)^{\frac{1}{2}}}$ **c)** $(\sqrt{81})^{\frac{3}{2}} \div 3^4$

14. Determine the exact value of each expression.

   a) $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$ **b)** $5^3 + 5^2$ **c)** $\frac{4}{(2^3 - 3)^{-2}}$

Chapter 5 Polynomials

15. Model each product using a diagram or algebra tiles. Determine the simplified expression representing the product.

   a) $(2x + 1)(x + 3)$

   b) $(x - 2)(x + 2)$

   c) $(x + 3)^2$

16. Multiply each product and then combine like terms.

a) $(a - 4)(a + 7)$

b) $(2x + 3)(5x + 2)$

c) $(-x + 5)(x + 5)$

d) $(3y + 4)^2$

e) $(a - 3b)(4a - b)$

f) $(v - 1)(2v^2 - 4v - 9)$

17. Determine a value for k that allows each trinomial to be factored over the integers.

a) $3x^2 + kx - 10$

b) $24x^2 + 47x - k$

18. Rachel's solution to the multiplication of a binomial and a trinomial is shown below.

$$(4x - 1)(2x^2 + 11x - 7) = 8x^3 + 44x - 24x - 2x^2 - 11x + 6$$
$$= 6x^2 - 9x + 6$$

a) Check Rachel's solution for $x = 2$.

b) Does Rachel have a correct solution? If not, identify her error and provide the correct product in simplified form.

19. Factor, if possible.

a) $9y^2 + 24y - 16$

b) $50x^2 - 60xy + 18y^2$

c) $x^2 + 9y^2$

20. It is possible to factor out a common factor from the expression $2x^2 + ky + 4$, where k is an integer. What can you state about the values of k? Explain.

21. Determine the greatest common factor in the terms of each polynomial.

a) $14x^2 - 21x$

b) $-10x^4 - 5x^3 + 15x^2$

c) $15a^3b(a - 1) - 12ab^2(a - 1)$

22. Express each polynomial as a product of its factors, if possible.

a) $x^2 + 8x + 9$

b) $2v^2 + 3v - 9$

c) $-2x^2 - 6x + 20$

d) $4y^2 - 25$

e) $20 - 21x + x^2$

f) $-15x^2 + x + 6$

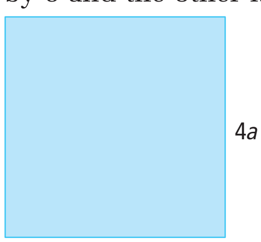
23. Julio was asked to factor the expression $2x^2 + 12x + 18$. His solution is shown.

$$\begin{aligned} 2x^2 + 12x + 18 &= 2(x^2 + 10x + 9) \\ &= 2(x + 1)(x + 9) \end{aligned}$$

a) Identify the error that Julio made.

b) Determine the correct factorization.

24. A square has a side length of $4a$. One dimension is increased by 6 and the other is decreased by 6.

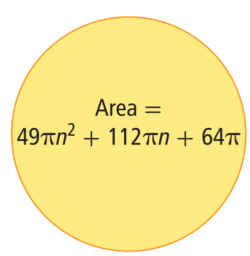


a) Write an algebraic expression to represent the area of the resulting rectangle.

b) Multiply this expression and combine like terms.

c) Write an algebraic expression for the difference between the area of the square and the area of the rectangle. Combine like terms.

25. What is the radius of a circle with an area of $49\pi n^2 + 112\pi n + 64\pi$?



2 Unit Test

Multiple Choice

For #1 to #5, choose the best answer.

1. Selena said, "A square root is the same as a perfect square."
Anoop said, "A cube root is always positive."
Danielle said, "A number is either rational or irrational, but not both."
Which of the students are correct?

A All three students are correct.

B Selena is correct, Anoop is incorrect, and Danielle is incorrect.

C Selena is incorrect, Anoop is correct, and Danielle is correct.

D Selena is incorrect, Anoop is incorrect, and Danielle is correct.

2. Which expression is equivalent to $\left(\frac{9}{4}\right)^{-2}$?

A $\frac{2}{3}$ **B** $\frac{3}{2}$ **C** $\frac{16}{81}$ **D** $\frac{81}{16}$

3. When the numbers $\sqrt{35}$, $\sqrt[3]{27}$, $\left(\frac{1}{3}\right)^{-2}$, $2\left(9^{\frac{1}{2}}\right)$ are written in ascending order, the correct sequence is

A $\sqrt[3]{27}$, $\sqrt{35}$, $2\left(9^{\frac{1}{2}}\right)$, $\left(\frac{1}{3}\right)^{-2}$

B $\left(\frac{1}{3}\right)^{-2}$, $2\left(9^{\frac{1}{2}}\right)$, $\sqrt[3]{27}$, $\sqrt{35}$,

C $\sqrt[3]{27}$, $\left(\frac{1}{3}\right)^{-2}$, $\sqrt{35}$, $2\left(9^{\frac{1}{2}}\right)$

D $\sqrt{35}$, $\left(\frac{1}{3}\right)^{-2}$, $2\left(9^{\frac{1}{2}}\right)$, $\sqrt[3]{27}$

4. Which equation is not correct?

A $(b + a)(b - a) = b^2 - a^2$

B $(a + b)^2 = b^2 + 2ab + a^2$

C $(a - b)^2 = a^2 - 2ab + b^2$

D $(a - b)(b - a) = a^2 - b^2$

5. Express the product of $3x^3 - 5$ and $3x^3 + 5$ in simplified form.

A $9x^6 - 25$ **B** $9x^3 - 25$

C $3x^3 - 5$ **D** $9x^9 - 25$

Numerical Response

Complete the statements in #6 to #9.

6. Given the trinomial $9x^2 + kx + 4$, the positive value for k that makes the expression a perfect square trinomial is ■.

7. When $\sqrt{80}$ is written in the simplified form $a\sqrt{b}$, the value of the radicand is ■.

8. You multiply the binomials $2x + 5$ and $x + 7$ and express the product in the form $ax^2 + bx + c$. The value of the coefficient of the x term is ■.

9. The value of the expression $\left(\frac{(2^{-3})(2^5)}{2^8}\right)^{-\frac{1}{3}}$ is ■.

Written Response

10. Simplify the expression $\left(\sqrt[3]{\sqrt{20}}\right)^{-1}$. Write it in exponential form with positive exponents only.

11. Multiply and then combine like terms.

a) $(x + 5y)(2x - y)$

b) $(2a - 3)(3a^2 + 2a - 7)$

c) $3x(x^2 - 2x + 4) - (x^2 + 5x - 1)$

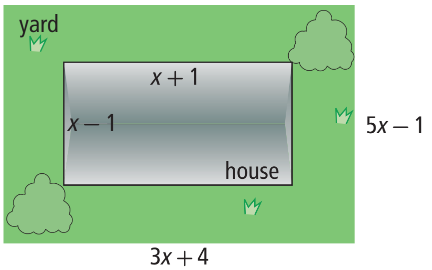
12. Factor.

a) $x^2 - 10x + 9$

b) $4a^2 - 5a - 6$

c) $16x^2 - y^2$

13. The diagram below represents a house located on a property.



a) Write an expression for the area of the house in simplified form.

b) Write an expression for the area of the yard in simplified form.

UNIT

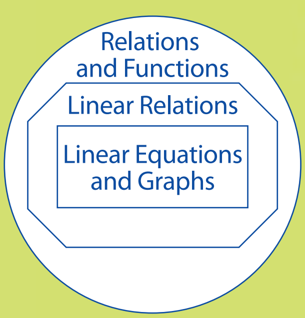
3 Relations and Functions

Canada requires all packaged foods to display the nutrition facts in both English and French. How might you use the information found on these labels? How can you determine the number of grams of protein in a meal if you double or triple the portion size?

Analysing the graph of a linear relation can provide information about the relationship and the particular situation. For example, consider a graph of the linear relation between the number of potatoes and the total mass of potatoes. The number of potatoes plotted would not increase to infinity. It would be equal to the maximum number that can fit on the truck. In this unit, you will learn how to interpret graphs of real-life situations. You will use several methods of graphing linear relations. You will also determine the equation of a line in different forms.

Your Relations and Functions Organizer

You can use this organizer to see how the concepts in this unit are connected. You will see this organizer on the first page of each chapter. The concepts covered in the chapter will be highlighted.



Looking Ahead

In this unit, you will solve problems involving …

- graphs of linear relations
- domain and range values for a situation
- function notation
- slope and rate of change
- linear equations written in various forms

Unit 3 Project

Forensic Discovery

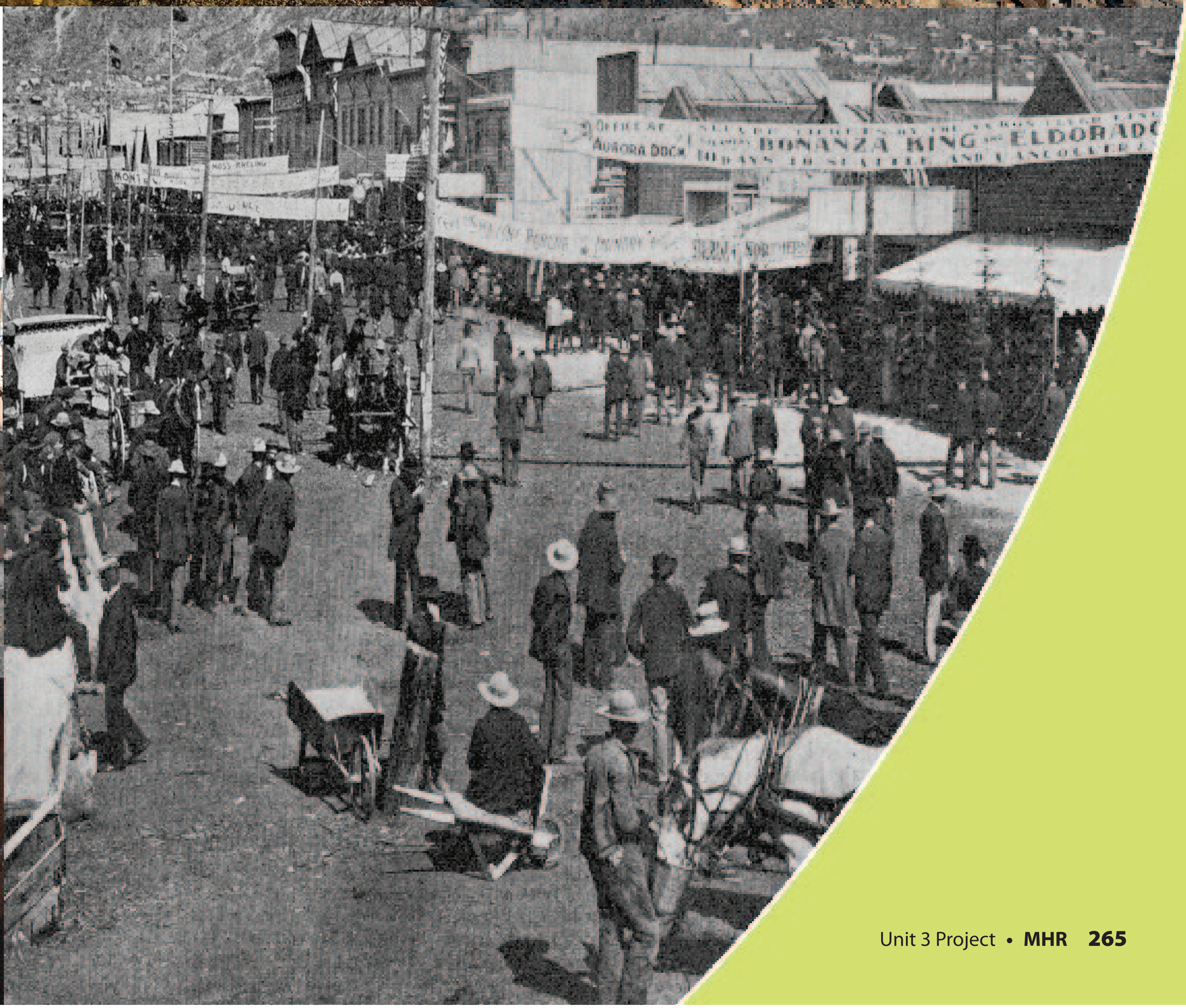
Gold was discovered in 1896 at Rabbit Creek (later named Bonanza Creek), YT. This discovery started the Klondike Gold Rush. Thousands of amateur gold seekers from the south raced to the Klondike River, near Dawson City. There were many routes to the gold fields. Most people had to travel over the difficult and dangerous White Pass Trail or the Chilkoot Trail. Only about 30 000 people completed the journey. Their spirit of adventure and determination contributed to the social and economic development of western Canada. The Klondike Gold Rush ended as quickly as it started. Many people left in 1899 when rumours spread of a gold strike at Nome, Alaska.

In the Unit 3 project, you will act as a modern forensic archaeologist. Your first case is to solve the mystery of what happened to three gold seekers and possibly find their gold treasure.

Unit Project questions and activities are included in Chapters 6 and 7. As you move through Chapter 6, you will learn about and use functions that contribute to forensic and related scientific inquiry. As you move through Chapter 7, you will collect and graph data and develop equations related to forensic science.

While completing your project, you will ...

- collect data and use functions and graphs to relate a person's height to the length of the humerus bone (Chapters 6 and 7)
- use graphs and functions related to radiocarbon decay to determine the age of fossils (Chapter 6)
- develop an equation relating temperature and altitude and use it to determine specific values (Chapter 7)


AURORA DOCK
BONANZA KING
ELDORADO

CHAPTER 6

Linear Relations and Functions

Our world is becoming increasingly complex. Large amounts of information, on virtually every topic imaginable, are readily available at our fingertips. The ability to interpret and analyse data has never been more necessary. There are many interesting relationships that you can explore. Have you ever wondered, "What would happen if ... ?" or "How does changing this affect ... ?" This is where the study of relations begins.

Big Ideas

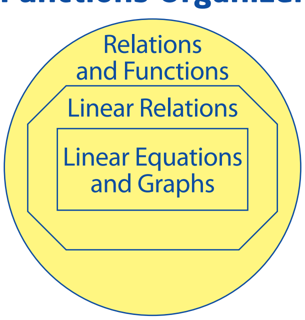
When you have completed this chapter, you will be able to ...

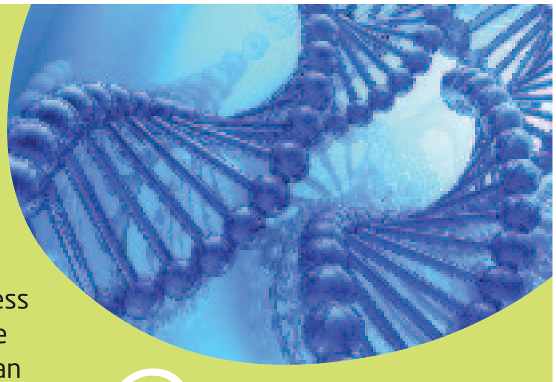
- create graphs that represent different situations and interpret graphs
- apply the various characteristics of linear relations to graphing
- determine an acceptable range of values for a situation
- work with function notation in a variety of ways
- work with slopes and solve problems involving rates of change

Key Terms

relation
linear relation
non-linear relation
discrete data
continuous data
independent variable
dependent variable
domain
range
function
function notation
vertical line test
slope

Your Relations and Functions Organizer



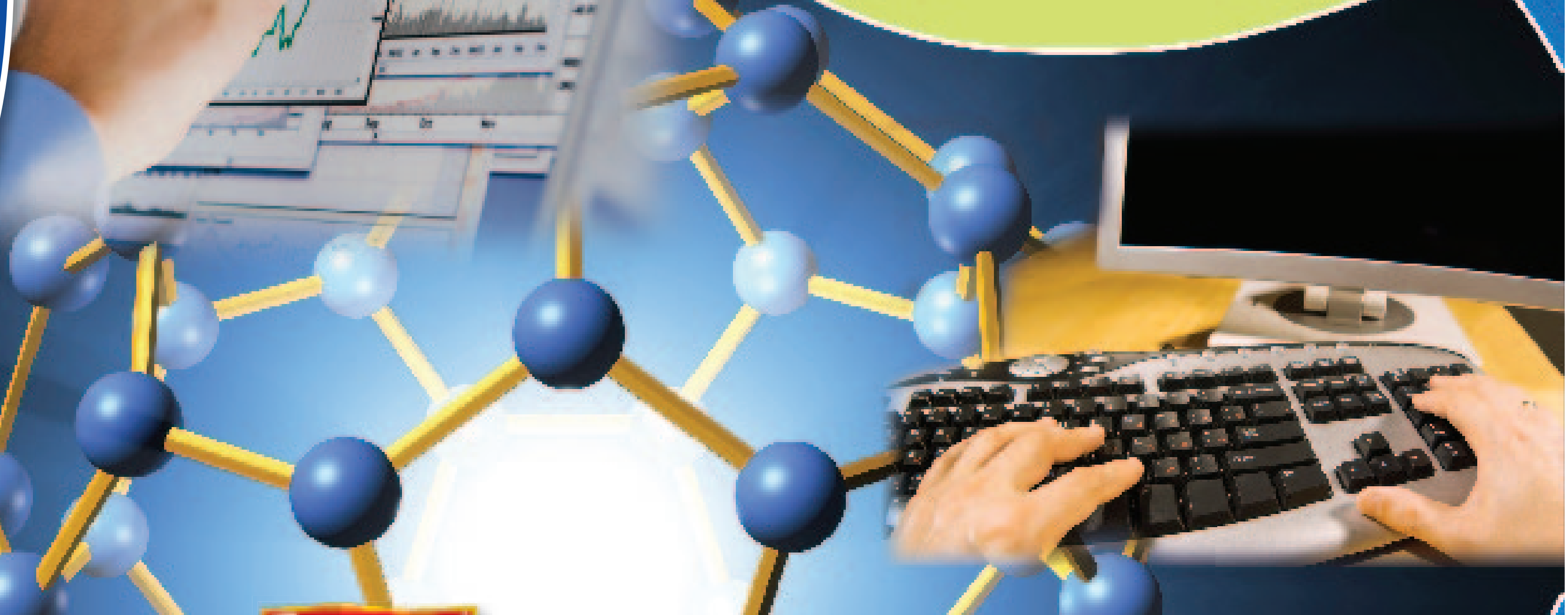


Forensic Scientist

The field of forensic science has been popularized on television and in movies and books. Forensic scientists play an important role in our society. They process and interpret physical evidence for the justice system. Forensic scientists can specialize in a variety of areas. These include accounting, anthropology, biology, chemistry, geology, medicine, and dentistry.

WWW Web Link

To learn more about forensic scientists, go to www.mhrmath10.ca and follow the links.



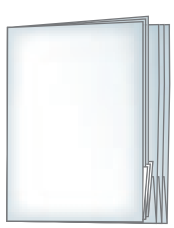
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 6.

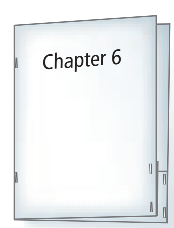
1. Take three sheets of 11 × 17 paper.

2. Make a pocket on each by folding the long bottom edge up 6 cm.

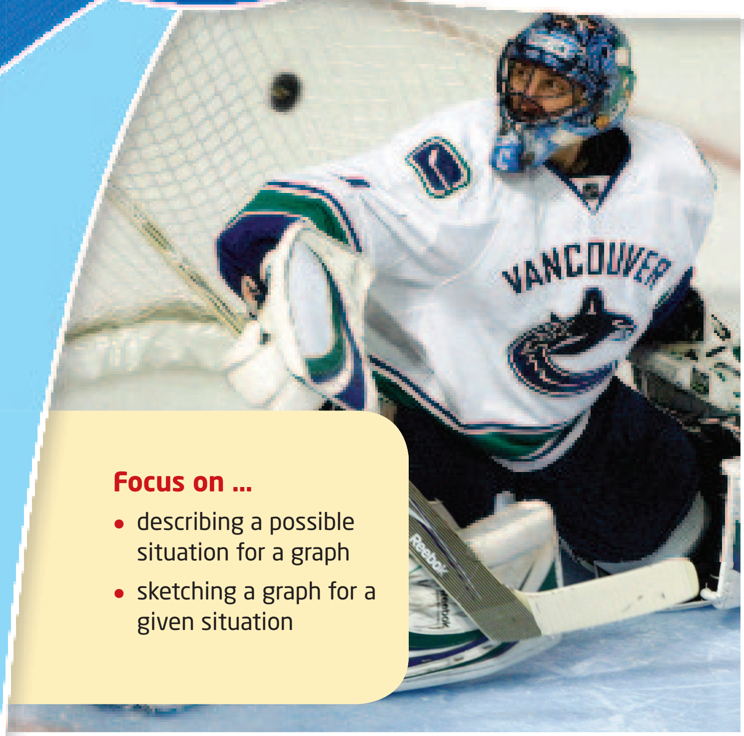
3. Stack all three sheets and then fold them in half to create a booklet.



4. To complete the booklet, staple the spine and staple the sides of each pocket.



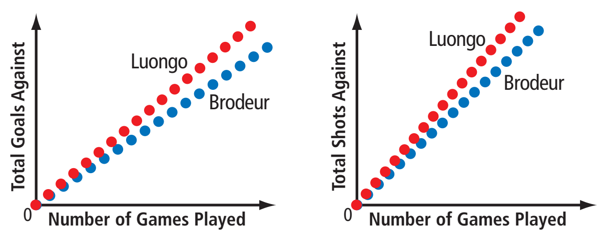
6.1 Graphs of Relations



Focus on ...

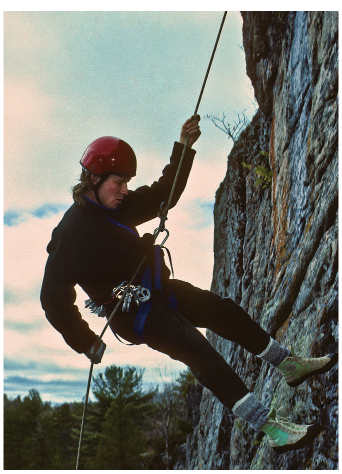
- describing a possible situation for a graph
- sketching a graph for a given situation

Graphs are often used to visually represent the relationship between two or more things. The graphs shown are derived from the career statistics for two NHL goalies, Martin Brodeur and Roberto Luongo. One graph shows the number of goals scored against each goalie. The other shows the number of shots each faced. If you compare the graphs, what conclusions might you make?



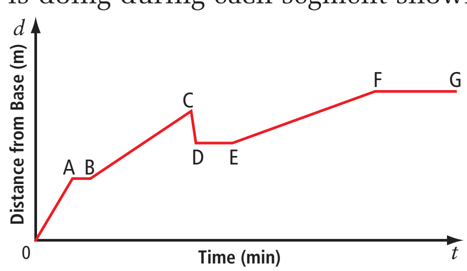
Materials

- ruler
- grid paper



Investigate Describing and Sketching Graphs

1. a) Work in pairs. The graph shows the distance a rock climber is from the base of a cliff as time passes. Using the words *climbing*, *resting*, or *descending*, describe what the climber is doing during each segment shown. Explain your choice.

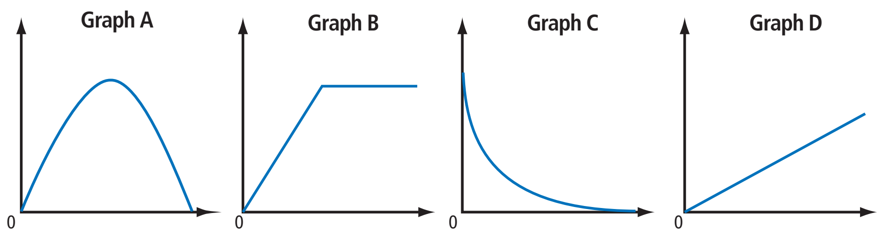


b) Is there more than one interpretation of the climber's actions during the times indicated by segments AB, CD, DE, and FG?

c) For any section that you listed as "climbing," how would you change the graph to show that the person is climbing faster? Explain your reasoning.

d) What would you add to the graph to show the climber's return to the bottom of the cliff?

2. Work in pairs. Match each graph with a situation from the list. Explain your choice. Suggest titles for each axis to show the quantities being compared.



a) the temperature of a cup of hot chocolate over time

b) a car accelerating to a constant speed

c) the distance a person walks during a hike

d) the height of a soccer ball kicked across a field

3. Work in small groups. Create a speed-time graph for the following scenario. Put speed on the vertical axis and time on the horizontal axis. Clearly describe each section of your graph. Then, pass your work to another group.

Connor is riding his skateboard along a path. Almost immediately after leaving home, Connor travels down a short steep hill. At the bottom, the path makes a turn. The remainder of the trip is on relatively flat land. Connor kicks to keep moving. He then stops before a railway crossing. He also practises a few tricks along the way. He completes a basic "ollie" and performs a second ollie over a speed bump. Finally, after travelling at a constant rate for the last part of the trip, Connor arrives at his destination.

4. a) Review and discuss another group's graph.

b) How is it similar to yours? How is it different from yours?

5. Reflect and Respond How might each situation be shown on a graph?

a) one quantity is changing at a constant rate in relation to the other quantity

b) the rate of change is constant and the change is happening quickly

c) one quantity is not changing

d) a change in one quantity is not constant

WWW Web Link

To conduct an interactive investigation of distance-time graphs, go to www.mhrmath10.ca and follow the links.

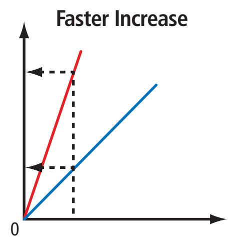
Did You Know?

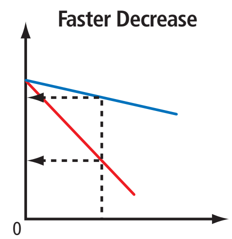
An ollie is a jump that is performed by tapping the tail of the skateboard to the ground. It was invented by Alan "Ollie" Gelfand in 1978. Many tricks start with, or incorporate, an ollie.

Link the Ideas

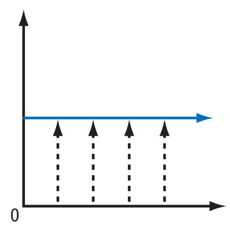
A graph is an effective way to show the relationship between two quantities. A constant rate of change is represented graphically by a straight line. The steepness of the line indicates the rate at which one quantity is changing in relation to the other.

A steeper line indicates a faster rate of vertical change on the red line than on the blue line. This change may indicate an increase or a decrease.





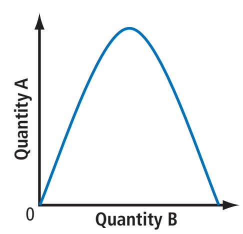
A horizontal line means that there is no rate of change.



Every value on the horizontal axis is related to the same value on the vertical axis.

Not all relationships are represented by a straight line. A curve shows that the rate of change is not constant.

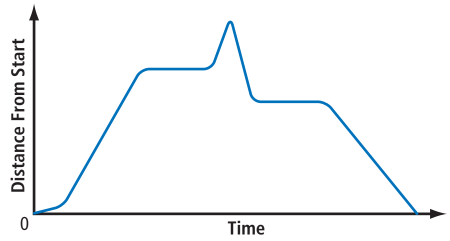
As quantity B increases, the increase in quantity A is gradual at first. It then becomes much greater.



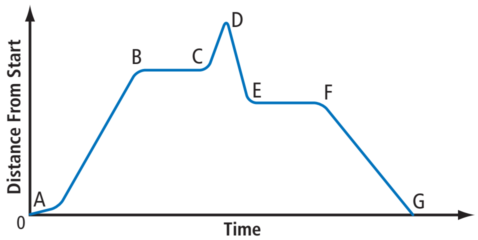
As quantity B increases, the increase in quantity A slows until quantity A reaches a maximum value. Then, quantity A decreases.

Example 1 Interpret a Graph

Wakeboarding has grown to be a popular water sport. The graph shows the distance that a wakeboarder is from her starting point on Last Mountain Lake in Saskatchewan. Describe what the boarder is doing.



Solution



AB: Since the distance is increasing, the wakeboard rider is moving away from her starting point. The change in distance starts slowly at first. It then reaches a constant rate.

Why does the distance increase slowly at first?

BC: Since the distance is not changing, the rider has either stopped or is on a path that keeps her at a constant distance from the starting point.

What kind of path would allow this?

CD: The change in distance increases so that the wakeboard rider is moving away from her starting point at a quicker rate.

DE: Since the distance is decreasing quickly, the rider is moving toward the starting point at a fast rate.

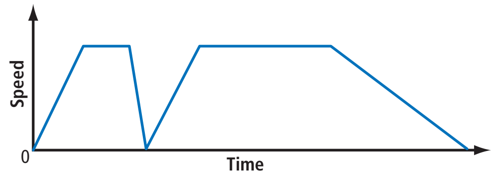
EF: Since the distance is not changing, the rider has either stopped or is on a path that keeps her at a constant distance from the starting point.

FG: The distance is decreasing to zero. The rider is returning to the starting point at a constant rate.

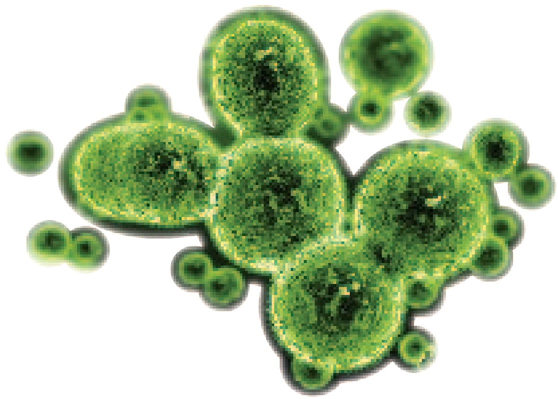
How do you know the rate is constant?

Your Turn

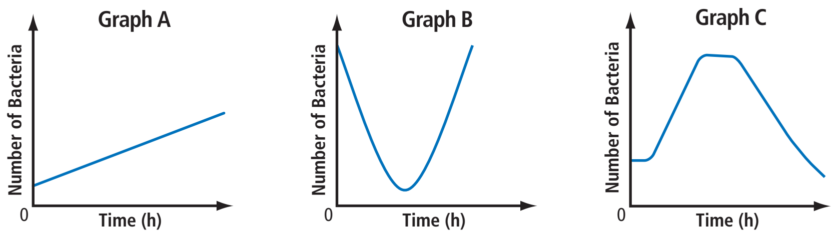
The graph shows the speed of the boat that is pulling a wakeboarder. Describe what the boat is doing.



Example 2 Interpret a Graph



Which graph best represents bacteria growth if the bacteria's food supply is limited? Explain your choice.



Solution

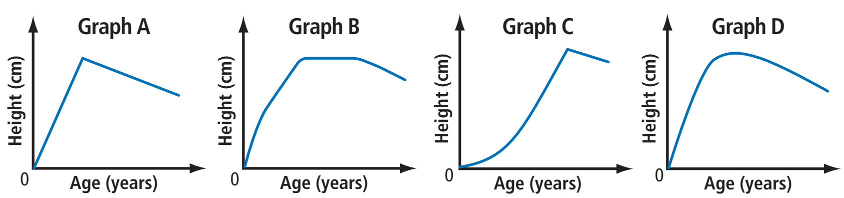
If the food supply is limited, the bacteria eventually will run out of food and die off. Graph A can be ruled out since it indicates continued growth.

Graph B is also not the correct choice. It shows the number of bacteria decreasing at the start while the food supply is high, reaching a low point, and then increasing.

Graph C is the correct choice. The increase in bacteria is initially slow but then goes through a period of rapid growth. The number remains stable for a while. Then the bacteria die off because there is no more food.

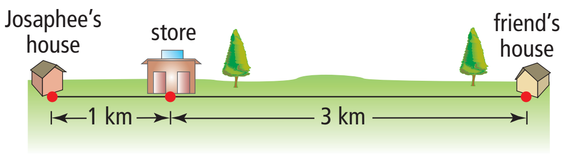
Your Turn

Which graph best represents a person's height as the person ages? Explain your choice.

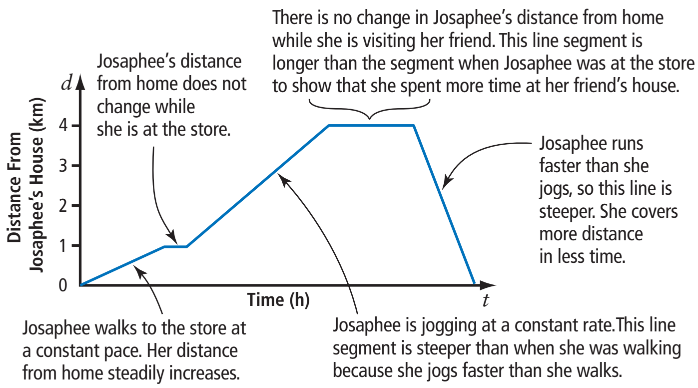


Example 3 Graph a Situation

Josaphee leaves her home and walks to the store. After buying a drink, she slowly jogs to her friend's house. Josaphee visits with her friend for a while and then runs directly home. Using the distances shown, draw a distance-time graph that shows Josaphee's distance from her house. Explain each section of your graph.



Solution



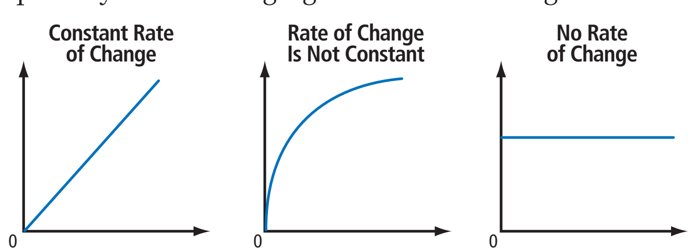
To make a distance-time graph, Josaphee's distance from her home is placed on the vertical axis. Time is placed on the horizontal axis.

Your Turn

For the same scenario and using the distances shown, draw a distance-time graph that shows Josaphee's distance from the store. Explain each section of your graph.

Key Ideas

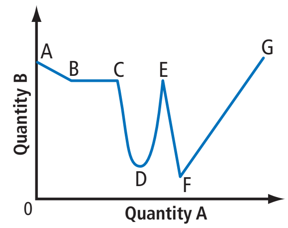
- When comparing two quantities, straight lines are used to indicate a constant change in the relationship. Curves are used when the rate of change is not constant. Horizontal lines are used if one quantity is not changing relative to a change in the other quantity.



Check Your Understanding

Practise

1. The graph shows how quantity B is changing relative to quantity A. Describe each section of the graph as representing a constant increase, a constant decrease, an increase that is not constant, a decrease that is not constant, or no change. Explain your answers.

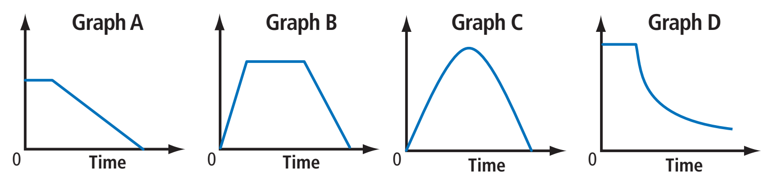


2. a) Match each scenario with its appropriate graph.

i) the speed of a train as it arrives at a station

ii) a football's distance above ground as the ball is kicked

iii) the number of un-popped kernels as a popcorn maker heats up and pops the corn

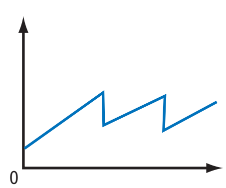


b) Describe a scenario for the graph that you do not use in part a).

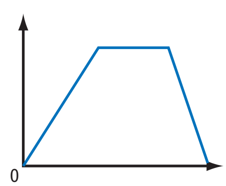
3. Sketch a copy of each graph. Label the axes using the choices given.

Choices for Vertical Axis	Choices for Horizontal Axis
Profit From Sales	Time
Speed of ATV	Ticket Price
Height of Grass	Distance Travelled

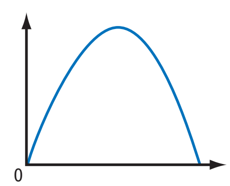
a)



b)

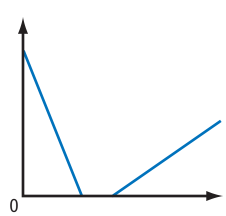


c)

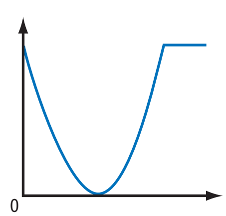


4. Describe a possible scenario for each graph. Tell what each axis represents in each case.

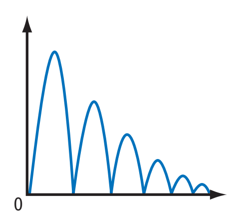
a)



b)



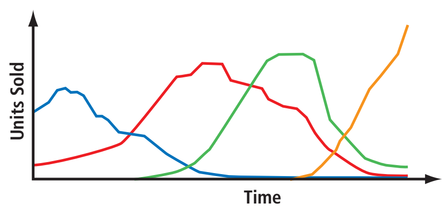
c)



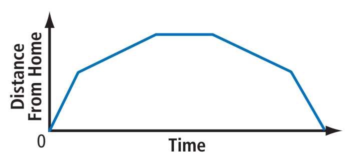
Apply

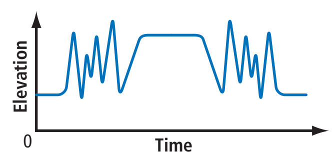
5. Paul boards Vancouver's SkyTrain at the Main Street station. He travels east on the Millennium line toward the Columbia station. He falls asleep around the 29th Avenue station and does not wake up until the Rupert station. Paul decides to stay on the train until Commercial Drive, where he transfers to another eastbound train. He takes this train to the Columbia station. Make a distance-time graph for Paul's journey. Sketch his distance from the Columbia station versus time.

6. Formats for distributing recorded music have changed through the years. Study the multi-line graph. Predict which line represents each format: vinyl albums, cassette tapes, compact discs, and digital downloads. Explain your choices.



7. Uriash enjoys snowmobiling. The two graphs give information about one of his rides. Use them to describe what Uriash did.





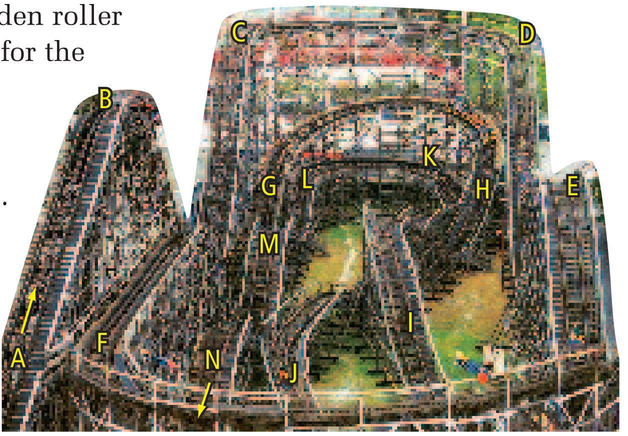
8. The table gives the approximate amount of water needed for various activities. Sketch a graph showing your water usage from the moment you wake up until you go to bed. On the vertical axis, record the amount of water that you use. Record time on the horizontal axis. Include a description.

Water Use	Amount
Toilet flush	6 L
Shower	10 L/min
Bath	68 L
Sink faucet	10 L/min
Dishwasher	27 L/load
Washing machine	99 L/load

Did You Know?

The first SkyTrain line, the Expo Line, was built in time for the Expo 86 World's Fair. It began operation in December 1985. The trains are made by a Canadian company, Bombardier.

9. The Coaster is a wooden roller coaster built in 1958 for the Pacific National Exhibition in Vancouver. Imagine yourself taking a ride. Follow all the ups, downs, twists, and turns. The letters on the track will help you follow the Coaster's path.



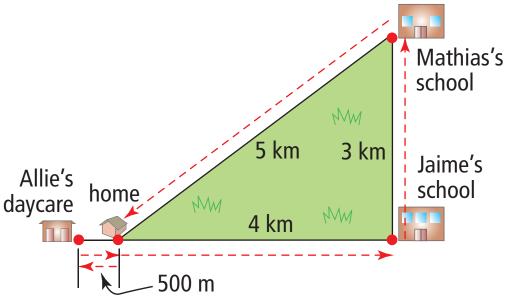
a) Sketch a height-time graph showing your height above the ground versus time for one complete ride.

b) Sketch a speed-time graph showing your speed versus time, for one complete ride.

Web Link

To view a video that shows how the Coaster was built or to view a video of a ride on the roller coaster, go to www.mhrmath10.ca and follow the links.

10. After arriving home from work, Cari leaves to pick up her daughter Allie from daycare. She walks to the daycare and then walks home with her daughter. One hour later, Cari and Allie leave home by car to pick up Cari's other children. They go first to Jaime's school and then to Mathias's school. The diagram shows the distances to each location and Cari's route, marked in red.



Sketch a distance-time graph of each scenario. Starting from the moment Cari leaves for the daycare, show

a) Cari's distance from her home

b) Allie's distance from home

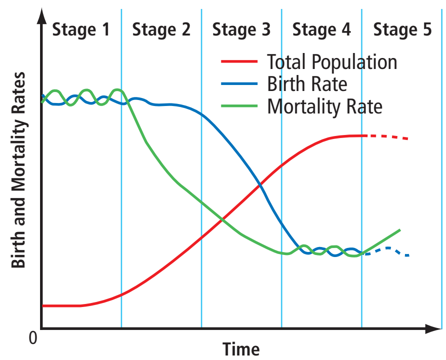
c) Jaime's distance from home

d) Mathias's distance from home

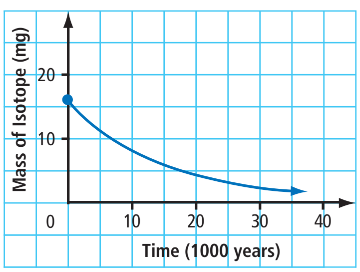
11. Create a speed-time graph for this scenario. A skydiver jumps from an airplane that is flying at a speed of 160 km/h. In about the first 10 s, the skydiver accelerates to a falling speed of 190 km/h. He stays at this speed because he has adopted the standard flat and stable, or "face to Earth," position. After another 30 s, the skydiver opens his parachute and quickly slows his descent to about 18 km/h. He maintains this speed until just before reaching the ground. Then he uses his parachute to slow down slightly, allowing him to make a smooth landing.

Extend

12. Demography is the study of human populations. The graph shows changes in birth and mortality rates over time and their effects on the total population. Study the birth and mortality rates. Describe and explain the changes in the total population at each of the five stages marked.

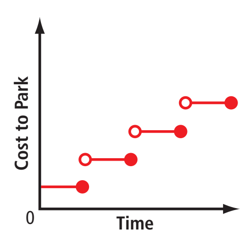


13. **a)** Half-life is the time required for half of a sample of a radioactive substance to decay. The graph shows a typical decay curve for an isotope. What is the half-life of this radioactive substance?



b) Bismuth-210 has a half-life of 5 days. Make a graph showing a decay curve for this substance. Show the first 20 days of decay.

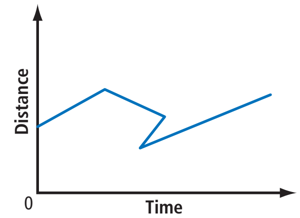
14. The graph shows fees to park a vehicle in a public parking lot in Calgary. Describe the rate scheme.



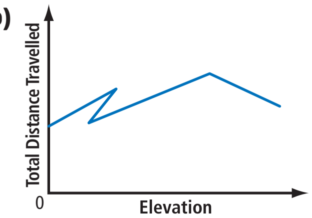
Create Connections

15. Explain why each graph represents an impossible situation.

a)



b)



16. Write a story less than one page in length that incorporates four general graphs: speed versus distance, temperature versus time, number of people versus time, and money versus number of people.

Materials

- CBL interface with a motion detector
- computer or graphing calculator with appropriate software

17. **MINI LAB** Use a motion detector connected to a computer or graphing calculator to collect and graph information about movement in your classroom.

Step 1 Load the data-collection program onto your computer or calculator.

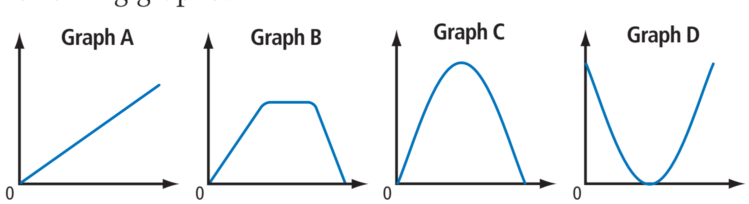
Step 2 Connect the CBL interface to the calculator or computer.

Step 3 Connect the motion detector to any of the sonic ports on the CBL interface.

Step 4 Attach the motion detector to a desk or table. Make sure there is a clear path in front of the detector. The device emits ultrasonic waves that fill a cone-shaped area about 15° to 20° off the centre line of the detector. Keep objects such as desks or chairs out of the cone, because the waves may detect them and record incorrect data.

Step 5 Start the data-collection program.

Step 6 The motion detector measures the time it takes for ultrasonic waves to travel to an object and return to the sensor. As a class, take turns walking in front of the sensor in a manner that produces each of the following graphs.



Step 7 Have a classmate walk in front of the sensor in any way he or she likes. Observe the motion carefully. Create a distance-time graph for the motion. Compare your graph to the one produced on the calculator or computer.

Step 8 Repeat step 7 for other members of the class.

6.2 Linear Relations

Focus on …

- determining if a relation is linear
- representing linear relations in a variety of ways
- explaining why data points should or should not be connected
- identifying the dependent and independent variables in a relation

Knowing the relationship between two quantities can be very useful. For example, it benefits a business to know how the number of units it sells is related to the product's price. It is important for a civil engineer to know the relationship between structural design and strength. Medical doctors are interested in the relationship between a patient's body mass index and heart health.

People have long studied relationships. The famous artist and scientist Leonardo da Vinci (1452–1519) studied the proportions in the human body. Why do you think this information is important to an artist?

Leonardo da Vinci

Materials

- measuring tape
- ruler
- grid paper or graphing technology

Investigate Relationships in the Human Body

1. Work with a partner.

a) Measure and record each other's height, in centimetres.

b) Measure and record the length, in centimetres, of each other's foot.

2. How many of your feet does it take to equal your height? How about your partner's?

3. a) As a class, share your results and determine the mean. Using this mean, predict a relationship between a person's height and foot length. Show your answer to the nearest whole foot.

b) Write this relationship as an equation. This is your model **relation**.

4. Estimate the shortest length and longest length of the feet of high-school students. Use these estimates to complete a table of values. Use increments of 1 cm for foot length. Determine the corresponding height using your equation from step 3b).

relation

- an association between two quantities
- can be presented in words, as an equation, as ordered pairs, as a table of values, or as a graph

5. Look at the height values in your completed table. Are they appropriate to represent the heights of high-school students? If not, make adjustments to your foot-length estimates.

6. a) Plot your results from step 4 to see a graph of this relationship. Record foot length on the horizontal axis and height on the vertical axis.
   b) Should you draw a line through the points on the graph? Why or why not?
   c) Is the resulting graph a straight line or a curve? Explain your answer.
   d) Determine the difference between each height value in your table. What do you notice?

7. **Reflect and Respond**
   a) What are different ways to present a relationship?
   b) If a relation is presented as a table of values, how do you determine if the relation is linear without creating a graph? Is the relationship between a person's height and shoe size linear? Explain your answer.
   c) Should a graph of this relation have the data points connected? Explain your answer.

Link the Ideas

Relations

A relation can be presented in a variety of ways. For example,

Words

Three times the length of your ear, *e*, is equal to the length of your face, *f*, (from chin to hairline).

Equation

$f = 3e$

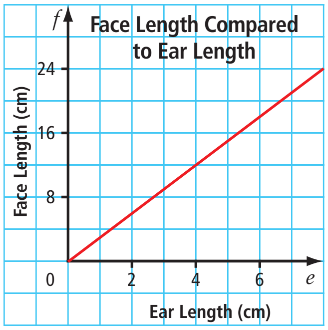
Ordered Pairs

(4, 12), (4.5, 13.5), (5, 15), (5.5, 16.5), (6, 18), (6.5, 19.5)

Table of Values

Ear Length, *e* (cm)	Face Length, *f* (cm)
4	12
4.5	13.5
5	15
5.5	16.5
6	18
6.5	19.5

Graph



Linear and Non-Linear Relations

There are a number of ways to determine whether a relation is a **linear relation** or a **non-linear relation**.

linear relation

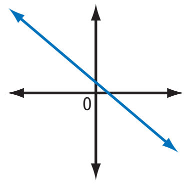
- a relation that forms a straight line when the data are plotted on a graph

non-linear relation

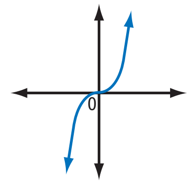
- a relation that does not form a straight line when the data are plotted on a graph

Linear relations have graphs that are straight lines.

Linear



Non-Linear



You can determine whether a relation is linear or non-linear from a table of values. In linear relations, values of y increase or decrease by a constant amount as values of x increase or decrease by a constant amount. Horizontal and vertical lines are exceptions.

Linear Relation

	x	y	
+1	2	8	+3
+1	3	11	+3
+1	4	14	+3
	5	17	

Non-Linear Relation

	x	y	
+1	2	8	+19
+1	3	27	+37
+1	4	64	+61
	5	125	

When a linear relation is written as an equation, it will contain one or two variables and its degree will be 1.

For an expression to have degree 1, what must be the maximum sum of the exponents of the variables for any term?

Linear Relations	**Non-Linear Relations**
$x = 7$	$2x + y^2 = 6$
$3m + 2n = -12$	$h = k^3$
$y = -\frac{2}{3}x + 5$	$xy = 3$

Discrete or Continuous Data

A graph of **discrete data** can only show points because the values in between them have no meaning. A graph of **continuous data** is a solid line or curve.

discrete data

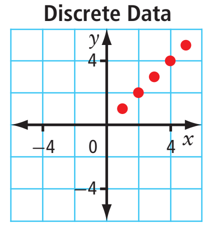
- data values on a graph that are not connected

continuous data

- data values on a graph that are connected

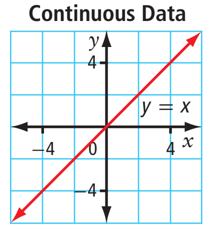
For example, a relation is defined by the set of ordered pairs {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)}. There are only five data points in the relation. These are discrete data. The graph has five unconnected points.

Discrete Data



For the relation defined by the equation $y = x$, there are an infinite number of possible ordered pairs. The points (1, 1), (2, 2), (3, 3), (4, 4), and (5, 5) satisfy this relation. So do many other points such as $\left(\frac{3}{2}, \frac{3}{2}\right)$ and $(-3.6, -3.6)$. These represent continuous data. On a graph, you show an infinite set with an unbroken, or continuous, line.

Continuous Data



Independent and Dependent Variables

independent variable
- the variable for which values are selected

dependent variable
- the variable whose values depend on those of the independent variable

In a relation with two variables, one is the **independent variable** and the other is the **dependent variable**.

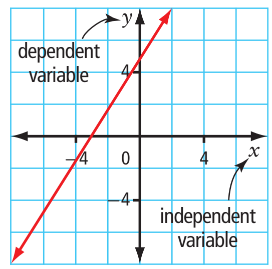
When a relation is expressed as a table of values, the values of the independent variable are listed in the first column. The values of the dependent variable are listed in the second column.

x	$y = 3x + 5$
−1	2
0	5
1	8
2	11

Choices for independent variable

Corresponding values of dependent variable

When a relation is expressed as a graph, the values of the dependent variable are plotted along the vertical axis. The values of the independent variable are plotted along the horizontal axis.



Example 1 Describe a Relation in a Variety of Ways

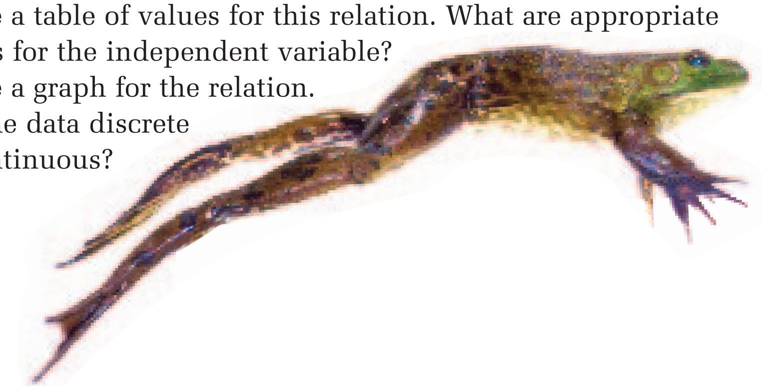
The Canadian National Frog Jumping Championship is part of Les Folies Grenouilles. This annual festival is in St-Pierre-Jolys, MB. The first champion, a frog named Georges, jumped a distance of just over 2 m in a single leap. Assume that Georges could maintain a distance of 2 m on every jump and that the total distance travelled from the start is measured after every jump. Consider the relationship between the number of jumps Georges takes and the total distance the frog travels.

a) Identify the relationship as linear or a non-linear. Explain how you know.

b) Create a variable to represent each quantity in the relation. Which is the dependent variable? Which is the independent variable?

c) Create a table of values for this relation. What are appropriate values for the independent variable?

d) Create a graph for the relation. Are the data discrete or continuous?



Solution

a) Since the distance that Georges covers on each leap is the same, the relation is linear.

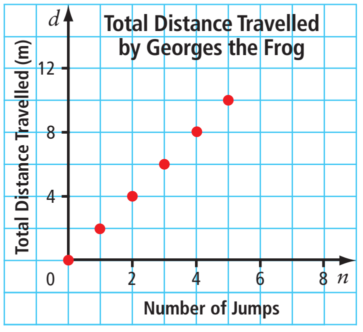
b) The total distance travelled depends on how many jumps the frog takes. Let n represent the independent variable, the number of jumps. Let d represent the dependent variable, the distance travelled.

c) Choose a realistic number of consecutive jumps that Georges might make. For example, the frog could make five jumps.

n	d
0	0
1	2
2	4
3	6
4	8
5	10

Why can the values of *n* only be whole numbers?

d) Display the independent variable, n, on the horizontal axis and the dependent variable, d, on the vertical axis. The data are discrete because there are only six possible values in the relation. Georges does not take partial jumps, so values for n such as 1.5 or 2.8 cannot be used.



Your Turn

Another popular event at *Les Folies Grenouilles* is the fireworks display. Assume that the event organizers send off 20 firework shells each minute.

a) Is the relationship between the total number of fireworks and the duration of the event linear or non-linear? Explain how you know.

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable?

c) Create a table of values for this relation. What are appropriate values for the independent variable?

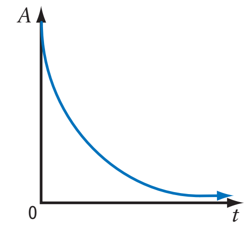
d) Create a graph for the relation. Are the data discrete or continuous?

Example 2 Determine Whether a Relation Is Linear or Non-linear

Consider each relation. Determine whether the relation is linear. Explain why or why not.

a) the relation described by {..., (−9, −10), (−7, −5), (−5, 0), (−3, 5), (−1, 10), ...}

b) The graph shows the relationship between the amount, A, of a radioactive isotope present and the age of a rock sample over time, t, in years.

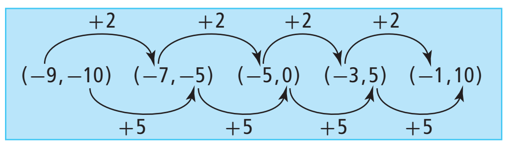


c) the relation described by the equation $m - 17 = 0.8n$

Solution

a) Method 1: Compare Changes in the Independent and Dependent Variables

Check to see if the independent variable increases or decreases at a constant rate and if, at the same time, the dependent variable increases or decreases at a constant rate.



The relation is linear. With each increase of 2 in the independent variable, the dependent variable increases by 5.

Method 2: Use a Table of Values to Compare Changes in Each Variable

	x	y	
	−9	−10	
+2	−7	−5	+5
+2	−5	0	+5
+2	−3	5	+5
+2	−1	10	+5

The relation is linear. Values of x (the independent variable) increase each time by 2. Values of y (the dependent variable) increase each time by 5.

b) The relation is not linear. The graph is not a straight line.

c) The degree is 1. The relation is linear.

How do you know that the degree is 1?

Your Turn

Determine whether each relation is linear. Explain why or why not.

a) the relationship between the cost to rent a dance hall and the number of people attending the dance, if the hall charges \$200 plus \$5 for each person who attends

b) the relation described by the equation $x^2 + y^2 = 25$

c) the relation described by the set of ordered pairs {(10, 12), (15, 4), (20, −4), (25, −12), (30, −20)}

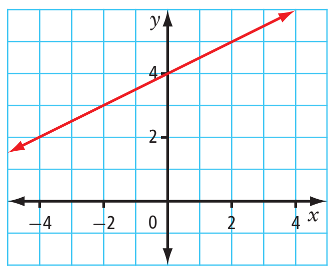
Example 3 Match Representations of a Linear Relation

Match each linear relation with possible representations in the selections that are given. Justify your choices.

a) The pressure, P, that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, d, below the surface.

b) $y = \frac{1}{2}x + 4$

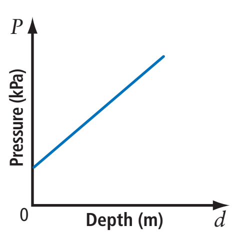
A



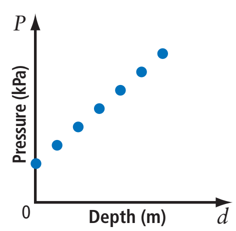
B

x	y
0	4
1	8
2	12
3	16

C



D

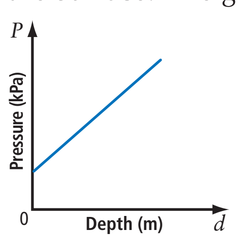


E One number is half another number increased by four.

F (0, 101), (25, 176), (50, 251), (75, 326), (100, 401), (125, 476)

Solution

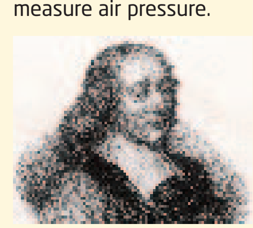
a) The pressure, P, that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, d, below the surface. The graph in choice D is a possible representation.



Increases in pressure are constant as depth changes. Therefore, the relation is linear. Also, since the values for the independent variable are not restricted to whole numbers of metres, the data are continuous.

Did You Know?

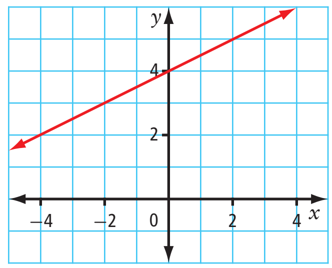
The kilopascal is an SI unit for measuring pressure. Its abbreviation is kPa. This unit represents a force of 1000 N per square metre. The pascal is named after the French mathematician Blaise Pascal. In the 1600s Pascal experimented with barometers, which are instruments used to measure air pressure.



Blaise Pascal

b) $y = \frac{1}{2}x + 4$

The graph in choice A shows this relation.
The representation in choice E also expresses this relation in words.



The equation represents linear data and continuous data.

The points on this graph satisfy the relation.

For example, (−3, 2.5), (−2, 3), and (0, 4) are all solutions to the equation.

Your Turn

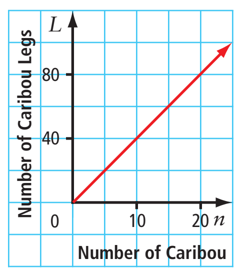
There is a linear relationship between the number of caribou, n, in a herd and the number of caribou legs, L. Which representations model this relation?

A $L = 4n$

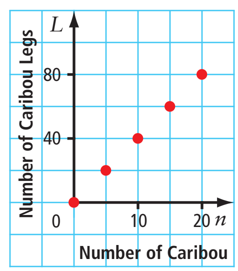
B (0, 0), (3, 12), (8, 32), (15, 60), (50, 200)

C $L = n + 4$

D



E

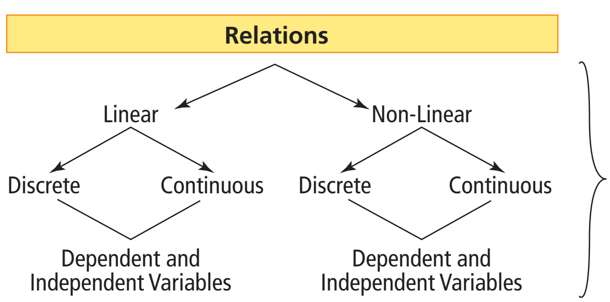


F

n	L
3	6
6	12
9	18
12	24

Key Ideas

- Relations can be represented in a variety of ways. You can use words, equations, tables of values, ordered pairs, or graphs.



These are different characteristics of relations, used to describe them in more detail.

Check Your Understanding

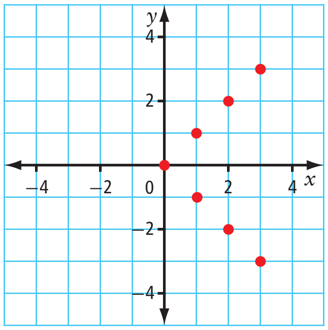
Practise

1. Convert each relation from its current representation to the one suggested.

a)

m	*n*
−2	5
−1	6
0	7
1	8
2	9
3	10
4	22

to ordered pairs

b) 

to a table of values

c) $P = 2d + 5$ to a graph

d) (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) to words, if the independent variable represents the number of children and the dependent variable represents the number of oranges eaten

2. Determine whether each relation is linear or non-linear. Explain your decision.

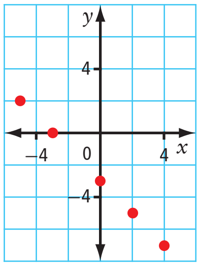
a) $C = 2\pi r$

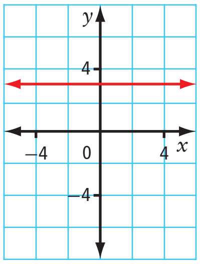
b) $A = s^2$

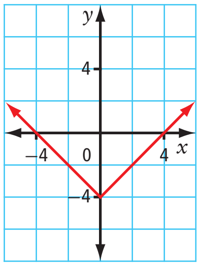
c) $y = 5x - 3$

d) (0, 0), (1, 1), (4, 2), (9, 3), (16, 4)

e) (5, 10), (10, 20), (15, 30), (20, 40), (25, 50)

f) 

g) 

h) 

3. For each relation, state the dependent variable and the independent variable.

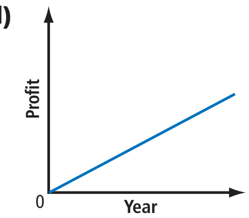
a) $A = \pi r^2$

b) $V = 7t + 2$

c)

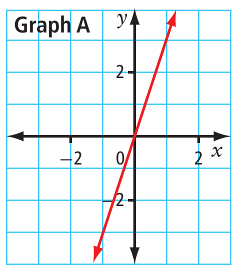
n	A
0	1
1	3
2	9
3	27
4	81

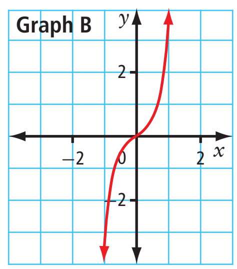
d)

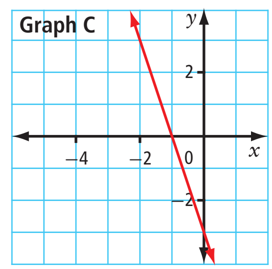


e) Five eggs, e, per chicken, c, are laid each week.

4. Consider the relation described by the equation $y = x^3 + 2x$ and the three graphs shown. Without performing any calculations, predict which graph matches this relation. Check your prediction against a partner's. Discuss your answers.







Apply

Did You Know?

A bushel is a unit used for measuring the volume of grain, fruit, vegetables, or other dry things. One bushel is equal to about 0.036 m^3.

5. More than half of all the wheat grown in western Canada is grown in Saskatchewan. One year, Saskatchewan farmers received \$6 per bushel for wheat. Consider the relationship between the total amount of money each farmer received and the number of bushels the farmer sold.

a) Is this a linear or non-linear relationship? Explain how you know.

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable? Explain how you know.

c) Assume that Saskatchewan wheat farms each produce up to 50 000 bushels of wheat. Create a table of values for the relationship.

d) Are the data discrete or continuous? Explain how you know.

e) Graph the relationship.

6. Give an example of a relation with each characteristic listed.
   a) a table of values that shows a non-linear relationship
   b) a graph that shows a non-linear discrete relationship
   c) an equation that shows a linear relation with independent variable m and dependent variable n

7. The refraction of light causes an object lying under water to appear closer to the surface than it actually is. The relationship between how deep a coin appears to be in a fountain and the coin's actual depth is given by the equation $D = 0.75d$, where D is the apparent depth and d is the actual depth.
   a) Is this relationship linear or non-linear? Explain.
   b) Identify the dependent variable and the independent variable for this relation.
   c) If the water is 2 m deep, how deep does the coin appear to be?
   d) Can you use the formula to determine the apparent depth of the coin if the water is 2.8 m deep? Does this indicate that the relation is continuous or discrete? Explain.

8. A killer whale is swimming at a speed of 6 km/h. Consider the relationship between the total distance, in kilometres, travelled by the whale and time, in hours.
   a) Assign variables to represent each quantity in the relation. Identify the dependent variable and the independent variable.
   b) Assume that the whale swims for 5 h without stopping. Create a set of ordered pairs for the relation.
   c) Is the relation continuous or discrete? Explain.
   d) Graph the relation.
   e) Is the relation linear or non-linear? Explain.

Did You Know?

Killer whales have long played a role in the legends and beliefs of coastal First Nations cultures. The Haida, for example, used the killer whale as a symbol for family. They did this because the whales stayed in families, travelling in large pods.

Killer Whale (Sgáan) by Don Yeomans, Haida artist

9. An action plan is in place to clean up a lake that has high levels of mercury. Tests show 9 mg of mercury per 1000 L of water. It is estimated that mercury levels can be reduced each year by 0.8 mg per 1000 L of water.

a) Create a table of values to represent the relationship between the amount, A, of mercury present and the number of years, t, since the clean-up plan started. A is the dependent variable and t is the independent variable in the relation.

b) According to this relation, how long will it take to rid the lake of mercury?

10. The Pacific Coast is the most earthquake-prone region in Canada, while Saskatchewan and Manitoba have the least probability of earthquakes.

Seismometer measuring seismic waves.

a) A magnitude scale describes the relative size of earthquakes. An earthquake with a magnitude value that is one higher than another earthquake is about 10 times more intense. For example, an earthquake measuring 7 is about 10 times as great as one with a magnitude of 6. Is the relationship between an earthquake's magnitude and its size linear or non-linear? Explain.

b) From northern Vancouver Island to the Queen Charlotte Islands, the oceanic Pacific plate is sliding to the northwest at about 6 cm per year. Starting with this year as 0, create a set of ordered pairs to show the relationship between the number of years from now and the amount of plate sliding for the next 8 years.

c) Graph the ordered pairs. Should the points be connected? Should you continue the pattern in both directions? Explain your answers.

Did You Know?

A seismometer is an instrument used to measure seismic waves. Information about the waves is then used to determine the magnitude of an earthquake.

Extend

11. Does each table of values represent a linear or a non-linear relation? Explain.

a)

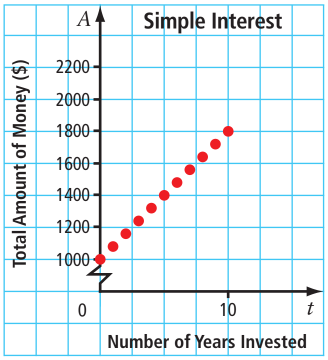
x	y
1	k
3	$2k$
5	$3k$
7	$4k$

b)

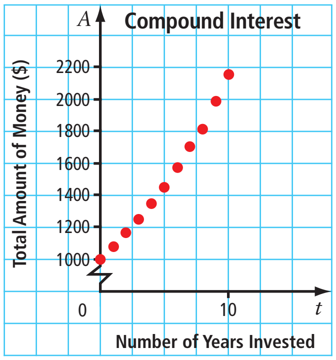
x	y
m	0
$m + 1$	$3n$
$m + 2$	$6n$
$m + 3$	$9n$

12. The graphs represent different investments based on investing $1000 today. The first graph shows simple interest of 8% per year. The second graph shows compound interest at 8% per year.

Graph A



Graph B



a) Which graph is linear? Which is non-linear? Explain.

b) In one interest calculation, you receive a set percent of your original investment each year. In the other interest calculation, you receive a set percent of the amount that you have in your account. Identify which graph represents each of these scenarios. Explain your reasoning.

Create Connections

13. Which method do you prefer to use when representing a relation? Explain.

14. Think about a relationship between two things that you are interested in learning more about. Describe two ways you could collect the information.

6.3 Domain and Range

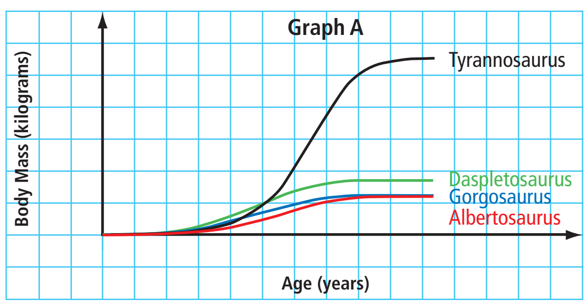
Scientists study our world to piece together the story of our past and make predictions about our future. Their writings are often accompanied by graphs to illustrate the data they are presenting. The more detail given with the graphs, the easier it is for the reader to interpret the information. One important aspect of a graph is the span of possible values for each quantity being compared.

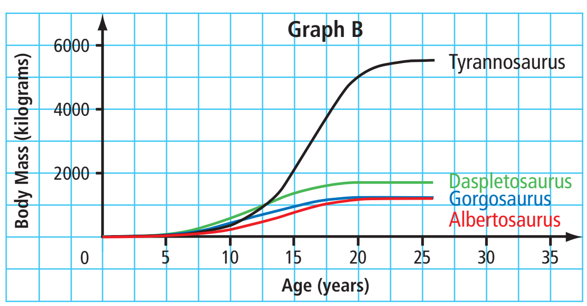
Focus on ...

- understanding the meaning of domain and range
- expressing domain and range in a variety of ways

Investigate Appropriate Values for the Dependent and Independent Variables

1. By studying changes in several specimens of tyrannosaurs, paleontologists can estimate the lifespan and growth rates of the species. The graphs show the relationship between body mass and age for four types of tyrannosaur.





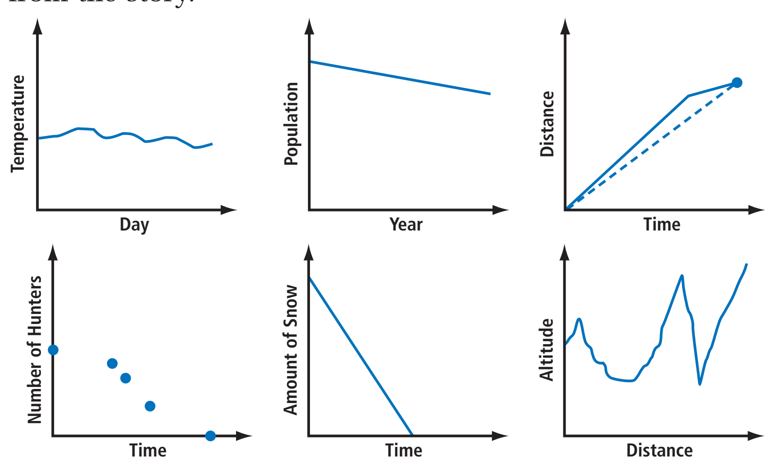
a) If you are given only Graph A, what conclusions can you make about the relationship between each tyrannosaur's body mass and its age?

b) What additional information can you tell if you are given Graph B instead?

2. Read the following story inspired by the Dene legend for the creation of the seasons.

> The first people of Earth had to endure **winter for twelve months** of the year. Most of the land was covered by massive, moving layers of ice and deep snow. No vegetation survived in the harsh gripping cold. All water was frozen in a land of endless cold.
>
> One day, when the first people were hunting, they came upon a bear with a sack around his neck. The curious hunters asked what was in the sack, and the bear growled that it was filled with the abundance of summer's warmth and light. The hunters coveted the sack, but the bear would not part with it. They decided to lure the bear to a great feast of moose and **caribou**, fill him with food, and, when he slept, steal the sack. The bear readily accepted their invitation for a feast, but arrived without the sack around his neck. The bear ate his fill and fell asleep. The frustrated chief ordered **four of the village's skilled hunters to follow the bear home** and steal the sack by any means. Peering inside the bear's large cave next morning, the hunters spotted the sack on the cave floor with two black bears guarding it. The courageous hunters sprang into the cave to demand the sack. **A fierce fight killed three of the hunters and mortally wounded the fourth**, but before he died, he grabbed the sack and unleashed the abundance of warmth and light. Instantly, the air became warm and the sky filled with bright sunlight. **The snow melted** into rivers and lakes, and the **hills and valleys** became covered with trees, flowers, and bushes. Strange birds flew in great numbers and built nests, and streams filled with fish. Every year since that time, summer has come to the Dene.
>
> *Author unknown*

The general graphs are related to the **bold text** in the story. In small groups, discuss how each graph depicts the information from the story.



3. a) Work with a partner. Choose any four of the graphs. Agree upon appropriate values for the dependent and independent variables. Clearly describe these possible values using words, lists, or number lines.

b) Compare your results with those of another group. Share your reasoning with them.

4. Reflect and Respond

a) Why did you not choose the two graphs that you did not use?

b) Did the other group choose any graphs different from yours? If so, do you agree with their choice of possible values for each axis? Explain.

c) How does adding values to each axis aid in the understanding of the graphs?

Link the Ideas

When comparing two quantities, the words **domain** and **range** are used to describe the values that are appropriate.

domain
- the set of all possible values for the independent variable in a relation

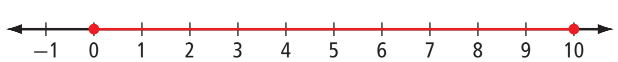
range
- the set of all possible values for the dependent variable as the independent variable takes on all possible values of the domain

In a set of ordered pairs, values for the domain are the first element of each pair. Values for the range are the second element. On a graph, values of the domain are plotted along the horizontal axis. Values of the range are plotted along the vertical axis.

There are a variety of ways to express the domain and range of a relation.

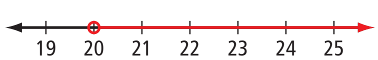
- **Words** can be used to describe the values that are allowed. For example, the domain is the set of all real numbers between 0 and 10, inclusive. The range is the set of all real numbers greater than 20.

- **Number lines** give a picture of the values that are allowed. For example, this number line represents all numbers between 0 and 10, inclusive.



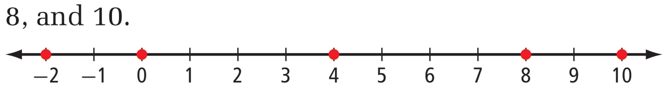
The solid circles at 0 and 10 indicate that these points are included.

This number line represents all numbers greater than 20.



The open circle at 20 indicates that the value 20 is not included. The arrow pointing to the right indicates that there is no upper limit.

This number line represents the discrete list of numbers –2, 0, 4, 8, and 10.



- **A list** is a useful way to give the domain and range for discrete data when there are not many numbers in the set.
  For the relation (0, 0), (1, 5), (3, 7), (5, 7), the domain is {0, 1, 3, 5} and the range is {0, 5, 7}.

- **Set notation** is a formal mathematical way to give the values of the domain and range.

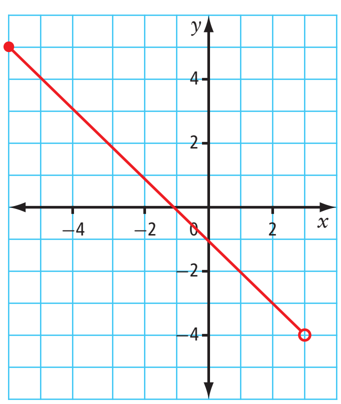
Set Notation	What It Means
The domain: $\{x \mid x \le 10, x \in \mathrm{R}\}$	{ } is the type of brackets used for a set. $\in$ means "is an element of". $\mid$ means "such that". The statement is read as follows: *x* is an element of the real numbers such that *x* is less than or equal to 10.
The range: $\{y \mid y > 20, y \in \mathrm{R}\}$	The statement is read as follows: *y* is an element of the real numbers such that *y* is greater than 20.

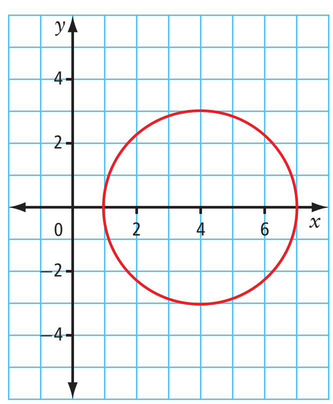
Recall that the symbols used for the number sets are
R for real numbers
Q for rational numbers
$\overline{\mathrm{Q}}$ for irrational numbers
I for integers
N for natural numbers
W for whole numbers

- **Interval notation** uses different brackets to indicate an interval.
  This style of bracket,], is used if the end number is included.
  This style of bracket,), is used if the end number is not included.
  The infinity symbol, ∞, is used if there is no end point.
  A domain of all numbers between 0 and 10, inclusive, would be given as [0, 10].
  A range of all numbers greater than 20 would be given as $(20, \infty)$.

Example 1 Determine the Domain and Range From a Graph

For each graph, give the domain and range. Use words, a number line, interval notation, and set notation.

a)


b)


Solution

a) From looking at the graph, you can see that the smallest value for x is -6. The largest value for x is up to, but not including, 3. The smallest value for y is down to, but not including, -4. The largest value for y is 5.

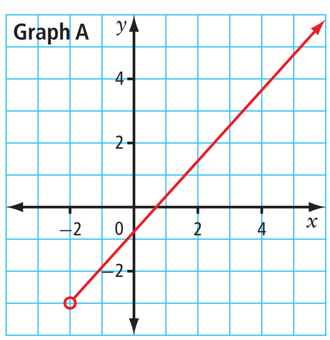
Domain	**Range**
Words All real numbers between -6 and 3, including -6 but not including 3	*Words* All real numbers between -4 and 5, not including -4 but including 5
Number Line 	*Number Line* 
Interval Notation $[-6, 3)$	*Interval Notation* $(-4, 5]$
Set Notation $\{x \mid -6 \leq x < 3, x \in \mathrm{R}\}$	*Set Notation* $\{y \mid -4 < y \leq 5, y \in \mathrm{R}\}$

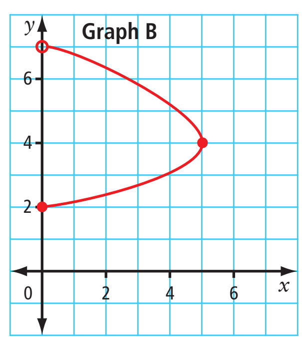
b) From looking at the graph, you can see that the smallest value for x is 1. The largest value for x is 7. The smallest value for y is -3. The largest value for y is 3.

Domain	**Range**
Words All real numbers between 1 and 7, inclusive	*Words* All real numbers between -3 and 3, inclusive
Number Line 	*Number Line* 
Interval Notation $[1, 7]$	*Interval Notation* $[-3, 3]$
Set Notation $\{x \mid 1 \leq x \leq 7, x \in \mathrm{R}\}$	*Set Notation* $\{y \mid -3 \leq y \leq 3, y \in \mathrm{R}\}$

Your Turn

For each graph, give the domain and range using words, a number line, interval notation, and set notation.





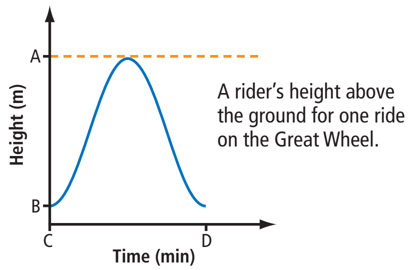
Example 2 Domain and Range for a Situation

The Great Wheel is being built in Beijing in the People's Republic of China. When finished, it will be the largest Ferris wheel in the world. The wheel will have a diameter of 193 m and will reach a maximum height of 208 m.

The graph shows a rider's height relative to the ground for a 20-min ride through one rotation.

a) What are the values of points A, B, C, and D, and what do they represent?

b) What are the domain and the range of the graph?



Solution

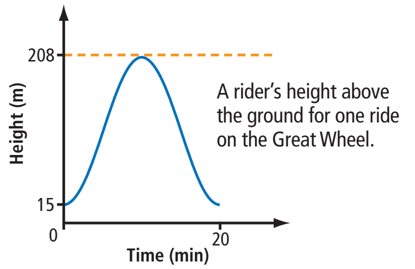
a) Point A is the highest point on the graph, so it must represent the highest point on the ride. Its value is 208 m.

Point B is the lowest point on the graph. It represents the location where a rider boards the Ferris wheel. Since the diameter of the wheel is 193 m, this point is 193 m below point A. By subtraction, the value of B is 15 m.

Point C is the origin, (0, 0). It represents ground level on the vertical axis and the starting time on the horizontal axis.

Why does the graph not begin at (0, 0)?

Point D represents the time it takes to complete one revolution. Its value therefore is 20 min.



Why is the graph in quadrant I?

b) The domain and range can be described in several ways.

Words:
Domain: All times are between 0 min and 20 min, inclusive.
Range: A rider's height above the ground is between 15 m and 208 m, inclusive.

Number Line:
Domain: Ride time, in minutes

Range: Height above the ground, in metres

Interval Notation:
Domain: Ride time, in minutes: [0, 20]
Range: Height above the ground, in metres: [15, 208]

Set Notation:
Domain: Ride time, in minutes: $\{t \mid 0 \leq t \leq 20, t \in \mathrm{R}\}$
Range: Height above the ground, in metres: $\{h \mid 15 \leq h \leq 208, h \in \mathrm{R}\}$

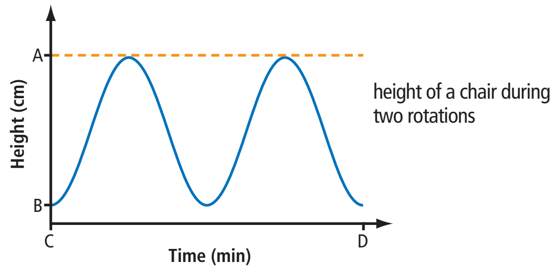
Web Link

To view a graph of a rider's height above the ground when on a Ferris wheel, go to www.mhrmath10.ca and follow the links.

Your Turn

A motorized model Ferris wheel has a radius of 22 cm. The support structure keeps the bottom of the wheel 3 cm above the base. It takes 10 s to complete one revolution. The graph shows the height of one of the chairs during two rotations of the wheel, starting at the lowest point.

a) What are the values of A, B, C, and D? What do they represent?
b) What are the domain and range of the graph? Express each in words, as a number line, in interval notation, and in set notation.

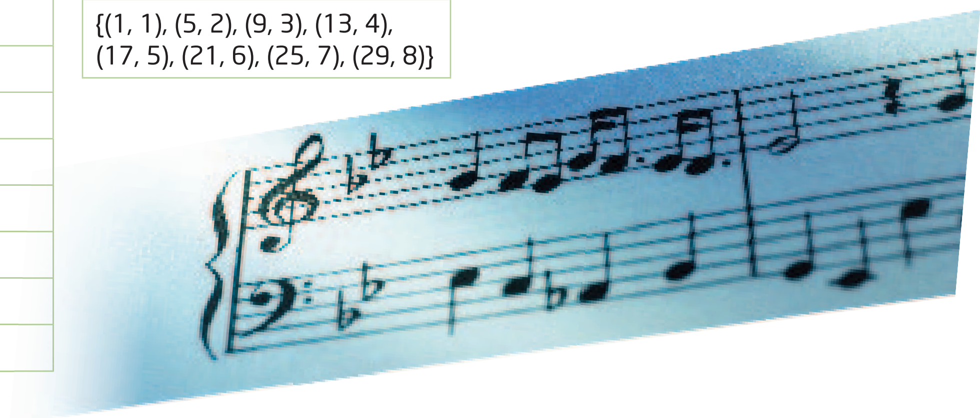


Example 3 Domain and Range for Discrete Data

Caitlin is marking time for some music by clapping on the first beat of every bar. The table of values and the set of ordered pairs show the relationship between the total number of beats and her total number of claps. Give the domain and range of the relation using words and a list.

Number of Beats	Number of Claps
1	1
5	2
9	3
13	4
17	5
21	6
25	7
29	8

Ordered Pairs (Beats, Claps)
{(1, 1), (5, 2), (9, 3), (13, 4), (17, 5), (21, 6), (25, 7), (29, 8)}



Solution

Words:

Domain: The total number of beats is given by the numbers 1, 5, 9, 13, 17, 21, 25, and 29.

Range: The total number of claps is given by the whole numbers between 1 and 8, inclusive.

List:

Domain: {1, 5, 9, 13, 17, 21, 25, 29}

Range: {1, 2, 3, 4, 5, 6, 7, 8}

Your Turn

Data for a relation are recorded in the table of values. Give the domain and range using set notation and lists.

a	b
−3	5
−2	6
−1	7
0	8
1	9
2	10

Example 4 Use Technology to Graph a Relation With a Restricted Domain

A variety of corn plant grows at an average rate of 4.5 cm per day from the start of the third week of growth to the end of the sixth week. The plant's growth can be modelled using the formula $h = 4.5a + 25$, where h is the height of the plant, in centimetres, and a is the age of the plant, in days. Use a graphing calculator to create a graph of a cornstalk's height from the beginning of week 3 to the end of week 6.

Solution

To get a view of the plant's height for the required time frame, you need to restrict the window settings of the graphing calculator displaying the relation $h = 4.5a + 25$.

Which variable is the independent variable? How do you know?

In the domain you want only the values for the age of the plant during the period from week 3 to week 6, inclusive. At the beginning of week 3, the plant is 14 days old. At the end of week 6, the plant is 42 days old. Therefore, the domain is [14, 42].

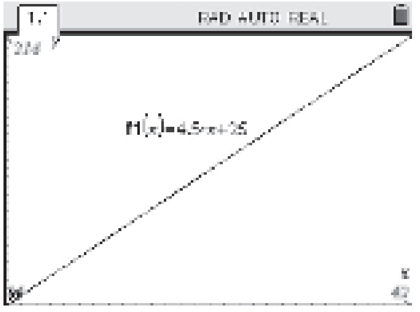
The range is the set of all possible plant heights during this growth period.

The model is valid only for these ages.

If $a = 14$, then $h = 4.5(14) + 25$ or $h = 88$.
If $a = 42$, then $h = 4.5(42) + 25$ or $h = 214$.
Therefore, the range is [88, 214].

Graph the relation on a graphing calculator.

- Enter the equation of the relation.
- Set the values for the window settings.
- Graph the relation.

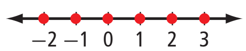


Your Turn

The same species of corn grows at an average rate of 5 cm per day from the start of week 7 until the end of week 9. The plant's growth in this period is modelled using the formula $h = 5a + 214$, where h is the height of the plant, in centimetres, and a is the age of the plant, in days. Use a graphing calculator to show a graph of the plant's height for these three weeks.

Key Ideas

- The domain of a relation is the set of all numbers for which the independent variable is defined.
- The domain of a relation may also be described as:
  - the set of first coordinates in a set of ordered pairs
  - the possible values in the first column of a table of values
  - the possible values on the horizontal axis of a graph
- The range of a relation is the set of all numbers for which the dependent variable is defined.
- The range of a relation may also be described as:
  - the set of second coordinates in a set of ordered pairs
  - the possible values in the second column of a table of values
  - the possible values on the vertical axis of a graph
- The domain and range can be expressed in different ways.

Words	All integers equal to or greater than −2 and less than or equal to 3
Number Line	
Interval Notation	$[-2, 3]$
Set Notation	$\{n \mid -2 \leq n \leq 3, n \in \mathrm{I}\}$
A List	$\{-2, -1, 0, 1, 2, 3\}$

Check Your Understanding

Practise

1. Describe the set of numbers indicated by each number line. Use words, interval notation, and set notation.

a) 

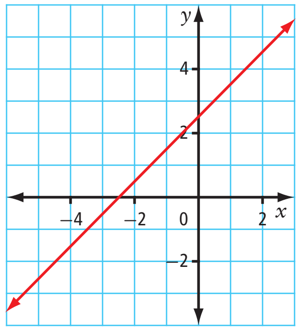
b) 

c) 

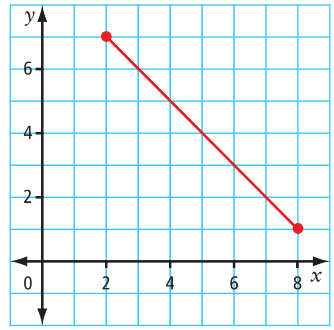
d) 

2. Give the domain and range of each graph. Use words, a number line, interval notation, and set notation.

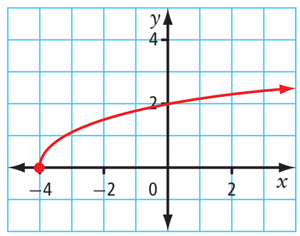
a)



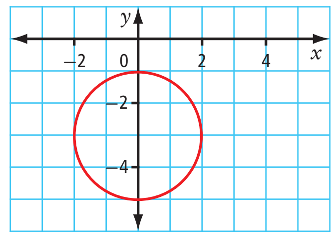
b)



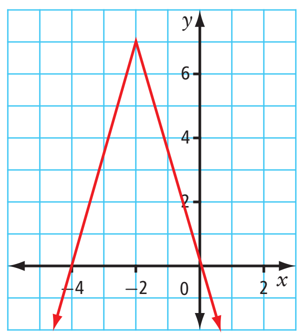
c)



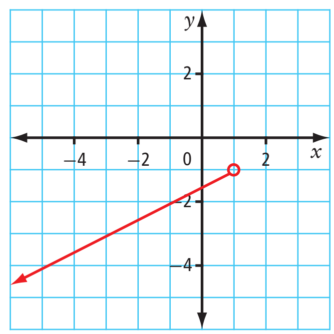
d)



e)

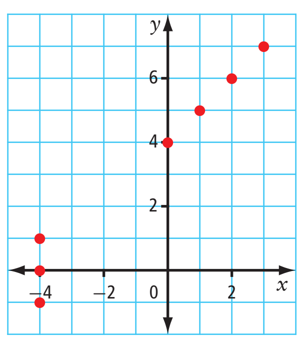


f)



3. Write the domain and the range of each relation as lists.

a)



b)

s	*v*
−4	5
−2	5
0	7
2	7
4	9
6	9

c) (50, 10), (100, 20), (150, 30), (200, 40)

4. A relation is given by the formula $k = 2.8m - 3.5$.

a) If the domain of the relation is [0, 25], what is the range?

b) Graph the relation on a graphing calculator. Record the window values you chose. Sketch the relation.

Apply

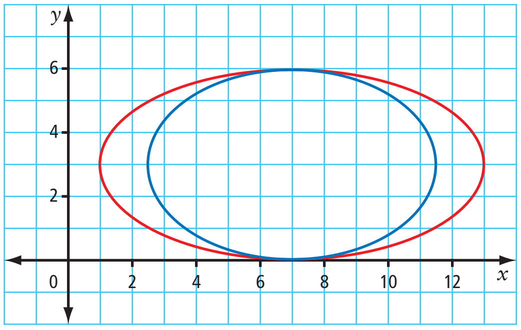
5. The table gives the average annual high temperature for a number of western and northern Canadian cities.

City	Average Annual High Temperature (°C)
Winnipeg	8.3
Regina	9.1
Edmonton	8.5
Calgary	10.5
Vancouver	13.7
Victoria	14.1
Whitehorse	4.5
Yellowknife	−0.2

a) Give the domain and the range for the relation.

b) Graph the relation.

6. A company offers two models of above-ground oval swimming pools. The ovals in the graph are drawn using the dimensions, in metres, of the pools.



a) Using interval notation, what are the domain and range of the blue oval?

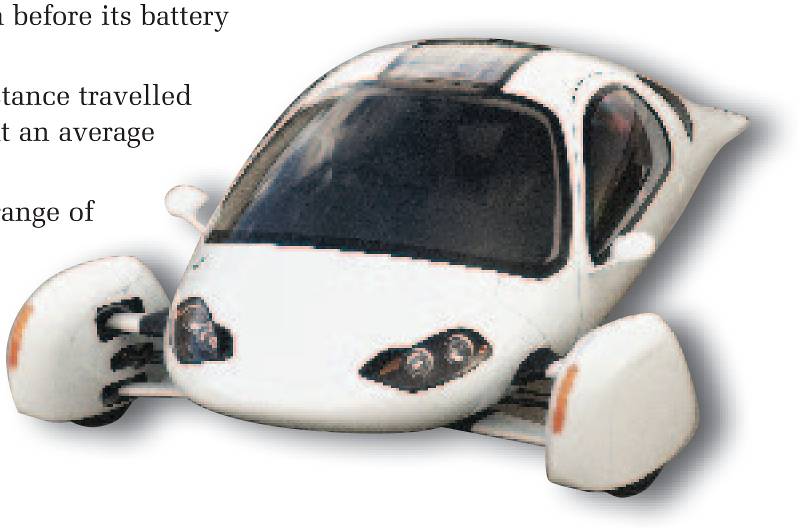
b) Using set notation, what are the domain and range of the red oval?

c) What are the actual dimensions of each pool?

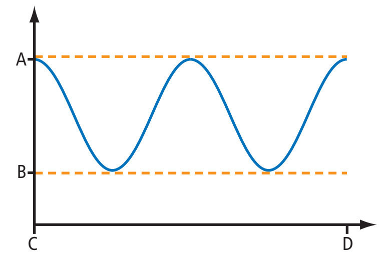
7. An electric car can travel 193 km before its battery needs to be recharged.

a) Create a graph to show the distance travelled over time if the car is driven at an average speed of 60 km/h.

b) Describe the domain and the range of your graph in two ways.



8. The graph shows the changes in tide levels for Bella Coola on the central coast of British Columbia for a 24-h period starting at 12:00 a.m. The greatest water depth, at high tide, is 15.9 ft. The least depth, at low tide, is 4.5 ft.



   a) Sketch a copy of the graph. Label each axis with appropriate dependent and independent variables.
   b) What are the values of A, B, C, and D? What do they represent?
   c) What are the domain and range of the graph? Express these in words, as a number line, in interval notation, and in set notation.

9. A hot-air balloon is flying at an altitude of 1236 m. It begins to descend at a rate of 10 m per minute.
   a) How long does it take the balloon to reach the ground?
   b) Assign variables to represent each quantity in the relation. Identify the independent variable and the dependent variable. Graph the balloon's progress from the moment it begins its descent until it reaches the ground.
   c) Does the graph continue in both directions? Explain.
   d) What do the domain and range represent in this context?
   e) Give the domain and the range using all forms appropriate for this situation.

Extend

10. The domain of a linear relation is given by $\{x \mid -2 \leq x \leq m, x \in \mathrm{I}\}$. The range, in order, is given by $\{2, 6, 10, k, 18\}$. Use a graphing approach to determine the smallest possible values of m and k.

11. James can store 8 GB of music on his phone. If a 3-min digital song requires 3.5 MB of storage space, what are the domain and range for the relationship between memory used and number of songs that are each 3 min in length?

Create Connections

12. Using everyday language, how do you describe domain and range to someone not familiar with the concepts?

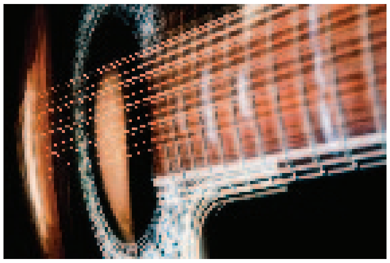
13. For a topic that interests you, such as sports, music, or entertainment, give an example of a relation that has a restricted domain and range. Why do these restrictions exist?

6.4 Functions

Focus on ...

- sorting relations into functions and non-functions
- using notation specifically designed for functions
- graphing linear functions

There is a special class of relations, called functions, where two quantities depend on each other in a particular way. The amount of tension on a guitar string determines the musical note played. The channel displayed on your television screen depends on the number you enter into the remote. Describe other examples from your daily life where two quantities depend on each other in a particular way.



Investigate Functions

Study the following relations. They are categorized as functions and non-functions.

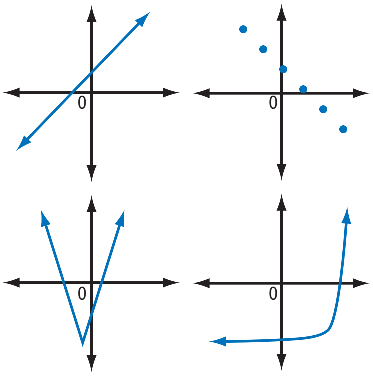
These 8 relations are functions.

x	y
5	10
6	15
7	20

x	y
11	3
21	3
31	3

$\{(-2, -5), (0, 4), (2, 13), (4, 22)\}$

$\{(10, 10), (12, 10), (14, 12), (16, 12)\}$



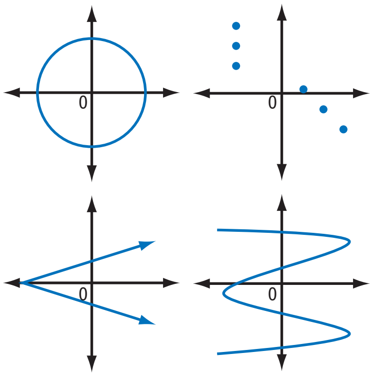
These 8 relations are not functions.

x	y
6	10
6	15
7	20

x	y
3	11
3	21
3	31

$\{(10, 10), (12, 10), (12, 14), (12, 16)\}$

$\{(7, 5), (7, 8), (9, 11), (11, 14)\}$



1. What is similar about the functions? What is similar about the non-functions? Discuss your findings with a partner.

2. Reflect and Respond

a) How can you tell whether or not a relation is a function?.

b) Share your explanation with your classmates. Once you have heard other explanations, revise yours to be more precise.

Link the Ideas

function

- a relation in which each value of the independent variable is associated with exactly one value of the dependent variable
- for every value in the domain there is a unique value in the range

function notation

- a symbolic notation used for writing a function
- $f(x)$ is read as "f of x" or "f at x".

Did You Know?

The term *function* was developed and used by German mathematician Gottfried Wilhelm Leibniz (1646–1716). The notation we use today was created by Swiss mathematician Leonhard Euler (1707–1783). Euler contributed to many branches of mathematics.

Functions can be written using **function notation**. The function $y = 4x + 1$ is written as $f(x) = 4x + 1$. The name of the function is f, with a variable name of x. In this example, $4x + 1$ is the rule that assigns a unique value for y for each value of x. Any letter may be used to name a function. Two examples are $v(t) = 9.8t^2$ and $A(r) = \pi r^2$.

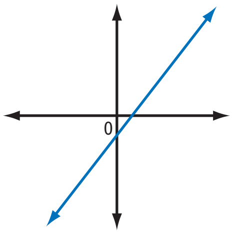
Function notation highlights the input/output aspect of a function. The function $f(x) = 4x + 1$ takes any input value for x, multiplies it by 4, and adds 1 to give the result. For example, if $x = 2$ is the input, then $f(2) = 9$ is the output.

$f(2) = 4(2) + 1$ — When $x = 2$, the value of the function is 9.
$f(2) = 9$ — The point (2, 9) is on the graph of the function.

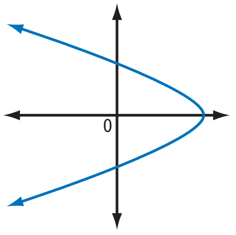
Example 1 Determine Whether a Relation Is a Function

For each pair of relations, decide which relation is a function and which is not a function. Explain your choice.

a) A



B



b) C

x	y
2	5
2	7
4	9
6	11

D

x	y
−3	3
−2	4
−1	3
0	4

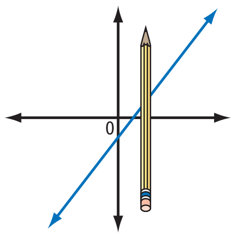
c) E {(1, 1), (2, 2), (3, 3), (4, 4)}
F {(1, 1), (1, 2), (1, 3), (1, 4)}

Solution

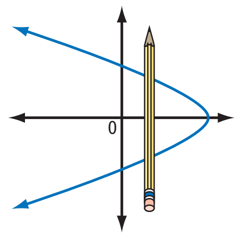
For each pair, check to see which relation has a domain value associated with more than one range value. These are not functions.

a) When a relation is given as a graph, look for any points on the graph that line up vertically. If points line up vertically, a value of x has more than one corresponding value of y. One method to check this is called the **vertical line test**.

A



B



You can use a pencil as your vertical line.

Relation A is a function, because it passes the vertical line test. Each domain value is used only once. Relation B does not pass the vertical line test. Many values of x have two values of y.

b) In relation C, the value of the relation when x equals 2 is associated with y-values of 5 and 7, so this is not a function. Relation D is a function, because each domain value is used only once.

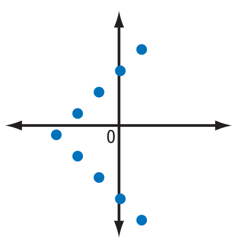
c) In relation E, each domain value is used only once, so it is a function. In relation F, the domain value 1 is used repeatedly. Therefore, relation F is not a function.

vertical line test
- a test to see if a graph represents a function
- if any vertical line intersects the graph at more than one point, the relation is not a function

Your Turn

Which of the following relations are functions? Explain your choices.

a)

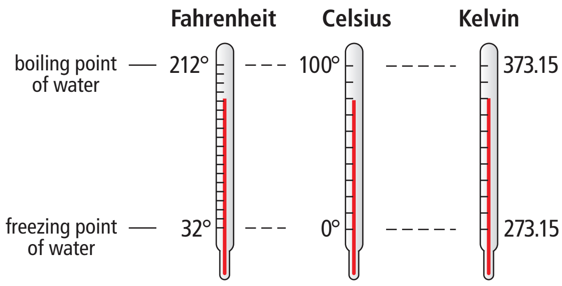


b) $\{(-2, 1), (0, 0), (2, 1), (5, 1)\}$

c)

x	y
1	3
2	3
3	4
4	4
5	4

Example 2 Work With Function Notation



The function $F(C) = 1.8C + 32$ is used to convert a temperature in degrees Celsius (°C) to a temperature in degrees Fahrenheit (°F).

a) Determine $F(25)$. Explain your answer.

b) Determine C so that $F(C) = 100$. Explain your answer.

Solution

a)
$$F(C) = 1.8C + 32$$
$$F(25) = 1.8(25) + 32$$
$$F(25) = 45 + 32$$
$$F(25) = 77$$

This means that 25 °C is the same as 77 °F.

b)
$$F(C) = 1.8C + 32$$
$$100 = 1.8C + 32$$
$$100 - 32 = 1.8C + 32 - 32$$
$$68 = 1.8C$$
$$\frac{68}{1.8} = \frac{1.8C}{1.8}$$
$$37.8 = C$$

This means that 100 °F is the same as 37.8 °C.

Your Turn

a) Determine $F(86)$. Explain your answer.

b) Determine C so that $F(C) = 98.6$. Explain your answer.

c) Another measurement scale for temperature that is used in science is the Kelvin scale. The function $K(C) = C + 273.15$ can be used to convert from degrees Celsius to kelvins. Determine $K(80)$ and explain your answer.

Did You Know?

The Kelvin temperature scale was introduced by William Thomson, 1st Baron Kelvin (1824–1907), a Scottish physicist. Units on the scale are called kelvins. The Kelvin scale does not use the word *degrees* or the symbol °.

Example 3 Graphing Linear Functions

Skye has a cell phone plan for a monthly fee of \$20 plus 15¢ for each text message to or from a number not on a list of favourites. Skye's monthly bill can be modelled by the relation $C = 0.15n + 20$, where C is the total charge, in dollars, and n is the number of additional text messages.

a) Write the relation in function notation.

b) Make a table of values. Graph the function if Skye sends up to four additional text messages.

c) If Skye's cell phone bill for a certain month is \$22.25, how many additional text message charges are there?

Solution

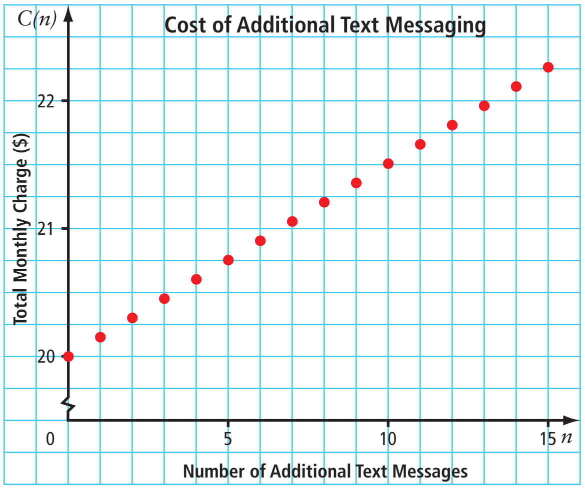
a) The independent variable is n, the number of additional text messages. The function is $C(n) = 0.15n + 20$.

b) Choose values for the independent variable, n. Use the formula to determine the total charge.

A table of values for the first five values of this function is shown.

n	$C(n)$
0	20.00
1	20.15
2	20.30
3	20.45
4	20.60

Continuing the pattern, the graph shows the first 16 values for the function.



Why are the points in this graph not connected?

c) Method 1: Use the Graph

Use the graph on page 309. Locate the value of \$22.25 on the total monthly charge axis. Read the corresponding number of text messages. From the graph, it can be seen that Skye's cell phone bill includes 15 additional text message charges.

Method 2: Use the Formula

The total charge is \$22.25.
Substitute $C = 22.25$ into the relation.

$$\begin{aligned} C(n) &= 0.15n + 20 \\ 22.25 &= 0.15n + 20 \\ 22.25 - 20 &= 0.15n + 20 - 20 \\ 2.25 &= 0.15n \\ \frac{2.25}{0.15} &= \frac{0.15n}{0.15} \\ 15 &= n \end{aligned}$$

Solve for *n*.

Skye's phone bill includes 15 additional text message charges.

Your Turn

Use the relation $y = 3x - 1$.

a) Write the relation in function notation using f for the name of the function.

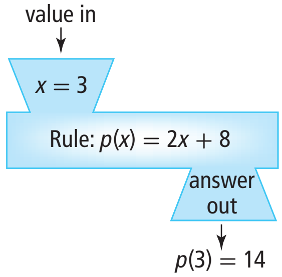
b) Make a table of values. Graph the function.

c) Determine the value of x if $f(x) = 53$.

Key Ideas

- All functions are relations but not all relations are functions.
- A relation is classified as a function if each value in the domain corresponds to exactly one value in the range.
- Each function has its own formula, or rule, that is often given using a special notation, called function notation.

  For example, $p(x) = 2x + 8$ shows that the function p takes an input value, multiplies it by 2, adds 8, and outputs the answer.



Check Your Understanding

Practise

1. Determine whether each relation is a function or is not a function. Give a reason for your answer.

a) (−1, 2), (0, 1), (1, 2), (2, 5)

b) (3, 12), (4, 12), (5, 14), (6, 14)

c) (1, 2), (2, 3), (3, 4), (4, 5), (5, 6)

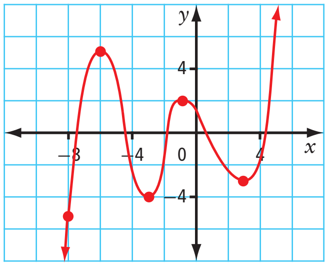
d)

x	y
0	0
1	−1
1	1
4	−2
4	2

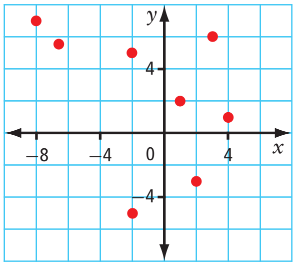
e)

Name	Age
Naomi	14
Wam	15
Brigid	14
Sharon	16
Arvind	15

f)



g)



2. The formula for the surface area, A, of a sphere with radius r is $A = 4\pi r^2$. Write this formula using function notation.

3. The cost to have artwork printed on T-shirts is given by the function $C(n) = 3n + 50$, where n is the number of shirts and C is the cost, in dollars. Write this function as a formula in two variables.

4. If $f(x) = 10x - 8$, determine

a) $f(2)$ **b)** $f(-3)$ **c)** x if $f(x) = 42$

5. If $h(x) = \frac{2}{3}x + 1$, determine

a) $h(9)$ **b)** $h(-3)$ **c)** x if $h(x) = -7$

6. Consider the function $p(x) = -4x + 2$.

a) What is the value of $p(0)$?

b) Determine x so that $p(x) = -2$.

7. Make a table of values and graph each function.

a) $g(x) = -3x + 5$ for the domain $\{-3, -2, -1, 0, 1, 2, 3\}$

b) $h(x) = \frac{x}{2}$ for the domain $\{x \mid -10 \leq x \leq 10, x \in \mathrm{R}\}$

Apply

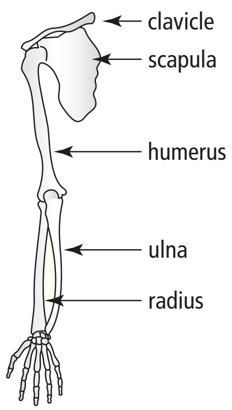
8. Mike currently has \$200 and saves \$20 each week. The function $M(w) = 20w + 200$ describes his saving pattern. Ali currently has \$200 and spends \$20 each week. The function $A(w) = 200 - 20w$ describes her spending pattern.

a) What does the variable w represent in each function?

b) Explain the meaning of $M(w)$ and $A(w)$.

c) What is the value of each function when $w = 4$? Explain your answer.

d) Determine the value of w when $A(w) = 0$. Explain your answer.



9. **Unit Project** Using skeletal remains, a forensic anthropologist can accurately determine the sex, race, age, and height of a person.

a) The height, h, in centimetres, of a male can be determined using the function $h(L) = 2.9L + 70.6$, where L is the length, in centimetres, of his humerus. Suppose you find a humerus of a male and measure the bone to be 36.87 cm in length. How tall was the man?

b) The function $h(L) = 2.8L + 71.4$ can be used to determine the height, h, in centimetres, of a female, where L is the length, in centimetres, of her humerus. Determine $h(36.87)$. What does $h(36.87)$ represent?

c) Height, h, in centimetres, can also be determined using the functions $h(L) = 3.3L + 86.4$ for a male and $h(L) = 3.3L + 81.3$ for a female. In these functions, L represents the length of the radius bone, in centimetres. What is an appropriate range for each of these functions? Explain.

d) Based on the range you determine in part c), what is the span of values (the domain) for the radius bone in males and in females?

e) Measure the length of your radius bone. Use the appropriate function to determine your height. How accurate is the prediction?

10. Weight on the moon is not the same as it is on Earth because of differences in the force of gravity. The function $m(E) = \frac{E}{6}$ can be used to approximate your weight, m, on the moon, where E represents your weight on Earth.

a) Does the function indicate that you would be heavier or lighter on the moon than on Earth? Explain.

b) If a person weighs 80 kg on Earth, how much would the person weigh on the moon?

c) How much would you weigh on the moon?

d) What is an appropriate domain for this function? Using this domain, make a table of values and graph the function.

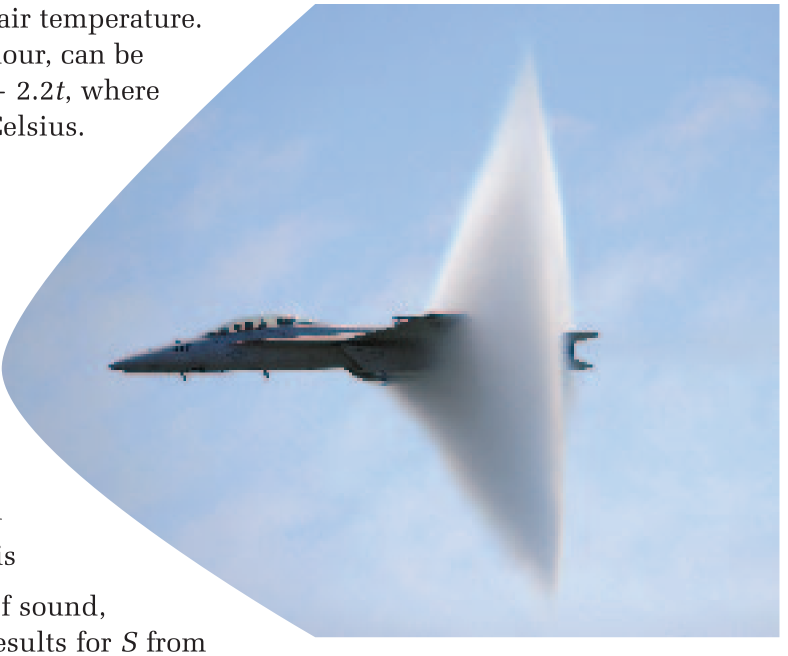
11. In air, the speed of sound depends on the air temperature. The speed of sound, S, in kilometres per hour, can be estimated using the function $S(t) = 1192 + 2.2t$, where t is the temperature of the air, in degrees Celsius.

a) What is the speed of sound in air at a temperature of 20 °C?

b) At an altitude of 10 km, the air temperature is about −55 °C. What is the speed of sound at this altitude?

c) A jet "breaks the sound barrier" when it flies faster than the speed of sound. The aircraft's speed is then typically referred to in Mach numbers. A jet's Mach number can be determined using the function $M(V) = \frac{V}{S}$, where V is the speed of the jet and S is the speed of sound, both in kilometres per hour. Use your results for S from parts a) and b) to write the Mach number function when the temperature is 20 °C, and when the temperature is −55 °C.

d) A jet is flying at 1800 km/h at an altitude of 10 km. What is the jet's Mach number?



Water vapour created as jet breaks sound barrier

Extend

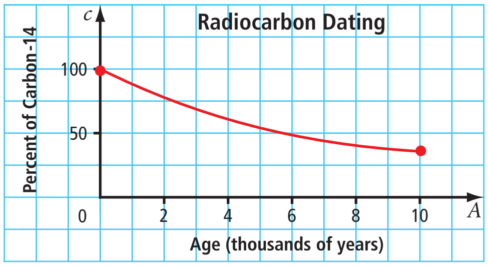
12. a) Sharon creates a function in the form $f(x) = \blacksquare x + \blacksquare$ for her classmates to solve. To figure out the actual equation, students give Sharon input values. She gives them the output from the function. The values are $f(1) = 5$, $f(2) = 8$, $f(-1) = -1$, and $f(-2) = -4$. What is the equation for Sharon's function?

b) Create a function of your own. Have someone determine the equation of your function by giving you input values and studying the pattern in your responses.

Did You Know?

First Nations hunters used a variety of approaches to hunt bison. Of all the methods the hunters devised, the most effective was the bison jump. At Head-Smashed-In Buffalo Jump, the cliff is about 10 m high. The oldest bones and stone tools at the jump are buried 10 m below this apron. This indicates that the cliff at one time was twice its current height.

13. **Unit Project** After an animal dies, the amount of radioactive carbon-14 in its bones declines. Archaeologists use this fact to determine the age of a bone based on the percent of carbon-14 remaining in the fossilized bones. The relation shows the age, A, in years, of an animal based on the percent, c, of carbon-14 remaining.



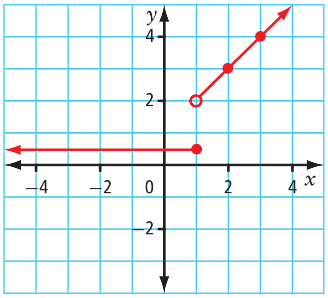
a) Is this relation a function? Why?

b) At Head-Smashed-In Buffalo Jump, in southwestern Alberta, the most recent bison bones found had 98% of the carbon-14 still remaining. From the graph, estimate the age of these bones.

c) The oldest bison bones found at Head-Smashed-In Buffalo Jump were about 5800 years old. Estimate the percent of carbon-14 still remaining in these bones.

14. **a)** Does the graph represent a function? Explain.

b) What is the value of $f(-4)$? $f(1)$? $f(3)$? $f(5)$?



15. The input for a function can be another function. If $h(x) = 2x - 5$, determine a simplified expression for each of the following.

a) $h(4x)$ **b)** $h(2x + 3)$ **c)** $h\left(\frac{x}{2} - 1\right)$

Create Connections

16. Explain, using examples, why some relations are not functions but all functions are relations. Draw a Venn diagram to illustrate the relationship between functions and relations.

17. Explain the difference between $f(2)$ and $f(x) = 2$.

18. Jean-Marie has never seen function notation. When he sees a question that asks him to determine the value of $f(x + 2)$, he gives his answer as $fx + 2f$.

a) How does Jean-Marie interpret the question?

b) Explain the meaning of this question to Jean-Marie in the context of functions.

6.5 Slope

Focus on ...

- determining the slope of a line
- using slope to draw lines
- understanding slope as a rate of change
- solving problems involving slope

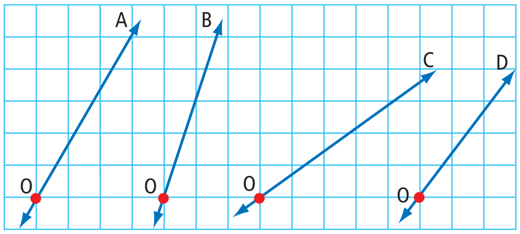
The national, provincial, and territorial parks of western and northern Canada feature some of the most beautiful backcountry in the world. To safely enjoy mountain adventures, specialized skills and knowledge, such as avalanche awareness, are essential. Though avalanches occur mostly in winter, they can happen at any time of the year. It is important to understand the many conditions that cause avalanches. The steepness, or slope, of a mountainside is one of them.

Materials

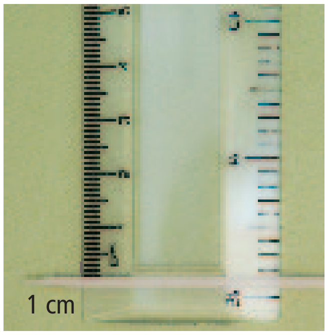
- grid paper
- plastic transparent ruler
- toothpick
- tape

Investigate Slope

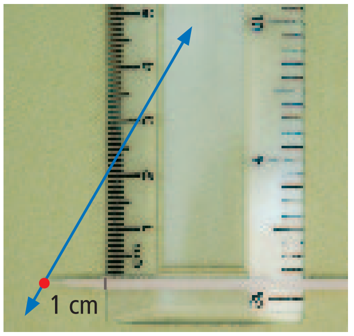
1. By observation, arrange the lines shown in order of steepness, from least steep to steepest. Explain your reasoning.



2. Convert a regular ruler into a "slope ruler" by taping a toothpick to the end of the ruler as follows:
   - Make a pencil mark 1 cm from the end of the toothpick.
   - Tape the toothpick so that its edge is aligned with the 0-cm mark of the ruler and the pencil mark is aligned with the edge of the ruler.



3. a) Use your slope ruler to measure and record the slope of each line. Place your ruler so that it is vertical and the end of the toothpick is on point O. Record the slope as the point where the line intersects the ruler.



b) How do the slope values that you measure relate to the steepness of each line?

4. On grid paper, plot a point and label it A. Measure 5 cm straight up (vertical) from point A and plot another point. Label this point B. Label the distance from A to B.

5. Measure 5 cm to the left (horizontal) of point A and 5 cm to the right (horizontal) of point A. Plot a point after each measurement. Label these points C and D. Label the distance from A to C.

6. a) Using a ruler, connect points C, B, and D. This triangle is a model mountain. Determine the ratio of the height at the centre of the mountain (A to B) to the horizontal distance from the centre to the base (A to C). That is, determine $\frac{AB}{AC}$.

b) Use your slope ruler to measure the slope of the mountain. How does the ratio compare to the slope given by the ruler?

7. a) Construct two other model mountains. Make one three times as high as the first one (AB = 15 cm) but with the same horizontal width (AC = 5 cm). Make the other the same height as the first one (AB = 5 cm) but with twice the horizontal width (AC = 10 cm).

b) Is each mountain steeper or less steep than the one in step 6?

c) Do you expect the slope value of each to be greater or less than the slope value in step 6?

d) Compare the ratio $\frac{AB}{AC}$ of each mountain to the slope value given by using the ruler.

8. Reflect and Respond

a) How do you draw a mountain with dimensions different from the model in step 6 but with the same slope? Draw this mountain. Check the slope by using your ruler.

b) A student uses a slope ruler and measures the slope of a model to be 8. If the model has a height of 32 cm at the centre, what is the distance from the centre to the base of the model?

The horizontal distance for this slope ruler is 1 cm. When the slope ruler measures a slope of 8, this represents the ratio of $\frac{8}{1}$.

c) Can a slope ruler give a measurement of $\frac{1}{4}$? If so, explain how.

d) Can a slope ruler show a negative value? If so, what does that value look like?

e) Suppose the horizontal distance for a slope ruler was 2 cm instead of 1 cm. What slope would a height reading of 8 cm represent on this slope ruler?

Link the Ideas

The **slope** of a line or line segment tells you how steep the line is. The sign of the slope value indicates the direction of the line.

slope
- the ratio of the vertical change, or rise, to the horizontal change, or run, of a line or line segment
- not expressed with units

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$\text{or } m = \frac{\text{rise}}{\text{run}} \text{ or } m = \frac{\Delta y}{\Delta x}$$

m is the variable used for slope and Δ is a symbol used to indicate change. The expression Δ*y* is read as "delta *y*."

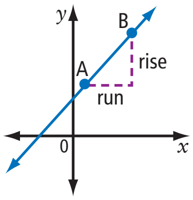
A line or line segment that rises from left to right has a positive slope.

Move from point A to point B:

$$\frac{\text{positive vertical change}}{\text{positive horizontal change}} \text{ results in a positive slope}$$

Move from point B to point A:

$$\frac{\text{negative vertical change}}{\text{negative horizontal change}} \text{ results in a positive slope}$$



slope is positive

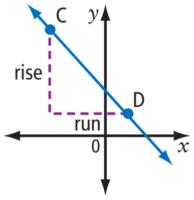
A line or line segment that falls from left to right has a negative slope.

Move from point C to point D:

$$\frac{\text{negative vertical change}}{\text{positive horizontal change}} \text{ results in a negative slope}$$

Move from point D to point C:

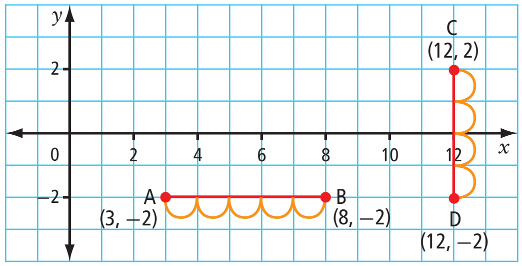
$$\frac{\text{positive vertical change}}{\text{negative horizontal change}} \text{ results in a negative slope}$$



slope is negative

Determining the Slope Using Points on a Line

When the distance from one point to another is along a horizontal line or a vertical line, you can find the distance by simply counting the spaces on the grid or by using subtraction. For example,

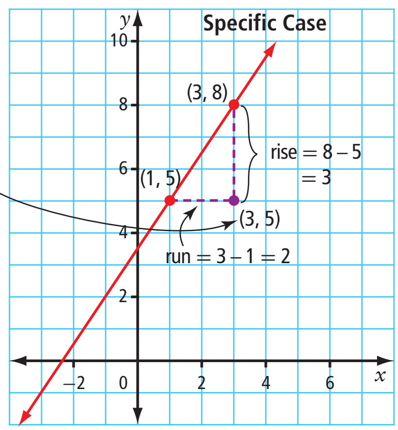


By counting, the distance AB is 5 units.
By subtraction, the distance AB is $8 - 3 = 5$ units.

By counting, the distance CD is 4 units.
By subtraction, the distance CD is $2 - (-2) = 4$ units.

Applying this idea, you can develop a formula to find the slope of any line.

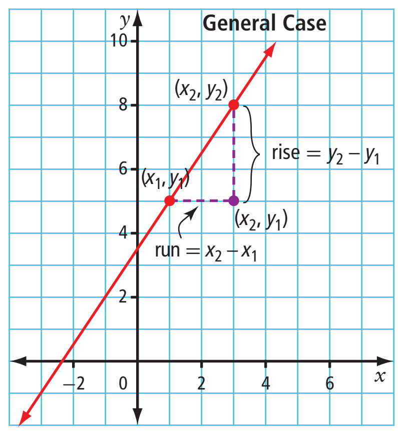
Notice how the coordinates of this point are related to the coordinates of the two points on the line.



Slope formula

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3}{2}$$



Slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$$

Applying the slope formula to line AB above shows that the slope of a horizontal line is 0.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{0}{8}$$

$$m = 0$$

Applying the slope formula to line CD above shows that the slope of a vertical line is undefined.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{6}{0}$$

$$m = \text{undefined}$$

Example 1 Classify the Slope of a Line

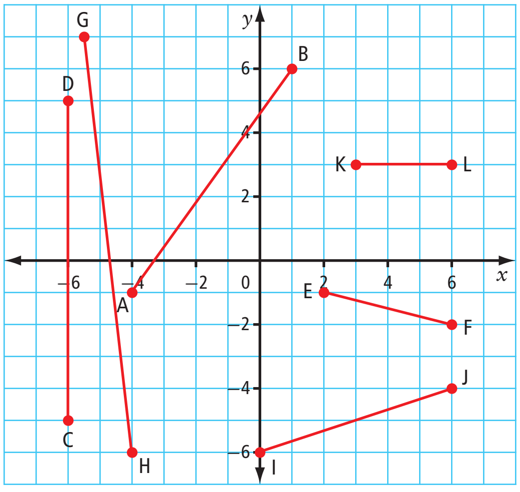
The North Shore in Vancouver is popular for hiking and biking. Bridges and stunt structures on trails are complex and often extremely challenging. They have a huge variety of slopes. Classify each slope marked on the photographs as either positive or negative.

Solution

Lines and line segments that rise from left to right have positive slopes. Therefore, line segment AB has a positive slope. Lines and line segments that fall from left to right have negative slopes. Therefore, line segments CD and EF have negative slopes.

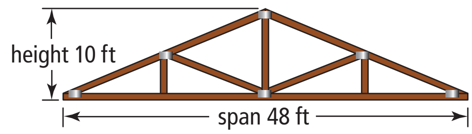
Your Turn

Classify the slope of each line segment as positive, negative, or neither.



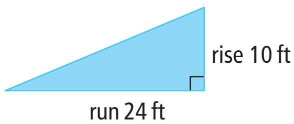
Example 2 Determine the Value of a Slope

When discussing a roof truss, carpenters refer to the *span* instead of the *width*. They talk about the *pitch* rather than the *slope*. Determine the pitch of the roof supported by the truss shown. Explain the meaning of your answer.



Solution

The pitch of the roof is its slope.



$m = \dfrac{\text{rise}}{\text{run}}$

$m = \dfrac{10}{24}$ How is the run determined?

$m = \dfrac{5}{12}$

The pitch of the roof is $\dfrac{5}{12}$. This means that the roof rises 5 units for every 12 units of horizontal distance.

Your Turn

Suppose that the roof truss in Example 2 has a height of 1 m and a span of 8 m. Determine the pitch and explain your answer.

Example 3 Determine the Slope of a Line Segment

What is the slope, m, of each line segment with the given end points?

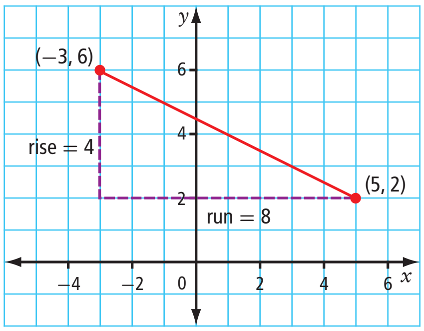
a) S(−3, 6) and T(5, 2)
b) H(4, 3) and K(4, 8)
c) M(−9, −7) and N(−1, −7)

Solution

Method 1: Use a Graph

Plot the points on grid paper. Count the rise and run.

a) Plot the points (−3, 6) and (5, 2).



$m = \dfrac{\text{rise}}{\text{run}}$

$m = -\dfrac{4}{8}$

$m = -\dfrac{1}{2}$

Recall that a line that falls from left to right has a negative slope.

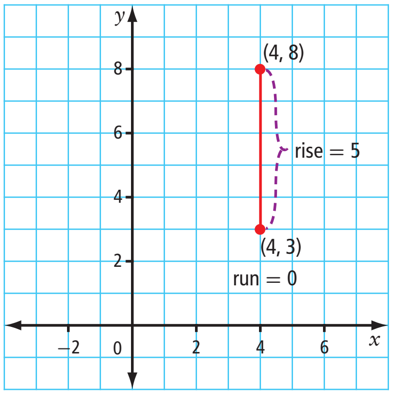
b) Plot the points (4, 3) and (4, 8).

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{5}{0}$$

m is undefined

Division by zero is not defined in the real number system.

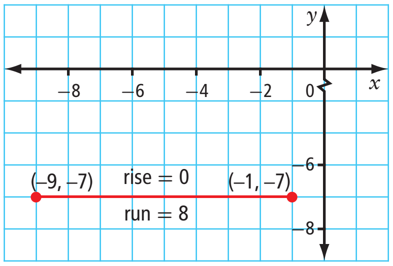


c) Plot the points (−9, −7) and (−1, −7).

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{0}{8}$$

$$m = 0$$



Method 2: Use the Slope Formula

Label the points and substitute into the formula.

a) S(−3, 6) T(5, 2) or T(5, 2) S(−3, 6)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - 6}{5 - (-3)} \qquad m = \frac{6 - 2}{-3 - 5}$$

$$m = \frac{-4}{8} \qquad m = \frac{4}{-8}$$

$$m = -\frac{1}{2} \qquad m = -\frac{1}{2}$$

b) H(4, 3) K(4, 8)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8 - 3}{4 - 4}$$

$$m = \frac{5}{0}$$

m is undefined

It does not matter which point is selected as (x_1, y_1); the value of the slope is unaffected.

c) N(−1, −7) M(−9, −7)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-7 - (-7)}{-9 - (-1)}$$

$$m = \frac{0}{-8}$$

$$m = 0$$

Your Turn

a) Use a graph to determine the slope of the line segment with endpoints P(−5, 6) and Q(1, 10).

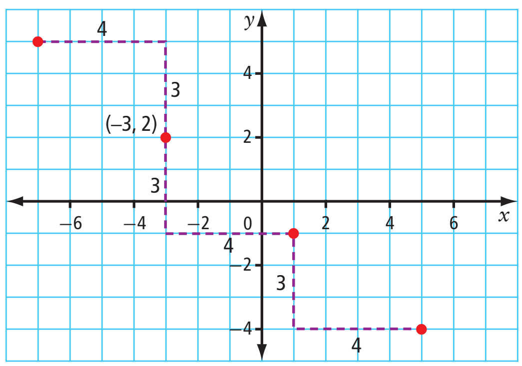
b) Use the slope formula to determine the slope of the line segment with endpoints W(2, −2) and X(−5, 5).

Example 4 Use Slope to Graph a Line

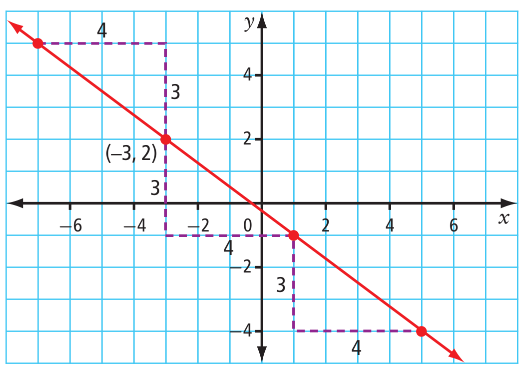
The point (−3, 2) is on a line that has a slope of $-\frac{3}{4}$. List three other points on the line. Graph the line.

Solution

The slope of the line gives the rise and run from one point to another. Plot the point (−3, 2). From this point, use the slope to locate other points on the line. Since the slope is negative, move down 3 units and right 4 units, or move up 3 units and left 4 units.



Three other points on the line are (−7, 5), (1, −1), and (5, −4). Now draw the line through the points.



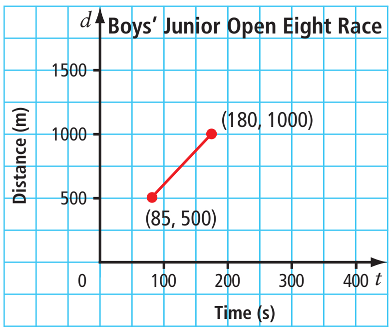
Move down 6 units and right 8 units from the point (−3, 2). What do you notice? Explain.

Your Turn

The point (−6, 1) is on a line that has a slope of $\frac{1}{3}$. List three other points on the line and graph the line.

Example 5 Slope as a Rate of Change

The Brentwood Regatta in Mill Bay, BC, is the largest junior rowing regatta hosted by a single school in North America. The races are all 1500 m in length. The graph shows the approximate times at the 500-m mark and the 1000-m mark for one of the boys' races. Determine the average rate of change for this portion of the race.



Solution

The slope of the line segment gives the ratio of the change in distance to the change in time. For this portion of the race,

$$\text{Rate of change} = \frac{\Delta d}{\Delta t}$$

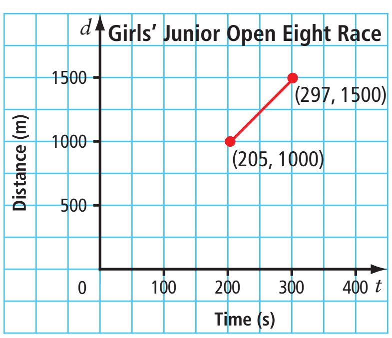
$$\text{Rate of change} = \frac{(1000 - 500)}{(180 - 85)}$$
$$= \frac{500}{95}$$

To help you interpret the meaning of the rate of change, look at the units that are used. This rate of change represents the rowers' average speed.

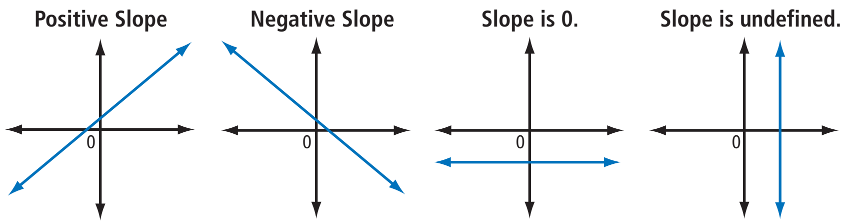
The average rate of change is approximately 5.26 m/s.

Your Turn

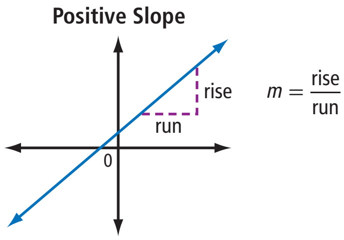
The graph shows the approximate times at the 1000-m mark and at the 1500-m mark for a rowing crew of the girls' junior open eight race at the Brentwood Regatta. Determine the average rate of change for this portion of the race.



Key Ideas

- 

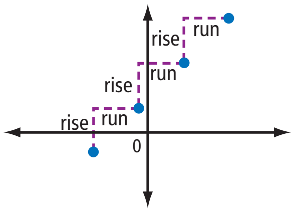
- The slope of a line is the ratio of the rise to the run.



- The slope of a line can be determined using two points on the line, (x_1, y_1) and (x_2, y_2).

  $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_2 \neq x_1$

- If you know one point on the line, you can use the slope to find other points on the line.



- The slope gives the average rate of change.

Time (s)	Distance (m)
1	4
2	7
3	10
4	13
5	16
6	19
7	22

Rate of change $= \frac{\Delta d}{\Delta t}$

Rate of change $= \frac{3}{1}$

The average rate of change is 3 m/s.

Time (s)	Distance (m)
1	4
3	10
5	16
7	22

Rate of change $= \frac{\Delta d}{\Delta t}$

Rate of change $= \frac{6}{2}$

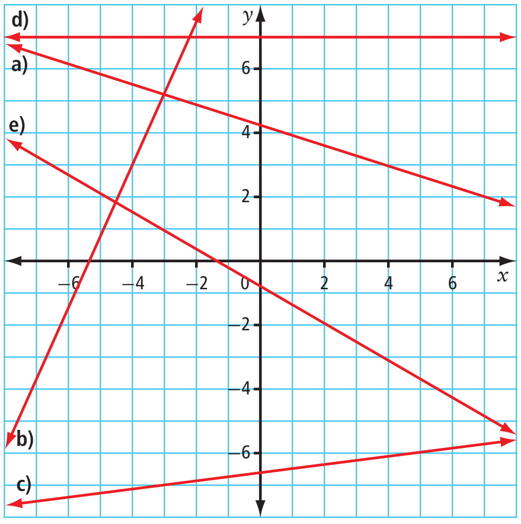
Rate of change $= \frac{3}{1}$

The average rate of change is 3 m/s.

Check Your Understanding

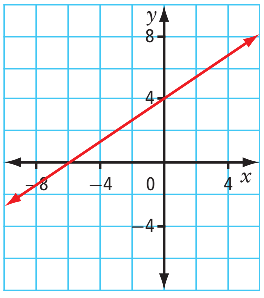
Practise

1. For each line, identify the slope as positive, negative, or zero.

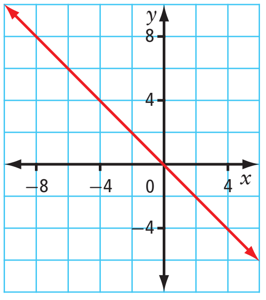


2. Determine the slope of each line.

a)



b)



3. Use the slope formula to determine the slope of the line passing through each pair of points.

a) (2, 4), (9, 8)

b) (1, 12), (6, 12)

c) (−2, −5), (1, −7)

d) (3, 6), (−3, −12)

e) (−9, 16), (−9, 25)

f) (3.9, 10.6), (10.3, 13.8)

4. Graph each line, given a point on the line and its slope.

a) (2, 3), $m = -\frac{2}{3}$

b) (−4, −4), $m = \frac{3}{5}$

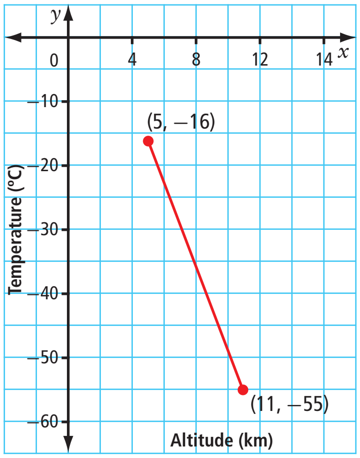
c) (5, −3), $m = -4$

d) origin, $m = \frac{1}{4}$

e) (−1, 6), $m = 0$

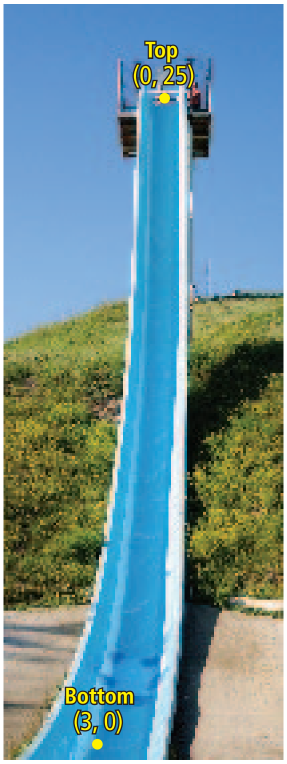
f) (4, 0), $m =$ undefined

5. The graph shows the air temperature at different altitudes above Earth's surface. Determine the average rate of change.



Apply

6. Time and height values (seconds, metres) are given for the Free Fall Slide in Kenosee, SK. Determine your average rate of change if you go down this slide.



7. **a)** Create a graph showing the melting of a 75-cm-high snow bank in spring. Plot the height, in centimetres, of the snow bank on the vertical axis and time, in days, on the horizontal axis. Draw a segment with a slope of −3, with one endpoint at (0, 75) and the other endpoint along the horizontal axis.
   b) What does each point on the graph represent?
   c) What does the endpoint along the horizontal axis represent?
   d) Explain the meaning of the slope in this situation.

8. Marjorie is having a wheelchair ramp built at the front entrance of her house.
   a) The rise to Marjorie's front door is 18 in. What is the shortest run, x, allowed for the ramp if the building code in her town sets a maximum slope of $\frac{1}{12}$?
   b) How long is the ramp?
   c) How long would the ramp be if Marjorie decides to have a gentler slope of $\frac{1}{16}$?

9. This sign on the Trans-Canada Highway indicates that a steep hill is ahead.

a) Written as a ratio, what is the slope of the hill, as indicated by the sign?

b) Describe this slope in words.

10. The Penny Ice Cap glacier in Auyuittuq National Park on Baffin Island, NU is melting. In 2009, some areas of the glacier were about 1000 ft thick. It is estimated that if the glacier continues to melt at its current rate, the ice cap could be 967 ft thick by 2020. What is the estimated rate of change in thickness?

Did You Know?

Auyuittuq National Park was established in 1976. To honour the Penny Ice Cap, the people of Pangnirtung gave the park its name, Auyuittuq. This means The Land That Never Melts.

11. In 1800, the wood bison population in North America was estimated at 168 000. The population declined to only about 250 animals in 1893. That year, Wood Buffalo National Park was established on the Alberta/Northwest Territories border. In 2006, there were about 5600 bison in the park.

a) What was the average rate of change in the bison population from 1800 to 1893? Describe the meaning of this rate.

b) What was the average rate of change in the bison population from 1893 to 2006? Describe the meaning of this rate.

12. The mountain pine beetle is infesting many forests in British Columbia and Alberta. In 2004, about 1000 infested trees were counted in Alberta. In 2007, the number of infested trees in the province was about 2.8 million.

a) Determine the average rate of change per year.

b) What does this rate of change represent?

c) What assumptions did you make? Predict the number of infested trees in Alberta in 2012.

Did You Know?

The roof of the speed skating oval for the Vancouver 2010 Winter Olympics is made almost entirely of wood. The wood is from pine trees that had been infested by the mountain pine beetle.

13. Since the speed of light is faster than the speed of sound, you see lightning before you hear the sound of the thunderclap. If a thunderstorm is 1100 m away, the sound of thunder is heard in 3.2 s. If the storm is 4950 m away, the sound reaches you in 14.5 s.

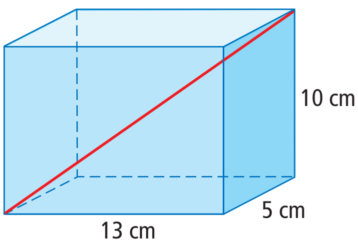
a) Determine the average rate of change, to the nearest metre per second.

b) What does this rate of change represent?

c) If you hear thunder 30 s after you see lightning, how far away is the storm?

Extend

14. What is the slope from the bottom front corner of the box to the top back corner of the box shown?



15. A metal cube has a side length of 5 cm. The cube is heated, causing it to expand uniformly to a side length of 5.01 cm.

a) Determine the volume of the cube before and after heating.

b) Determine the average rate of change of the cube's volume with respect to its side length.

16. The points that a line passes through are given as algebraic expressions, $(-4x, 7x^2)$ and $(8x, 15x^2)$. Determine a simplified algebraic expression for the slope of the line.

Create Connections

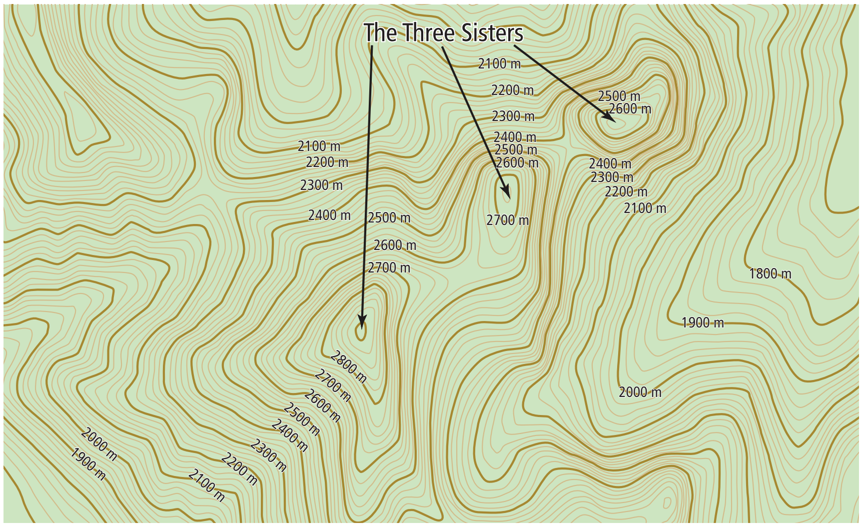
17. Explain why the slope of a line is *constant*. Use the terms *rise* and *run* in your explanation.

18. Matthew measured the slope of a ramp to be $\frac{1}{16}$. He then used trigonometry to determine the angle that the ramp made with the ground.

a) Describe how he did this.

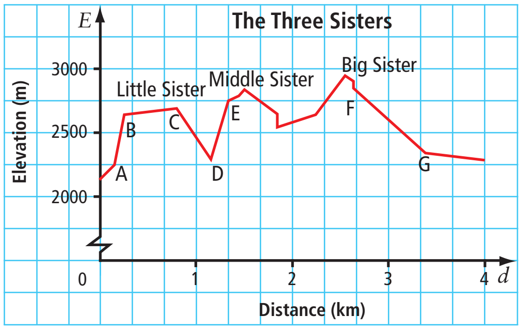
b) Determine the angle.

19. MINI LAB Topographic maps show hills and valleys using contour lines. Contour lines connect points of equal elevation. Contour lines are usually labelled with the elevation above sea level, as shown in this sample map.



Step 1 The map shows contour lines and selected elevations for The Three Sisters mountains in southern Alberta. If the change in elevation between two adjacent lines is 20 m, what is the approximate height of each peak?

Step 2 The diagram is a simplified cross profile, or side elevation, of the Three Sisters. Compare the diagram and the map. Which peak on the map represents Big Sister?



Step 3 The slope of a mountain will vary from the bottom to the top. Explain how to estimate the average slope of Middle Sister.

Step 4 Suppose the greatest risk for an avalanche occurs when the slope is between 0.5 and 1.7. From the diagram, determine the approximate slope of the following sections of the Three Sisters: AB, BC, CD, DE, and FG. Which section(s) pose the greatest avalanche risk?

Step 5 The risk of an avalanche is reduced if the slope of the mountainside is less than 0.5 or greater than 1.7. Explain why this statement is true.

Web Link

To learn more about avalanche awareness and safety, go to www.mhrmath10.ca and follow the links.

6.1 Graphs of Relations, pages 268–278

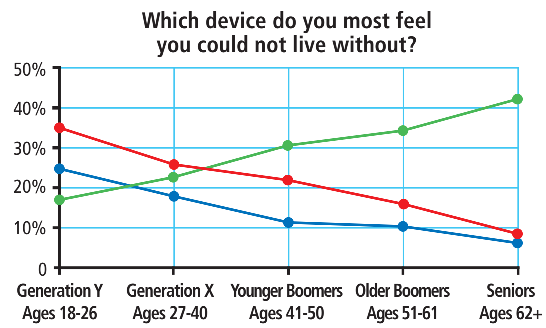
1. Consider the following scenario.

Your car's gas tank is about half full. You drive to meet Ahmed and Jackie at the movie theatre which is on the way to Ahmed's house. There is a gas station located on the edge of town just past Ahmed's house. After the movie, you drive your friends home. First you drop off Ahmed. On your way to Jackie's house, you realize that you have only one quarter of a tank of gas left, so you stop at the gas station and fill up. You drive Jackie home and then you drive to your home.

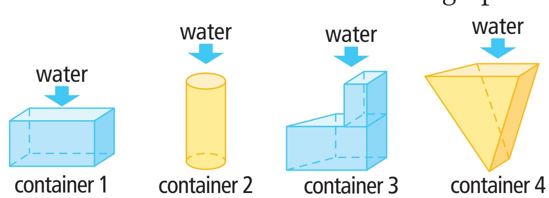
a) Draw a map to represent this scenario. Then, based on your map, sketch a graph showing your distance from home versus time.

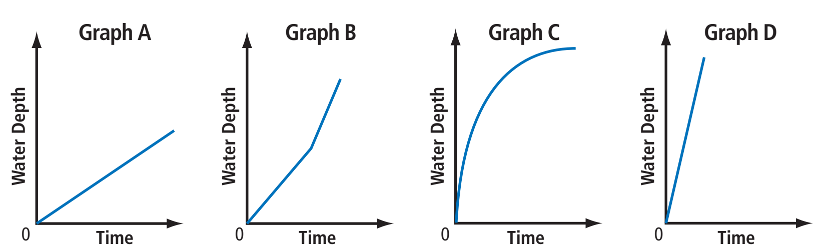
b) Sketch a graph showing the amount of gas in the car's gas tank versus time.

2. The lines on the graph represent three devices: televisions, computers, and cell phones. Which line do you think represents each device? Explain your answer.



3. The graphs show the depth of water as various containers are being filled. Match each container to its graph. Explain your choice.





6.2 Linear Relations, pages 279–291

4. Is each relation linear or non-linear? Explain your choice.

a) the orbit of a planet around the sun on an elliptical path, in terms of time and distance of the planet from the sun

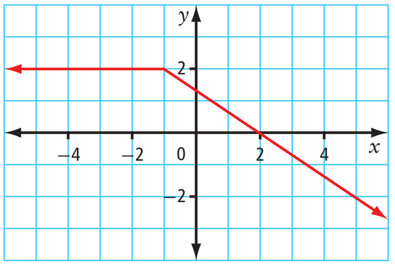
b) (−10, −5), (−5, 0), (0, 5), (5, 10)

c) $y = 2x^2 + 3x - 1$

d)

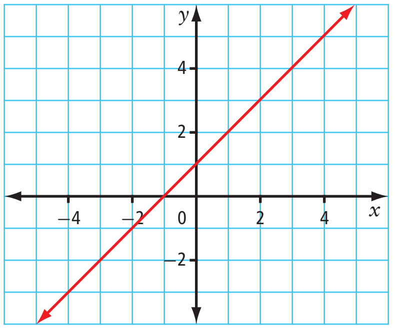
x	y
82	16
91	20
100	25
109	31
118	38

e)



5. Which of the following represent the same relation?

A



B

x	y
−2	0
0	2
2	4
4	6

C One number is double another.

D (−2, 0), (0, 2), (2, 4), (4, 6)

6. Katya wants to sell a camper trailer. The cost to place an advertisement in a newspaper is $37.95 for three lines of text and a picture, plus $7 for each additional line of text. Consider the relation of total cost versus number of lines of text.

a) Is this relation linear? Explain.

b) Identify the dependent variable and the independent variable.

c) Create a set of ordered pairs to represent 3, 4, 5, 6, and 7 lines of text in Katya's advertisement.

d) Is this relation discrete or continuous?

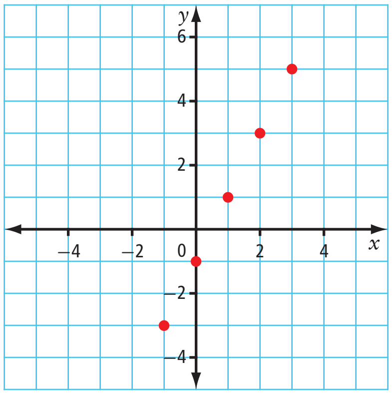
e) Graph the relation.

6.3 Domain and Range, pages 292–304

7. State the domain and range of each relation.

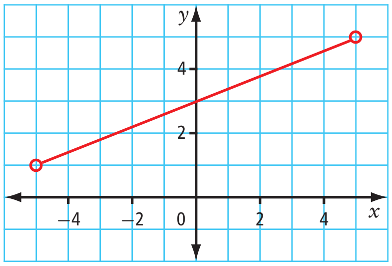
a) $(-9, 5)$, $(-5, 5)$, $(0, 5)$, $(0, 8)$, $(2, 8)$

b)

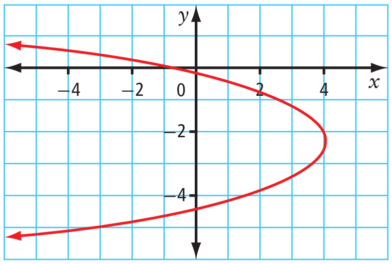


8. Express the domain and the range of each relation as a number line, in interval notation, and in set notation.

a)



b)



6.4 Functions, pages 305–314

9. The circumference of a circle is given by the function $C(d) = \pi d$, where d is the diameter of the circle. Write this function as a linear formula in two variables.

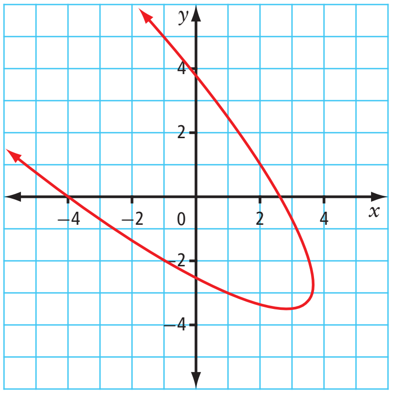
10. The formula for volume of a cube with radius r is $V = r^3$. Write this formula using function notation.

11. Which relations are functions? Explain how you know.

a)

x	y
1	1
2	2
3	3
4	4
5	5

b)



c)

Hair Colour	Gender
brown	male
black	male
black	female
blonde	female
red	female

d) (2.3, 5.1), (8.6, 9.4), (8.6, 9.2), (9.5, 10.0)

12. The function $M(E) = \frac{E}{2.7}$ can be used to approximate your weight, M, in kilograms, on Mars, where E, in kilograms, is your weight on Earth.

a) Suppose you weigh 66 kg on Earth. How much would you weigh on Mars?

b) How much would a Martian who weighs 14 kg on Mars weigh on Earth?

13. For her local Run for the Cure, Amber donates \$50 of her own money. She also collects \$25 pledges. The function $P(n) = 25n + 50$ represents the total funds she contributes.

a) Determine an appropriate domain and range. Then, use a table of values to graph the function.

b) Determine the value of $P(8)$. Explain the meaning of your answer.

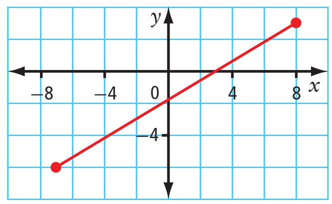
c) Prizes are awarded to students who collect more than \$675. How many pledges must Amber collect to receive a prize?

d) Explain why this situation depicts a function.

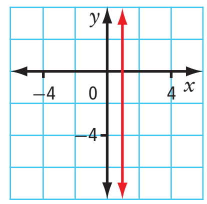
6.5 Slope, pages 315–329

14. Determine the slope of each line segment or line.

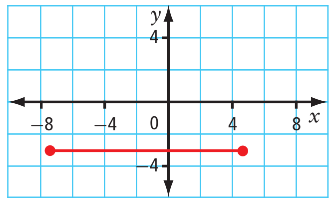
a)



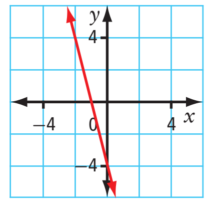
b)



c)



d)



> **Did You Know?**
>
> Blackstrap Mountain was made in 1969–1970 for the 1971 Canada Winter Games in Saskatoon.

15. Blackstrap Mountain in Blackstrap Provincial Park, SK, is an artificial hill. The hill is approximately 90 m high with an average rate of change from top to bottom of $\frac{1}{5}$. If the coordinates (0, 90) represent the top of Blackstrap Mountain, what coordinates represent its bottom?

16. Use the slope formula to determine the slope of the line passing through each pair of points.

a) (−3, 8) and (13, 12)

b) (5.6, −8.2) and (−0.4, 3.8)

17. Carl is running in a 10-km race.

a) Create a graph showing Carl's progress during the race. Plot distance, in kilometres, as the dependent variable and time, in minutes, as the independent variable. Start your graph at the point (0, 0) and draw a line with a slope of $\frac{1}{4}$.

b) What does each point on the graph represent?

c) At what point does your line end? What does this point represent?

d) Explain the meaning of slope in this situation.

18. In Manitoba, the number of people aged 12 and older who have asthma was 63 028 in 1996 and 73 427 in 2005.

a) What was the average rate of change per year?

b) What does this rate of change represent?

c) If the number of people in Manitoba living with asthma continues to increase at this rate, how many people in Manitoba will have asthma in 2017?

6 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. Which of these relations is *not* linear?

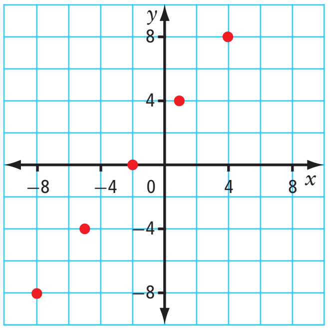
A $y + 3 = \frac{2}{3}x$

B (2, 3), (4, 6), (6, 9), (8, 12)

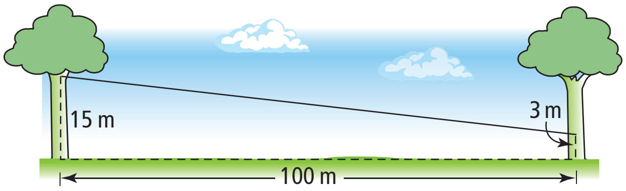
C

a	*d*
−2	3
−1	4
0	6
1	8
2	11

D

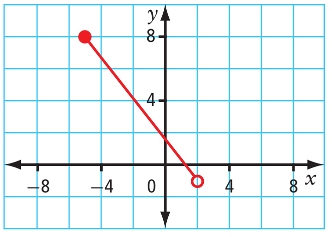


2. A zip line is attached to two trees as shown. What is the slope of the line?



A $\frac{10}{12}$ **B** $\frac{3}{10}$ **C** $-\frac{6}{5}$ **D** $-\frac{3}{25}$

3. Determine the domain of the relation in the graph.



A $\{-5, -4, -3, -2, -1, 0, 1, 2\}$

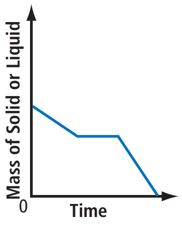
B $[-5, 2)$

C $\{x \mid -5 < x \leq 2, x \in \mathrm{R}\}$

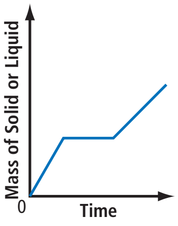
D all numbers between −5 and 2, inclusive

4. As ice is heated, it changes state from solid to liquid and then to gas. Which graph could represent the amount of solid or liquid remaining as heat is applied to a block of ice?

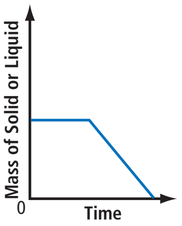
A



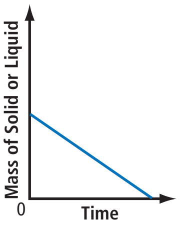
B



C

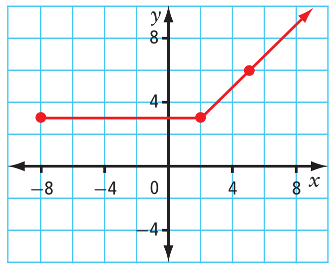


D

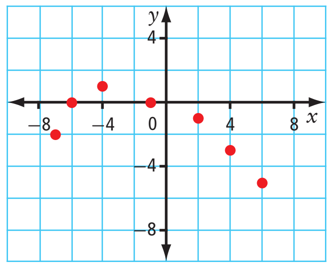


5. How many of these relations are functions?

i)



ii)



iii) (1, 8), (2, 12), (5, 12), (7, 18)

iv) (−9, −9), (−3, −3), (−1, 0), (9, 9)

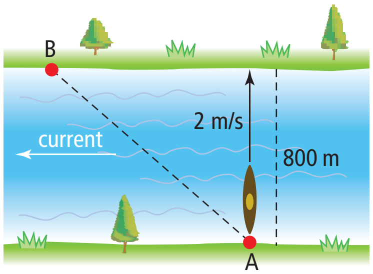
A 1 **B** 2 **C** 3 **D** 4

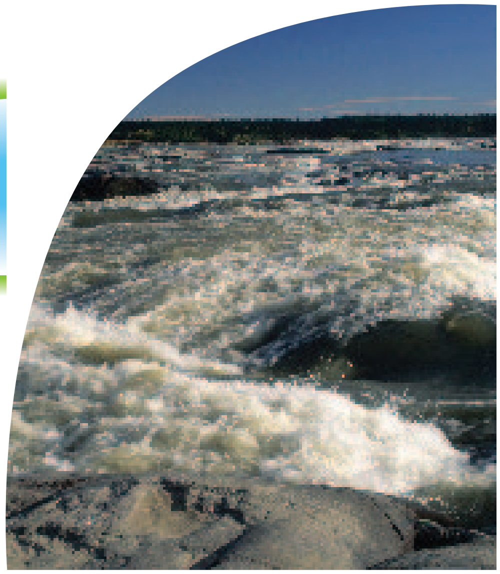
Short Answer

6. **a)** Determine the slope of the line that passes through the points (−2, −3) and (5, 4).

   b) Give the coordinates of another point on this line.

7. The Slave River rapids between Fort Fitzgerald, AB, and Fort Smith, NT, are popular for experienced whitewater kayakers. To cross an 800-m-wide section of the river, a kayaker points his kayak perpendicular to the current. He maintains this position as he paddles at a rate of 2 m/s. However, the current affects the kayaker's actual course. He reaches the other side in 6 min 40 s (400 s) at a point 2000 m downstream from his original position.





   a) What is the kayaker's average rate of change from point A to point B?

   b) In the context of this question, what does this rate represent?

8. A boat travelling at 5 m/s begins to accelerate. Its new speed, S, in metres per second, is modelled by the function $S(t) = 5 + 2.5t$, where t is the length of time, in seconds, that it accelerates.

   a) What does $S(6)$ represent?

   b) Determine the value of $S(6)$.

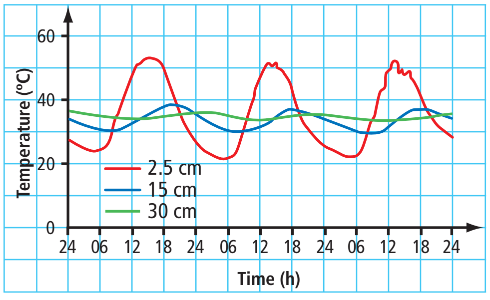
   c) How long does the boat accelerate in order to reach a speed of 23 m/s?

9. Select and graph the function that represents continuous data.

   Function A: $f(x) = 2x - 5$, where the domain is $\{x \in \mathrm{R}\}$
   Function B: $h(x) = 3x + 4$, where the domain is $\{-2, -1, 0, 1, 2\}$

Extended Response

10. The graph shows soil temperature over 3 days at different depths.



   a) All three curves have the same domain. Express this domain in words.

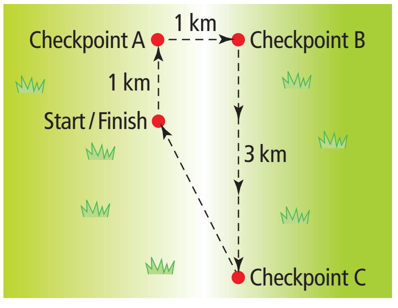
   b) Express the range for the curve representing a depth of 2.5 cm using a number line.

   c) Express the range for the curve representing a depth of 15 cm using interval notation.

   d) Express the range for the curve representing a depth of 30 cm using set notation.

   e) What conclusions can you make from these relations?

11. Dan is training for the Canadian Orienteering Championships in Spruce Woods Provincial Park, MB. Create a distance-time graph showing Dan's distance from the start as he completes the practice course shown.



CHAPTER 7

Linear Equations and Graphs

We make sense of our world by understanding how things are related. One common relationship is the linear relation. This is often expressed as an equation. Linear equations are involved in almost every career and field of study. Archaeologists and paleontologists use linear equations to learn about past civilizations and forms of life. Police detectives and forensic analysts use linear equations to solve crimes. Developers and civil engineers use linear equations to predict future trends.

Big Ideas

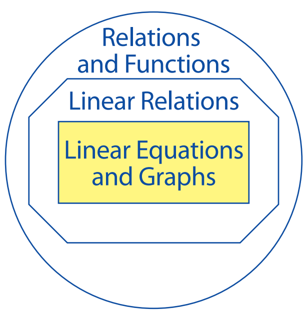
When you have completed this chapter, you will be able to ...

- graph a line using a variety of methods
- determine the equation of a line using characteristics of the line
- convert the equation of a line from one form to another
- model and solve real-life problems using a linear equation

Key Terms

y-intercept
slope-intercept form
parameter
general form (of a linear equation)
x-intercept
slope-point form
parallel lines
perpendicular lines

Your Relations and Functions Organizer

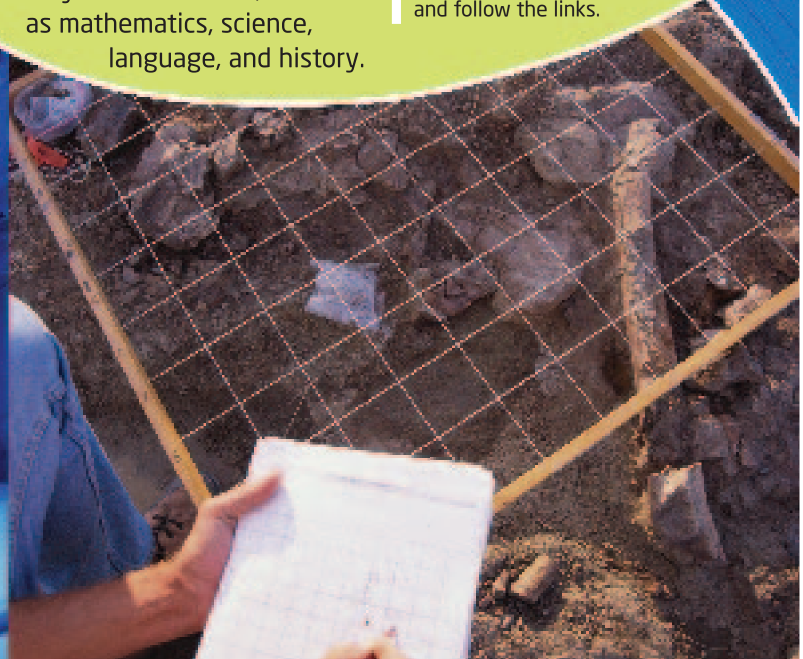


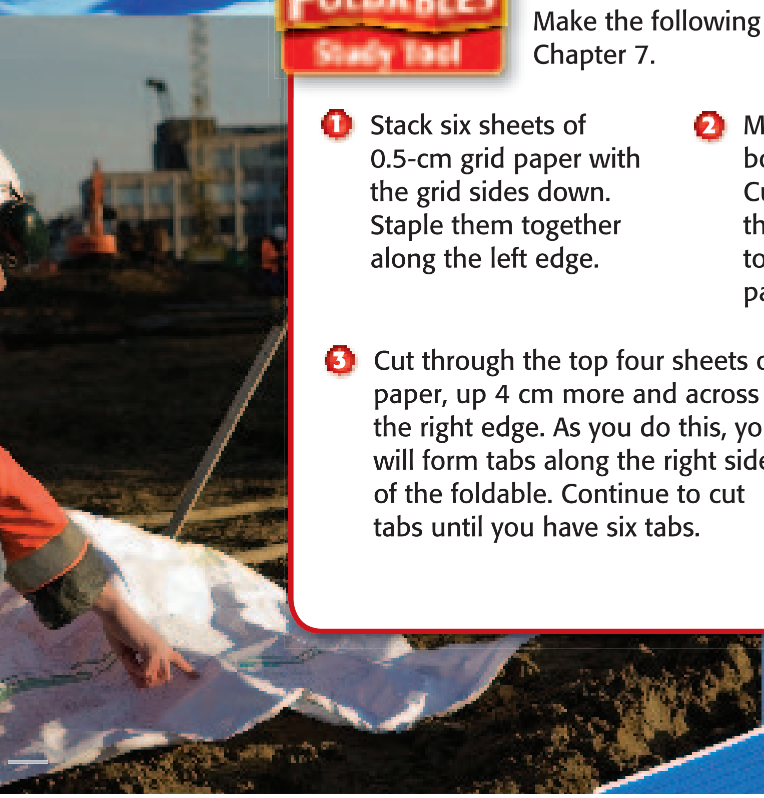
Archaeologist

Archaeologists recover and analyse materials and artifacts from the past. They examine tools, debris, architecture, artwork, physical remains, and landscape features. The information they gather provides clues about traditions, beliefs, customs, languages, and practices. They use these clues to reconstruct human cultures. Archaeologists use skills from many different fields, such as mathematics, science, language, and history.

Web Link

To learn more about archaeology, go to www.mhrmath10.ca and follow the links.



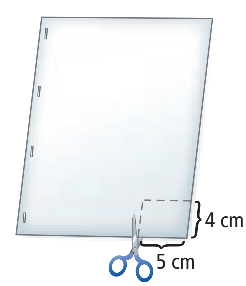


FOLDABLES Study Tool

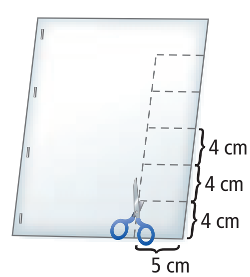
Make the following Foldable™ to take notes on what you will learn in Chapter 7.

1. Stack six sheets of 0.5-cm grid paper with the grid sides down. Staple them together along the left edge.

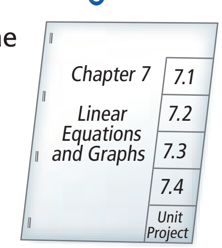
2. Make a mark 5 cm in from the bottom right corner of the top page. Cut through the top five sheets from this mark 4 cm up and then across to the right edge. Do not cut the last page because this is your back cover.



3. Cut through the top four sheets of paper, up 4 cm more and across to the right edge. As you do this, you will form tabs along the right side of the foldable. Continue to cut tabs until you have six tabs.



4. Label the tabs as shown.



7.1 Slope-Intercept Form

Focus on ...

- identifying the slope and y-intercept of a straight-line graph
- determining a linear equation using slope and y-intercept
- rewriting a linear relation in slope-intercept form
- graphing equations in slope-intercept form
- solving problems using equations in slope-intercept form

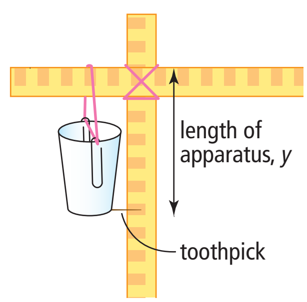
Many relationships can be modelled by the graph of a straight line. For example, a farmer purchases chicken feed pellets by mass, yet dispenses the feed by volume. When the farmer analyses the feeding of chickens, the relationship between the volume of feed pellets in a dispenser and the mass of the feed may be modelled by a linear equation. Linear equations can be written in different forms. Each form can provide specific information about the graph.

Materials

- two metre sticks
- elastic band
- foam cup
- paper clips, string, or tape
- toothpick or straightened paper clip
- six identical marbles or other items of equal mass
- ruler
- grid paper

Investigate the Graph of a Linear Equation

1. a) Suspend a metre stick between two chairs or desks. Attach an elastic band across the diameter of a foam cup using paper clips. Loop the other end of the elastic band around the metre stick. Poke a toothpick horizontally through the bottom edge of the cup.



b) Position another metre stick vertically beside the suspended cup. Measure the length of the apparatus, y, in centimetres, from the top of the elastic band to the bottom of the cup. Use the toothpick to help you locate the reading on the metre stick.

2. a) Place one marble in the cup. Let the cup come to rest. Measure and record the length of the apparatus. One at a time, place the other marbles in the cup. After adding each marble, measure the length of the apparatus again.

b) On grid paper, plot the individual data points. Plot the number of marbles along the x-axis. Using a ruler, draw a straight line that represents the data.

3. Identify the independent and dependent variables.

4. Determine the slope of the line. What are the units of the slope? What does the slope of the line represent in this situation?

5. a) Identify the point where the line intersects the y-axis. What does this point represent in this situation?

b) Starting at this point, explain how to use the slope to determine the length of the apparatus when one, two, or three marbles are in the cup.

6. Use the ***y*-intercept** and slope to determine what the length of the apparatus would be if ten marbles were in the cup. Does your graph support your answer?

7. a) Write the equation of the line in **slope-intercept form**. Let y represent the length of the apparatus, in centimetres. Let x represent the number of marbles in the cup.

b) Use the equation to estimate the length of the apparatus if the cup contains 15 marbles.

8. Reflect and Respond Suppose you use a cup that is 4 cm taller but has the same mass as the foam cup used.

a) How would this change the data you collected?

b) Describe how the graph would change.

c) Write an equation in slope-intercept form that represents this situation.

9. Suppose you use items that are double the mass of the marbles used.

a) How would this change your data?

b) Describe how the graph would change.

c) Represent this situation using an equation written in slope-intercept form.

***y*-intercept**

- the y-coordinate of the point where a line or curve crosses the y-axis
- the value of y when $x = 0$

slope-intercept form

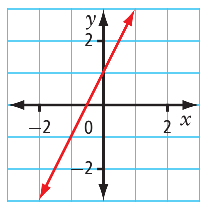
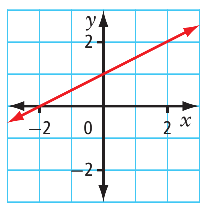
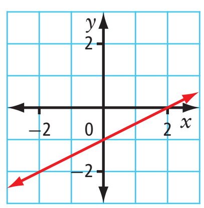
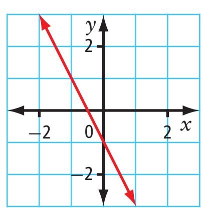
- the equation of a line in the form $y = mx + b$, where m is the slope of the line and b is the y-intercept

Link the Ideas

To write the equation of a straight-line graph, you can use the following two constants:

- the rate of change or slope, m
- the y-intercept. If $(0, b)$ is the point where the line crosses the y-axis, then b is the y-intercept.

The equation of a non-vertical straight-line graph can be written in slope-intercept form. The equation is $y = mx + b$, where m represents the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ and b represents the y-intercept.

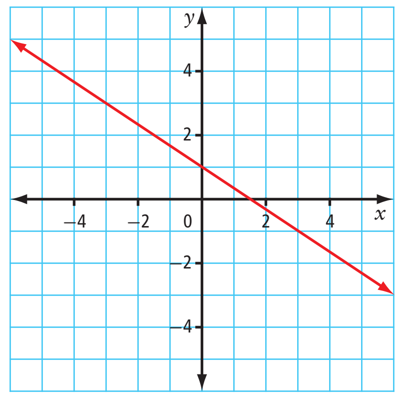
<table>
<tr><th>Table of Values</th><th>Graph</th><th>Slope, m</th><th>y-intercept, b</th><th>Equation, $y = mx + b$</th></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>5</td></tr></table></td><td>
</td><td>$m = \frac{\Delta y}{\Delta x}$
$m = \frac{2}{1}$
$m = 2$</td><td>1</td><td>$y = 2x + 1$</td></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>2</td><td>2</td></tr><tr><td>4</td><td>3</td></tr></table></td><td>
</td><td>$m = \frac{\Delta y}{\Delta x}$
$m = \frac{1}{2}$</td><td>1</td><td>$y = \frac{1}{2}x + 1$</td></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>−1</td></tr><tr><td>2</td><td>0</td></tr><tr><td>4</td><td>1</td></tr></table></td><td>
</td><td>$m = \frac{\Delta y}{\Delta x}$
$m = \frac{1}{2}$</td><td>−1</td><td>$y = \frac{1}{2}x + (-1)$</td></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>−1</td></tr><tr><td>1</td><td>−3</td></tr><tr><td>2</td><td>−5</td></tr></table></td><td>
</td><td>$m = \frac{\Delta y}{\Delta x}$
$m = \frac{-2}{1}$
$m = -2$</td><td>−1</td><td>$y = -2x + (-1)$</td></tr>
</table>

Example 1 Write the Equation of a Line in Slope-Intercept Form

a) What are the slope and y-intercept of the line shown in the graph?

b) Write the equation of the line in slope-intercept form, $y = mx + b$.

c) Use graphing technology to check your equation.



Solution

a) The y-intercept is 1. Therefore, $b = 1$.
Using the points (0, 1) and (3, −1), the slope is

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 - 1}{3 - 0}$$

$$m = -\frac{2}{3}$$

The slope is $-\frac{2}{3}$ and the y-intercept is 1.

What do you know about the slope if the line falls from left to right?

How else could you determine the slope?

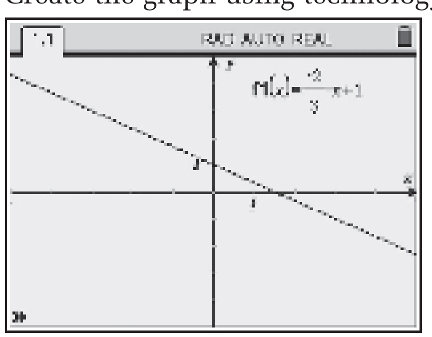
b) Substitute the values of m and b into the slope-intercept form of an equation.

$$y = mx + b$$

$$y = -\frac{2}{3}x + 1$$

The equation of the line in slope-intercept form is $y = -\frac{2}{3}x + 1$.

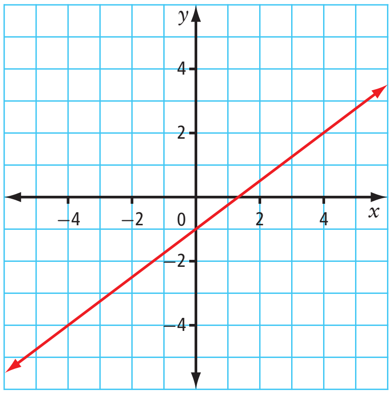
c) Create the graph using technology.



How can you confirm that this is the equation of the line that passes through the points (0, 1) and (3, −1)?

Your Turn

a) What are the slope and y-intercept of the line shown in the graph?



b) What is the equation of the line in slope-intercept form, $y = mx + b$?

c) Use graphing technology to check your equation.

Example 2 Convert an Equation to Slope-Intercept Form

A students' council rents a portable dunk tank as a fund-raising activity. Students pay for the chance to hit a target with a ball and dunk a teacher into a tank of water.

The relationship between the number of balls thrown, x, and the profit, y, in dollars, may be represented by the equation $3x - 2y - 600 = 0$.

a) Rewrite the equation in slope-intercept form.

b) State the slope of the line. What does the slope represent?

c) Identify the y-intercept. What does it represent?

d) The break-even point is the point at which the money raised equals the money spent. How many balls must the students sell to reach the break-even point?

Solution

a) Rearrange the equation into the form $y = mx + b$. To do this, isolate the variable y.

$$3x - 2y - 600 = 0$$
$$3x - 2y - 600 + 2y = 0 + 2y$$
$$3x - 600 = 2y$$
$$\frac{3}{2}x - 300 = y$$

The equation written in slope-intercept form is $y = \frac{3}{2}x - 300$.

$3x - 600 = 2y$
Dividing all terms by 2
gives $\frac{3x}{2} - \frac{600}{2} = \frac{2y}{2}$.
So, $y = \frac{3}{2}x - 300$.

b) The slope of the line is $\frac{3}{2}$ or 1.5. It represents income of \$1.50 per ball. The slope is positive because the money is coming in.

c) The y-intercept is -300. It represents a cost of \$300 to rent the portable dunk tank. It is negative because the money is paid out as an expense.

d) At the break-even point, students do not make or lose money. So, the profit is zero.

Substitute $y = 0$ into the equation and solve for x.

$$y = \frac{3}{2}x - 300$$
$$0 = \frac{3}{2}x - 300$$
$$300 = \frac{3}{2}x - 300 + 300$$
$$2(300) = \cancel{2}^{1}\left(\frac{3}{\cancel{2}_{1}}x\right)$$
$$600 = 3x$$
$$200 = x$$

To reach the break-even point, they must sell 200 balls, at a rate of \$1.50 per ball. They will make money if they sell more than 200 balls. They lose money if they sell fewer than 200 balls.

Your Turn

Parents of members of the cheerleading squad rent a hall. They arrange a talent show as a fundraiser. The relationship between the number of tickets sold, x, and the profit, y, in dollars, may be represented by the equation $12x - y - 840 = 0$.

a) What is the slope of the line? What does the slope represent?

b) Identify the y-intercept. What does it represent?

c) How many tickets must the parents sell to reach the break-even point?

Did You Know?

In 1998, the Canadian Armed Forces purchased four submarines from Britain. The submarines were named after Canadian port cities. The *Victoria* is based at Esquimalt, BC. It is part of the Maritime Forces Pacific fleet.

parameter

- a variable that has a constant value in a particular equation

Example 3 Model and Solve a Problem Using an Equation in Slope-Intercept Form

Submarines must withstand tremendous pressure exerted on all sides by the water. The table shows the linear relationship between pressure and water depth.

Depth (m)	Pressure (kPa)
0	100
25	350
50	600
75	850

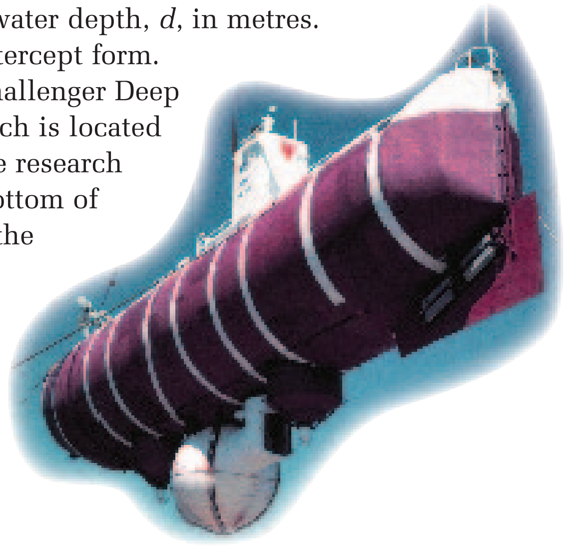
a) Sketch a graph of the data.

b) What is the slope of the line? What does it represent?

c) Determine the value of the **parameter** b. What does this value represent?

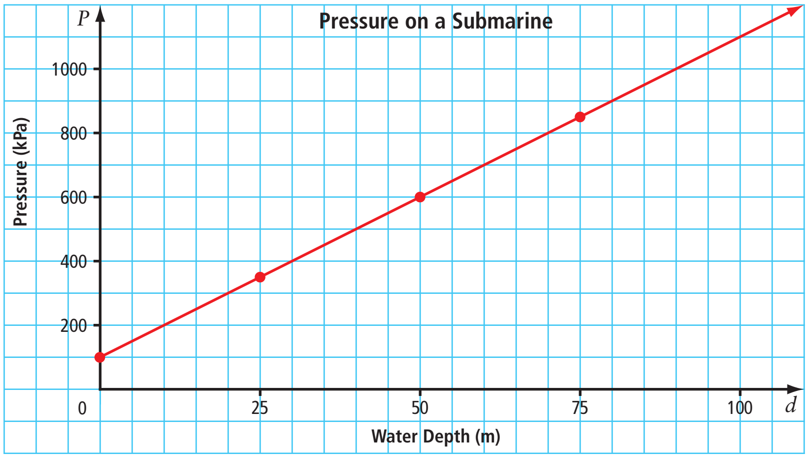
d) Write an equation that models the relationship between pressure, P, in kilopascals, and water depth, d, in metres. Express the equation in slope-intercept form.

e) The deepest point on Earth is Challenger Deep in the Mariana Trench. This trench is located in the Pacific Ocean. In 1960, the research submarine *Trieste* reached the bottom of Challenger Deep. At this depth, the walls protecting the two crew members had to withstand a pressure of 109 300 kPa. What is the approximate depth of Challenger Deep?



Solution

a) The independent variable is depth. The dependent variable is pressure. Plot the coordinate pairs (0, 100), (25, 350), (50, 600), and (75, 850). Both depth and pressure can be any real number, so join the points with a straight line.



b) Determine the ratio of vertical change to horizontal change.

$m = \dfrac{\Delta y}{\Delta x}$

$m = \dfrac{250}{25}$

$m = 10$

The slope of the line is 10. This means that for every metre you descend, the pressure increases by 10 kPa.

What values could you use to calculate Δy and Δx?

c) The parameter, b, represents the y-intercept, which is equal to 100. The air pressure is 100 kPa at the surface of the water, where water depth is 0 m.

d) Substitute $m = 10$ and $b = 100$ into $y = mx + b$.

$y = 10x + 100$

If P represents pressure and d represents depth, then the equation of the line is $P = 10d + 100$.

e) Substitute 109 300 for P.

$$\begin{aligned} P &= 10d + 100 \\ 109\,300 &= 10d + 100 \\ 109\,200 &= 10d \\ 10\,920 &= d \end{aligned}$$

The approximate depth of Challenger Deep is 10 920 m.

Your Turn

Asha has selected a hotel for her wedding reception. The cost involves a fee for the deluxe ballroom and a buffet charge that depends on the number of guests. This is shown in the table.

Number of Guests	Cost ($)
0	425
25	1800
50	3175
100	5925

a) Sketch a graph of the data in the table.
b) What are the slope and y-intercept of the line? What does each parameter represent?
c) Write an equation that describes the relationship between the cost and the number of guests. Express the equation in slope-intercept form.
d) What is the cost for 140 guests?
e) Asha would like the total cost to be no more than $15 000. What is the maximum number of guests that can attend?

Did You Know?

To boil water, some First Nations people used to dig a bowl-shaped pit in the ground. Then, they lined the bottom with buffalo hide. They added water and red-hot rocks to the pit, until the water boiled.

Example 4 Determine an Unknown Parameter

An archaeologist simulates a First Nations method of boiling water by adding hot rocks to an earthen pit filled with water. As the rocks cool and lose their heat, the archaeologist replaces them with new rocks from the fire. Suppose the water temperature rises at a constant rate. The temperature of the water at the start of the experiment is 10 °C.

Cooking in a Fire Pit by Shayne Tolman
Painting on display at Head-Smashed-In Buffalo Jump Interpretive Centre likely represents 2500-year-old Besant culture.

The equation $W = mt + 10$ models how the temperature of the water, W, in degrees Celsius, increases at a constant rate of m degrees Celsius per minute for t minutes.

a) After 5 min, the water temperature is 19 °C. Determine the value of the parameter m. What does m represent?

b) How long will it take for the water to boil?

Solution

a) Represent a temperature of 19 °C after 5 min as the point (5, 19). Substitute the coordinates of the point (5, 19) into the given equation. Solve for m.

$$W = mt + 10$$
$$19 = m(5) + 10$$
$$19 - 10 = 5m$$
$$9 = 5m$$
$$\frac{9}{5} = m \text{ or } m = 1.8$$

The parameter m represents the rate at which the temperature of the water increases per minute. The water temperature increases at a rate of 1.8 °C/min.

b) Solve the equation for t when W is 100 °C.

What does $W = 100$ °C represent?

$$W = 1.8t + 10$$
$$100 = 1.8t + 10$$
$$90 = 1.8t$$
$$50 = t$$

The water boils after 50 min.

Did You Know?

The coordinates of a point on a graph satisfy the equation of the graph. Conversely, a point whose coordinates satisfy an equation is a point on a graph of the equation.

For example,
The point (1, 8) is a point on the graph of $y = 3x + 5$.
Substituting $x = 1$ and $y = 8$.
$8 = 3(1) + 5$
$8 = 8$ True.

Conversely, since the point (1, 8) satisfies the equation, (1, 8) is a point on the graph of the equation $y = 3x + 5$.

Your Turn

A decorator's fee can be modelled by the equation $F = 75t + b$. In the equation, F represents the fee, in dollars, t represents time, in hours, and b represents the cost of the initial consultation, in dollars.

a) Suppose the decorator spends 4 h working for a client and charges the client \$450. Determine the value of the parameter b.

b) How many hours does the decorator work if a client is charged \$975?

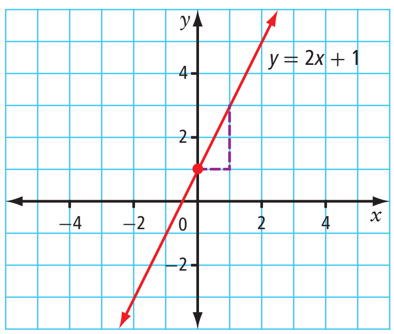
Key Ideas

- The slope-intercept form of a linear equation is $y = mx + b$, where m represents the slope and b represents the y-intercept.

$y = 2x + 1$

slope = 2 y-intercept = 1

$\frac{\text{rise}}{\text{run}} = \frac{2}{1}$ The graph passes through (0, 1).



Check Your Understanding

Practise

1. What are the slope and y-intercept of each line?

a) $y = -5x + 4$ **b)** $y = \frac{3}{4}x + 1$

c) $y = x - 7$ **d)** $y = -4x$

e) $y = -3$ **f)** $y = 0.5x - 1.25$

2. Consider the line $y = -3x + 2$.

a) What are the slope and y-intercept of the line?

b) Explain how you could sketch the graph of this line using the slope and y-intercept.

3. Sketch the graph of each line using the slope and y-intercept. Use graphing technology to check your graphs.

a) $y = 2x - 3$ **b)** $y = -4x + 8$

c) $y = -x + 1$ **d)** $y = \frac{5}{2}x - 4$

e) $y = -\frac{3}{4}x + 2$ **f)** $y = 5$

4. Omar determines the slope and y-intercept of the line $3x - 2y - 8 = 0$. His work is shown.

$3x - 2y = 8$	Step 1
$-2y = -3x + 8$	Step 2
$y = \frac{3}{2}x + 4$	Step 3
The slope of the line is $\frac{3}{2}$ and the y-intercept is 4.	Step 4

a) In which line did Omar first make an error?

b) Correct Omar's work.

5. Express each equation in slope-intercept form. Write the slope and y-intercept of each line.

a) $2x + y = 6$

b) $3x + y + 9 = 0$

c) $5x + 6y = 8$

d) $6x - y = 4$

e) $7x - y + 9 = 0$

f) $8x - 4y = 3$

6. Write the equation of each line in the form $y = mx + b$.

a) slope $= -3$, y-intercept $= 2$

b) slope $= \frac{5}{6}$, y-intercept $= -4$

c) slope $= -0.75$, y-intercept $= -5$

d) slope $= 1$, y-intercept $= -7$

e) slope $= -1$, y-intercept $= 0$

f) slope $= 0$, y-intercept $= \frac{1}{3}$

7. Use the following equations to answer each question. Justify your answers.

Equation A: $y = 3x - 5$
Equation B: $y = -4x + 1$
Equation C: $y = x + 2$
Equation D: $y = -\frac{1}{2}x$

a) Which lines slope *up* from left to right?

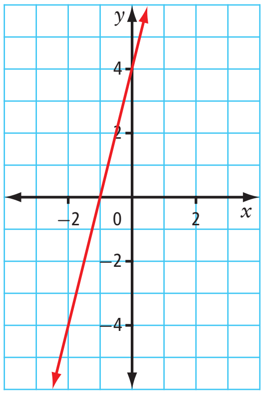
b) Which lines slope *down* from left to right?

c) Arrange the lines from greatest to least y-intercepts.

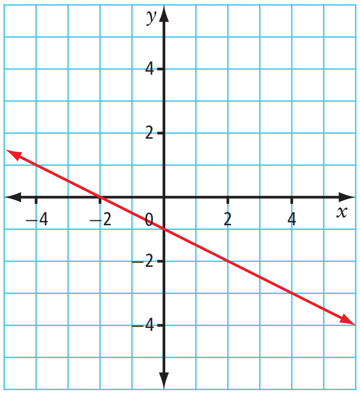
d) Which lines pass through the origin?

8. What are the slope and y-intercept of each line? Write the equation of each line in slope-intercept form.

a)



b)



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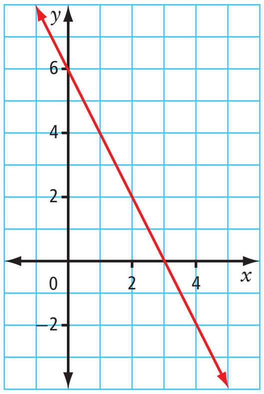
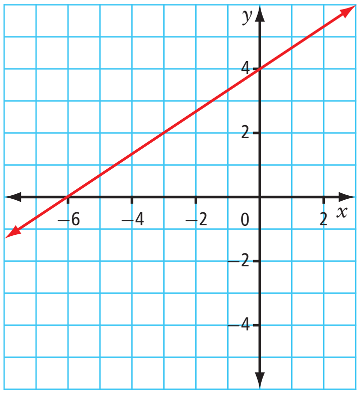
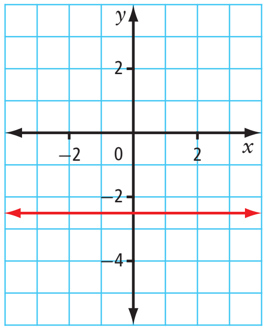


d)



e)



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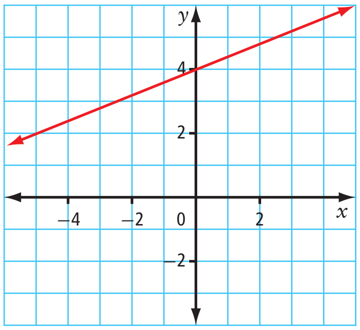


9. Consider the equation $y = 3x + b$. What is each value of b if a graph of the line passes through each point?

a) $(4, 9)$ **b)** $(-4, 6)$

c) $(3, -2)$ **d)** $(-1, -8)$

10. For the equation $y = mx - 2$, what is each value of m if the line passes through each point?

a) $(3, 1)$ **b)** $(-2, 8)$

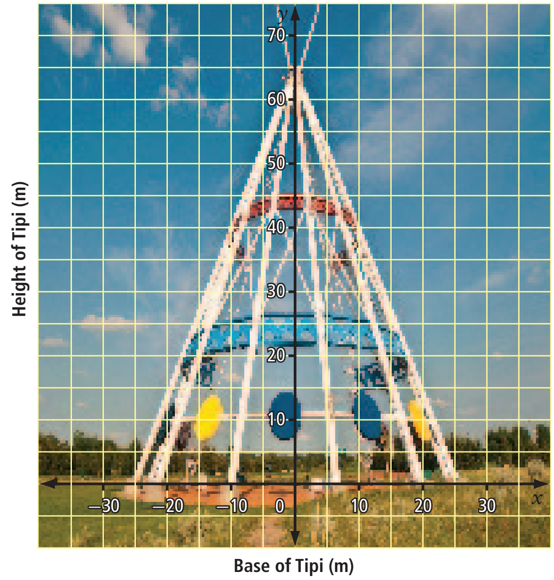
c) $(4, -8)$ **d)** $(-6, -1)$

Did You Know?

The Saamis Tipi, a stylized structure, is located in Medicine Hat, AB. It was originally constructed for the 1988 Olympics as a tribute to First Nations peoples.

11. A photograph of the Saamis Tipi is shown on a metre grid. Write the equation of each line representing the extreme left post and the extreme right post. Express the equations in slope-intercept form.

Dimensions of the Saamis Tipi



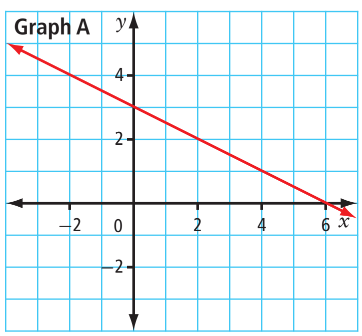
12. State the slope and y-intercept of each equation. Then, identify the graph that matches the equation. Use graphing technology to check your answers.

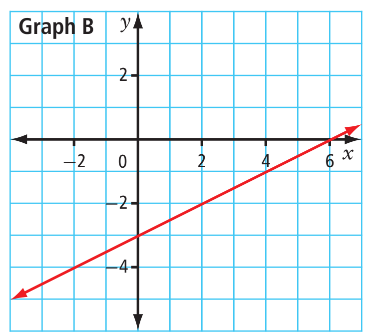
a) $y = -2x + 3$

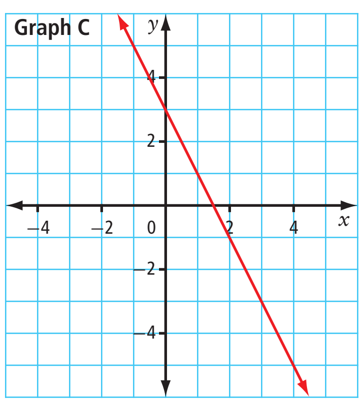
b) $y = 2x - 3$

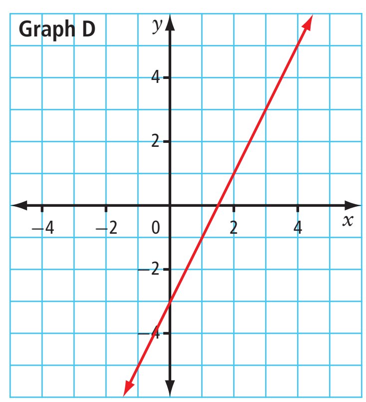
c) $y = \frac{1}{2}x - 3$

d) $y = -\frac{1}{2}x + 3$









13. Write an equation to represent each situation.

a) The cost, C, to take n students to the theatre is \$300 to rent a bus and \$6.25 per ticket.

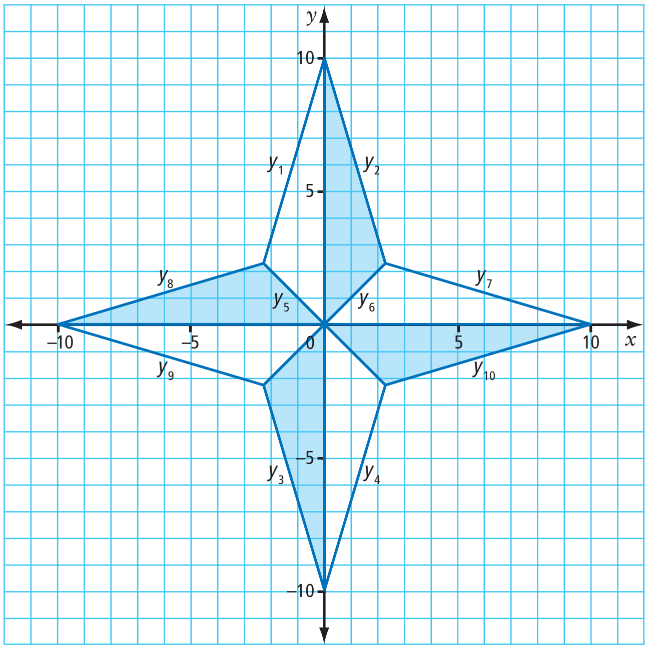
b) The taxi fee, T, is \$3.60 to start plus \$1.48 for each kilometre travelled, x.

c) A rewritable Blu-ray disc has 1024 MB of data stored on it. When new data is added to the disc, the total data, D, in megabytes, stored on the disc at time t seconds increases at a rate of 54 MB/s.

d) A water delivery truck is filling the water tank in Simeonie's house. The truck arrived with 2500 L of water. The number of litres of water, L, remaining in the truck at time t minutes decreases at a rate of 120 L/min.

Apply

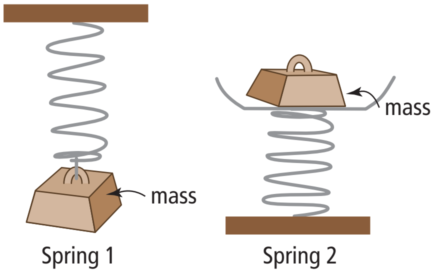
14. At the centre of the North Atlantic Treaty Organization (NATO) emblem is a compass that may be created using the ten line segments, labelled y_1 to y_{10}, in the figure. Work with a partner to determine the equation of each line segment. Express your equations in the form $y = mx + b$. For each equation, do not consider the boundaries of the line segment.



Did You Know?

Canada has been a member of NATO since NATO's formation in 1949. The main goal of NATO is to safeguard the freedom and security of its member countries.

15. A group of students tested how different masses changed the lengths of two different coil springs. The results of their experiments are summarized in the table.



Mass (g)	Spring 1 Length (cm)	Spring 2 Length (cm)
0	8	24
4	14	18
8	20	12
12	26	6

a) For each spring, write an equation to model how spring length, L, in centimetres, changes with mass, x, in grams. Express each equation in slope-intercept form.

b) What does a negative slope represent in the experiment?

Did You Know?

Many animals that roamed Earth at the time of the last glacial period are now extinct. These include the sabre-toothed cat, mammoth, mastodon, and giant beaver, bison, and bear. Other animals that roamed the area that is now known as western Canada during this time include the horse, camel, antelope, and ground sloth.

16. About 12 000 years ago during the last glacial period, giant bison roamed the plains of North America. Using fossil bones, paleontologists can estimate the size of these huge animals. The equation $y = 2.4x - 7.9$ approximates the relationship between an adult male bison's front limb length, y, in centimetres, and the length of its humerus bone, x, in centimetres.

Painting of giant bison by Ludo Beogaet

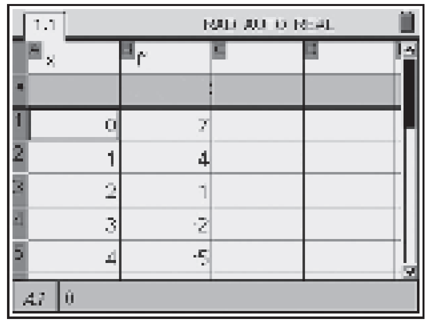
a) Sketch a graph of the equation $y = 2.4x - 7.9$.

b) Use graphing technology to check your graph. Then, estimate, to the nearest centimetre, the front limb length of each bison:

- an extinct giant bison with a fossil humerus bone length of 40.2 cm
- a modern North American bison with a humerus bone length of 32.6 cm

c) By what percent was the giant bison taller than the modern bison?

17. Marge uses graphing technology to graph a line. A table of values for the line is shown. What is the equation of the line? Express the equation in slope-intercept form.



x	y
0	7
1	4
2	1
3	-2
4	-5

18. **Unit Project** Exposure to cold weather can cause frostbite and hypothermia. Mountain climbers, sky divers, and other high-altitude enthusiasts must protect their skin because air temperature decreases as altitude increases. This rate of decrease in temperature is nearly constant, up to about 11 000 m.

An airplane taking off in the Yukon Territory recorded the following temperatures.

Altitude (m)	Temperature (°C)
0	12
4000	−13.6
8000	−39.2

a) Sketch a straight-line graph of the data.

b) What is the slope of the line expressed as a fraction in lowest terms? What does the slope represent?

c) What is the y-intercept? What does it represent?

d) Write an equation that describes the relationship between temperature, T, in degrees Celsius, and altitude, A, in metres.

e) Mount Logan is in Kluane National Park and Reserve, YT. At a height of 5959 m, Mount Logan is the highest mountain in Canada. It is the second highest mountain in North America. Predict the temperature a climber would experience at the peak of Mount Logan on the day that the airplane collected the data. Assume minimal wind.

f) Most people are at risk of frostbite within 10–20 min of exposure to temperatures below −20 °C. Predict the altitudes at which the temperature will be below −20 °C.

Did You Know?

The coldest recorded temperature in North America was −63 °C on February 3, 1947. It was recorded in the village of Snag, YT. Temperatures at higher altitudes can get even colder.

19. Write the equation of a line with the same slope as the line $x + 4y + 8 = 0$ and the same y-intercept as the line $2x - 3y + 12 = 0$.

Extend

20. When graphed, the equations $x + 4y = n$ and $5x - 2y = 10$ have the same y-intercept. What is the value of n?

21. A graph of $6x - ny = 8$ and $2x + 3y = 12$ shows two lines that have the same slope. What is the value of n?

22. A line has a y-intercept of 3 and forms an angle of 45° with the x-axis. Write the equation of this line in the form $y = mx + b$.

23. Consider the equation $2x + y = 200$. When graphed, how many points (x, y) on the line will have natural numbers for both the x-coordinate and y-coordinate?

Create Connections

24. Explain how to determine the slope and y-intercept of a line given each representation.
    a) a table of (x, y) values
    b) a graph of the line
    c) an equation of the line in the form $y = mx + b$

25. Explain how you could use the slope and y-intercept of a line to
    a) write the equation of the line
    b) create a graph of the line

Materials
- graphing technology or grid paper and ruler

26. **MINI LAB**

Step 1 Use technology to graph each set of equations on the same display. Then, sketch a graph of each set.

a)	b)	c)
$y_1 = x + 5$	$y_1 = x - 1$	$y_1 = -x + 2$
$y_2 = -2x + 5$	$y_2 = 2x - 2$	$y_2 = -x + 4$
$y_3 = 3x + 5$	$y_3 = -3x + 3$	$y_3 = -x + 6$

Step 2 Describe the similarities and differences between each set of lines. Lines that share at least one characteristic are called a *family of lines*. Why might each set be considered a family of lines?

Step 3 Write the equation of another line that belongs to each family. Use graphing technology to check your answers.

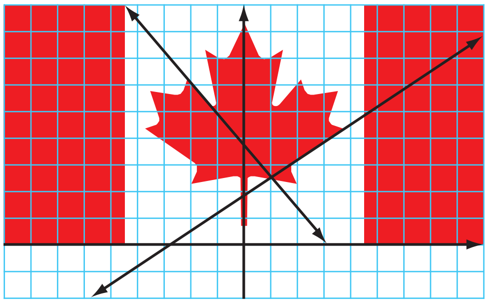
Step 4 Create your own family of lines. Write the equations of the lines in slope-intercept form. Share with a partner and describe each other's family of lines.

7.2 General Form

Focus on ...

- converting a linear equation to general form
- using intercepts to graph a line
- relating the intercepts of a graph to the situation
- solving problems using equations in general form

The slope-intercept form of an equation is one of the most common ways to write the equation of a line. Suppose you place a transparent grid on the Canadian flag. Most, but not all, of the line segments in the flag could be expressed as equations in slope-intercept form. Can you think of a type of line that cannot be expressed in the form $y = mx + b$? In this section, you will explore a general way of writing the equation of any line.

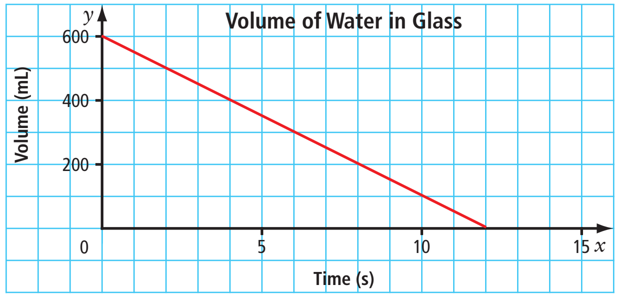


Materials

- bottle of water
- stopwatch
- grid paper and ruler or graphing technology

Investigate Intercepts and General Form

Leora quenches her thirst after a soccer game by drinking a large glass of water at a constant rate. The straight-line graph on the following page shows how the volume of water in the glass changes with time.



1. Identify the domain and range in this situation.
2. Identify the slope of the line segment. What does the sign of the slope mean? What does the slope represent?
3. What is the y-intercept of the line segment? What does the y-intercept represent?
4. **a)** What is the equation of the line in slope-intercept form?
   b) Rearrange the terms in the equation so that the right side is zero. This is the **general form** of an equation.

general form
- the equation of a line in the form $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number. This means that A will always be positive.

x-intercept
- the x-coordinate of the point where a line or curve crosses the x-axis
- the value of x when $y = 0$

5. **a)** Identify the **x-intercept** of the line. What does the x-intercept represent?
   b) Write the coordinates of the x-intercept and the y-intercept.
6. Time yourself while drinking a 500-mL bottle of water slowly at a constant rate.
   a) Sketch a straight-line graph to show how the volume of water remaining in the bottle changes with time.
   b) Compare your graph to Leora's graph. How can you tell who drank more, who finished first, and who drank at a faster rate?
   c) Write the equation of your graph in both slope-intercept form and general form.
7. **Reflect and Respond**
   a) Recall that a point on a line satisfies the equation of the line. Are the x-intercept and y-intercept points on the line?
   b) When asked to determine the x-intercept, what coordinate value do you always know? When asked to determine the y-intercept, what coordinate value do you always know? How can you use this information to help you determine the x-intercept or y-intercept?
   c) Describe strategies for determining the y-intercept and x-intercept of a line if the equation is given in general form.
   d) Create two linear equations. Then, identify the y-intercept and x-intercept of each line. Use graphing technology to check your answers.
8. What form is the equation $x - 5 = 0$ expressed in? Can you write the equation in another form? Explain.

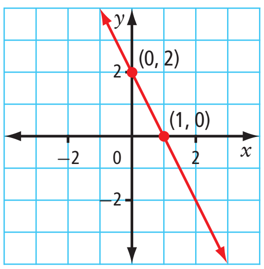
Link the Ideas

The general form of a linear equation is $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number.

You can convert a linear equation from one form to another by applying the rules of algebra.

The x-intercept of a line is the x-coordinate of the point where the line crosses the x-axis. The y-intercept is the y-coordinate of the point where the line crosses the y-axis. To sketch a linear equation, you can draw a line joining the intercepts, $(x, 0)$ and $(0, y)$.

The line in the graph below has an x-intercept of 1 and a y-intercept of 2.



Example 1 Convert an Equation to General Form

Rewrite the equation $y = -\frac{2}{3}x + 6$ in general form, $Ax + By + C = 0$.

Solution

$$y = -\frac{2}{3}x + 6$$

$$3(y) = 3\left(-\frac{2}{3}x + 6\right)$$ **Why are both sides multiplied by 3?**

$$3y = \cancel{3}^{1}\left(-\frac{2}{\cancel{3}_{1}}x\right) + 3(6)$$

$$3y = -2x + 18$$

$$2x + 3y - 18 = 0$$

The equation written in general form is $2x + 3y - 18 = 0$.

Your Turn

Rewrite the equation $y = \frac{3}{4}x - 2$ in general form.

Example 2 Sketch a Graph Using Intercepts

For the linear equation $2x - 3y - 6 = 0$,

a) state the x-intercept of a graph of the equation

b) state the y-intercept

c) use the intercepts to graph the line

Solution

a) To determine the x-intercept, substitute $y = 0$. Then, solve for x.

$$\begin{aligned} 2x - 3y - 6 &= 0 \\ 2x - 3(0) - 6 &= 0 \\ 2x - 6 &= 0 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Why do you substitute $x = 0$ to find the y-intercept?

The x-intercept is 3. So, the line crosses the x-axis at the point (3, 0).

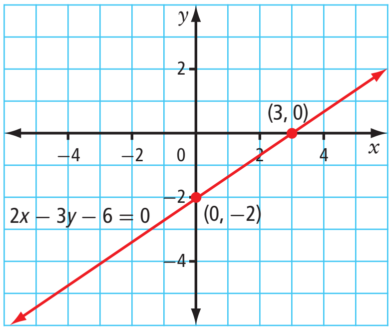
b) To determine the y-intercept, substitute $x = 0$. Solve for y.

$$\begin{aligned} 2x - 3y - 6 &= 0 \\ 2(0) - 3y - 6 &= 0 \\ -3y - 6 &= 0 \\ -6 &= 0 + 3y \\ \frac{-6}{3} &= y \\ -2 &= y \end{aligned}$$

Why do you substitute $y = 0$ to find the x-intercept?

The y-intercept is -2. The line crosses the y-axis at the point $(0, -2)$.

c) Locate the points (3, 0) and $(0, -2)$ on the grid. Then, draw a line passing through these points.



How else could you graph an equation given in general form?

Your Turn

Consider the linear equation $4x + 5y - 20 = 0$.

a) What is the x-intercept of a graph of the equation?

b) What is the y-intercept?

c) Use the intercepts to graph the line.

Example 3 Identify Intercepts of Horizontal or Vertical Lines

Sketch each linear relation and identify the intercepts. Then, state the domain and range.

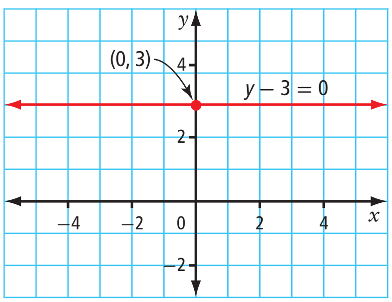
a) $y - 3 = 0$

b) $x + 4.5 = 0$

c) $y = 0$

Solution

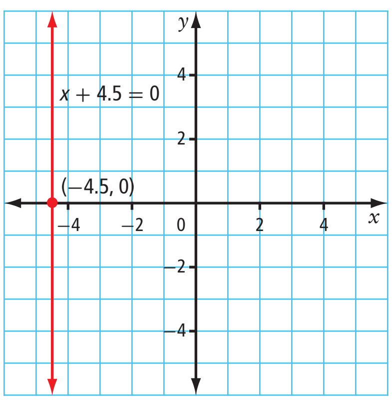
a) The equation $y - 3 = 0$ can be written in slope-intercept form as $y = 0x + 3$.
The graph is a horizontal line with slope zero.
The line crosses the y-axis at the point (0, 3).



What would the values in a table representing this equation show?

The y-intercept is 3. There is no x-intercept.
The domain of the line $y - 3 = 0$ is $\{x \in \mathrm{R}\}$.
The range of the line is $\{3\}$.

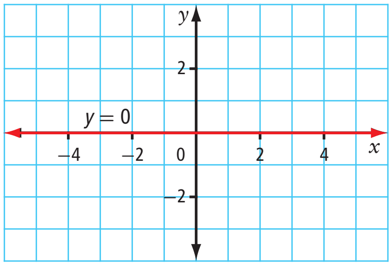
b) The equation $x + 4.5 = 0$ expressed in general form is $x + 0y + 4.5 = 0$. The coefficient of y is zero.
The value of x is always -4.5.
The graph is a vertical line and crosses the x-axis at $(-4.5, 0)$.



The x-intercept is -4.5. There is no y-intercept.
The domain of the line $x + 4.5 = 0$ is $\{-4.5\}$.
The range of the line is $\{y \in \mathrm{R}\}$.

c) The equation $y = 0$ is a horizontal line, which represents the entire x-axis. The graph always intersects the x-axis. Therefore, there are an infinite number of x-intercepts. The y-intercept is 0.

The domain of the line $y = 0$ is $\{x \in \mathrm{R}\}$.
The range of the line is $\{0\}$.



What do the coordinates of points on this line have in common?

Your Turn

Sketch each linear relation and identify the intercepts. What are the domain and range for each relation?

a) $x - 3 = 0$
b) $x = 0$
c) $y + 2 = 0$

Example 4 Interpret Intercepts

Spencer has 66 GB of disk space left on his laptop to fill with television shows and movies that he purchases on-line.

a) Suppose a one-hour show uses 1.1 GB of disk space and a movie uses 4.4 GB. Write a linear equation that represents the number of television shows, T, and movies, M, that Spencer can store on his laptop.
b) Determine the T-intercept of a graph of the linear equation. What does the T-intercept represent?
c) What would the M-intercept be? What does the M-intercept represent?
d) If Spencer stores 16 television shows, how many movies does he have space for?

Solution

a) The equation is $1.1T + 4.4M = 66$.

What does each term represent?

Simplify the equation.

Write the coefficient of T as a whole number.

$11T + 44M = 660$

Reduce the equation to lowest terms.

$T + 4M = 60$

How else could you reduce the original equation to lowest terms with the coefficient of T as a whole number?

The equation $T + 4M = 60$ represents the number of television shows and movies that Spencer can store on his laptop.

b) To determine the T-intercept, substitute $M = 0$. Solve for T.

$$\begin{aligned} T + 4M &= 60 \\ T + 4(0) &= 60 \\ T &= 60 \end{aligned}$$

The T-intercept is 60. So, if Spencer stores no movies, he can store 60 television shows.

c) To determine the M-intercept, substitute $T = 0$. Solve for M.

$$\begin{aligned} T + 4M &= 60 \\ 0 + 4M &= 60 \\ M &= 15 \end{aligned}$$

The M-intercept is 15. So, Spencer can store 15 movies if he does not store any television shows.

d) Substitute $T = 16$ and solve for M.

$$\begin{aligned} T + 4M &= 60 \\ 16 + 4M &= 60 \\ 4M &= 44 \\ M &= 11 \end{aligned}$$

Spencer has space on his laptop for 11 movies.

Your Turn

Brooke wants to save \$336 to decorate her bedroom. She has two part-time jobs. On weekends, she works as a snowboard instructor and earns \$12 per hour. On weeknights, she earns \$16 per hour working as a high-school tutor.

a) Write an equation to represent the number of hours Brooke needs to work as a snowboard instructor, S, and as a tutor, T.

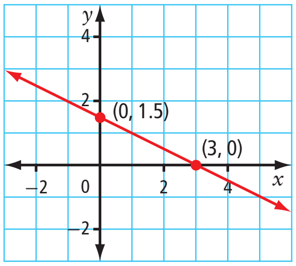
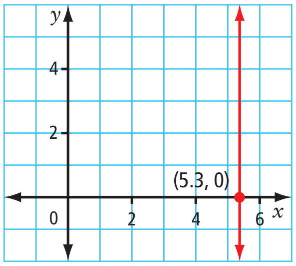
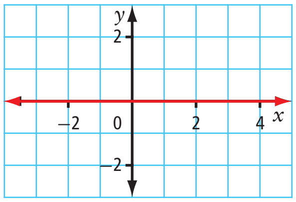
b) What is the S-intercept of a graph of the equation? What does the S-intercept represent?

c) What would the T-intercept be? What does it represent?

d) Suppose Brooke works 8 h as a snowboard instructor. How many hours will she need to work as a tutor?

Key Ideas

- The general form of a linear equation is $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number.
- To graph an equation in general form, determine the intercepts, then draw a line joining the intercepts; or convert to slope-intercept form.
- To determine the x-intercept, substitute $y = 0$ and solve. To determine the y-intercept, substitute $x = 0$ and solve.
- A sketch of a linear relation may have one, two, or an infinite number of intercepts. A line that represents an axis has an infinite number of intercepts with that axis. A horizontal or vertical line that does not represent an axis has only one intercept.

Equation	x-Intercept(s)	y-Intercept(s)	Graph
$x + 2y - 3 = 0$	$x + 2y - 3 = 0$ $x + 2(0) - 3 = 0$ $x = 3$	$x + 2y - 3 = 0$ $(0) + 2y - 3 = 0$ $2y = 3$ $y = 1.5$	
$x = 5.3$	$x = 5.3$	no y-intercept	
$3y = 0$	infinite number of x-intercepts	$3y = 0$ $y = 0$	

Check Your Understanding

Practise

1. Jasmine was asked to convert the equation $y = -\frac{3}{2}x + 4$ to general form. Her work is shown.

$$y = -\frac{3}{2}x + 4$$
$$2y = -3x + 4$$
$$3x + 2y - 4 = 0$$

Identify Jasmine's error. Then, correct her work.

2. Express each equation in general form, $Ax + By + C = 0$.

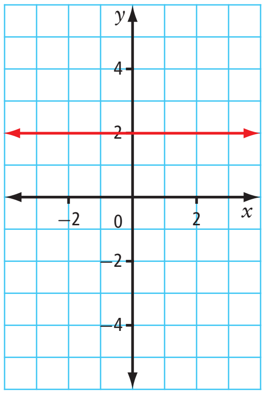
a) $y = 7x - 5$
b) $y = -x + 8$
c) $y = \frac{3}{2}x + 4$
d) $y = -\frac{3}{5}x - 2$
e) $y = 0.25x - 0.3$
f) $y = -\frac{5}{2}x + \frac{1}{8}$

3. Determine the intercepts of each line. Then, graph the line.

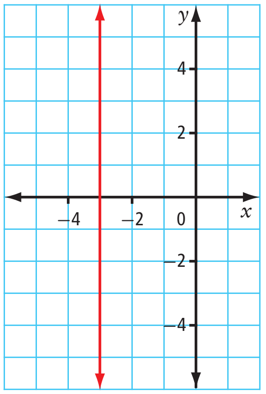
a) $2x + y - 9 = 0$
b) $4x - y - 8 = 0$
c) $x - 2y + 10 = 0$
d) $3x - 8y - 24 = 0$
e) $4x + 5y + 6 = 0$
f) $x = 4$
g) $y = 0$
h) $4x - 12 = 0$

4. For each line, state the domain, range, intercepts, and slope. What is the equation of each line, in general form?

a)



b)



5. Graph each line using the given intercepts. What is the equation of each line?

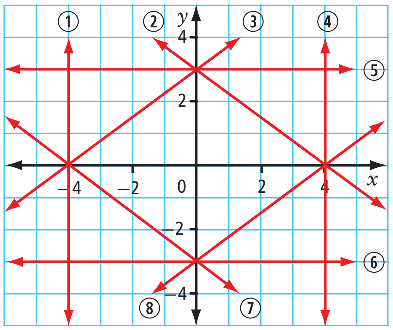
a) an x-intercept of 3 and no y-intercept

b) a y-intercept of -5 and no x-intercept

c) an infinite number of x-intercepts

d) an infinite number of y-intercepts

6. Match each equation with a line labelled in the figure.



a) $3x + 4y = 12$

b) $x = 4$

c) $3x - 4y = 12$

d) $y - 3 = 0$

e) $3x - 4y + 12 = 0$

f) $y = -3$

g) $3x + 4y + 12 = 0$

h) $x + 4 = 0$

Apply

7. Write an equation, in general form, for each line described.

a) a vertical line passing through the point (3, 5)

b) a horizontal line passing through the point (-2, 6)

c) the x-axis

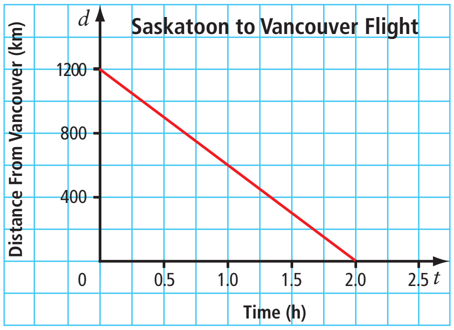
d) the y-axis

8. Write an equation, in general form, of a line that does not have a y-intercept and passes through the point (3, 6).

9. Having fibre in your diet helps with digestion, heart health, and maintaining a healthy weight. Courtney wants to increase her fibre intake by 21 g per day. She plans to do this by adding bran buds and one vegetable to her diet each day. Write an equation in general form that describes this situation. Courtney mixes 125 mL of bran buds in her cereal each day. What volume of green peas would she need to eat during the day? What volume of baked beans would she need to eat? Express your answers in whole millilitres.

Food	Grams of Fibre per 125 mL
Green peas	4
Baked beans	7
Bran buds	16

10. An airplane flies directly from Saskatoon, SK, to Vancouver, BC. The graph shows the relationship between the distance from Vancouver, d, in kilometres, and the flying time, t, in hours.



a) State the intercepts of the line segment. What does each intercept represent?

b) State a suitable domain and range of the graph.

c) Determine the slope of the line. What does the slope represent?

d) Write the equation of the line in general form.

e) For how many hours has the plane been flying when it is 200 km from Vancouver?

f) What is the distance from Vancouver when the plane has been flying for 45 min?

11. Luc swims as part of an active and healthy lifestyle. The number of calories burned by a swimmer of Luc's body weight is shown in the table.

Swimming Style	Calories Burned Per Minute
Backstroke	8
Butterfly	11

a) Write a linear equation to describe the number of minutes Luc would need to swim backstroke, x, and butterfly, y, to burn 440 cal.

b) What are the intercepts of the line? What does each intercept represent?

c) Suppose Luc swims butterfly for 16 min. How long will he need to swim backstroke in order to burn 440 cal in total?

12. Sanding trucks spread a mixture of sand and salt on roads to improve traction in winter. The density of the salt is 1200 kg/m^3. The density of the sand is 1800 kg/m^3.

a) Write a linear equation to represent the volume, in cubic metres, of salt, x, and of sand, y, in a mixture with a mass of 10 000 kg.

b) For temperatures below -12 °C, the volume of sand in 10 000 kg of the mixture is 5.22 m^3. What is the volume of salt in the mixture?

c) By mass, what percent of the mixture is salt?

13. Advance tickets for a local concert sold for \$8 each. Tickets at the door were \$12 each. The revenue from ticket sales was \$1120.

a) Write a linear equation relating the number of advance tickets, a, to the number of tickets sold at the door, d. Express your equation in general form.

b) Describe the steps you would follow to graph this equation. Do not graph.

c) Suppose twice as many advance tickets were sold as tickets at the door. How many of each type of ticket were sold?

14. What is the value of the unknown parameter in each equation?

a) $Ax + 5y - 6 = 0$, passing through $(-3, 2)$

b) $2x + By + 7 = 0$, passing through $(4, -5)$

c) $4x - 3y + C = 0$, passing through $(-2, -6)$

Extend

15. The equation of a line is $x + 3y - 24 = 0$. Write the coordinates of a point on the line for each of the following conditions.

a) The x-coordinate is equal to the y-coordinate.

b) The x-coordinate is three times as great as the y-coordinate.

c) The y-coordinate is four greater than the x-coordinate.

16. The equation $6x + By + 5 = 0$ describes a line with a slope of $\frac{3}{2}$. What is the value of B?

17. What is the area of the triangle bounded by each set of lines?

a) the line $x + 2y = 10$, a line with an infinite number of x-intercepts, and a line with an infinite number of y-intercepts

b) a line with an infinite number of x-intercepts, the line $2x - y = 6$, and the line $x = 10$

Create Connections

18. a) Which form of a linear equation do you prefer to graph: $y = mx + b$ or $Ax + By + C = 0$? Why?

b) Describe a situation when you might work with a linear equation in the other form.

19. The equation of a line is $2x + y - 8 = 0$.

a) Explain how you could determine the x-intercept.

b) Explain two different ways to determine the y-intercept.

20. The general form of an equation is $Ax + By + C = 0$. Identify each line as horizontal, vertical, or oblique.

a) $A \neq 0$, $B = 0$, and C is a real number.

b) $A = 0$, $B \neq 0$, and C is a real number.

c) $A \neq 0$, $B \neq 0$, and C is a real number.

21. **MINI LAB** Explore the effects of changing parameters on a graph of $Ax + By + C = 0$.

Materials
- graphing technology or grid paper and ruler

Step 1 For each group of equations, graph the three linear relations on the same axes. Use graphing technology or sketch the graphs by hand.

a) $x + y - 2 = 0$
$x + y - 6 = 0$
$x + y - 8 = 0$

b) $x + 2y - 6 = 0$
$2x + 2y - 6 = 0$
$3x + 2y - 6 = 0$

c) $2x + y - 6 = 0$
$2x + 2y - 6 = 0$
$2x + 3y - 6 = 0$

d) $x + y - 2 = 0$
$2x + 2y - 4 = 0$
$3x + 3y - 6 = 0$

Step 2 Compare the three graphs in each group of equations. What parameter(s) have changed? Explain how changing one or two parameters affects the slope, x-intercept, and y-intercept of the graph.

Step 3 How does changing the parameters A, B, and C affect the graph of a linear equation in general form?

7.3 Slope-Point Form

Focus on ...

- writing the equation of a line from its slope and a point on the line
- converting equations among the various forms
- writing the equation of a line from two points on the line
- solving problems involving equations in slope-point form

You can measure the length of an arena with a ruler, use scissors to cut your grass, or loosen a screw with a paperclip. All of these are possible, but are they using the best tool for the job? In mathematics and in life, using an inappropriate tool may prevent you from completing your task or make it take longer to finish. You have explored tools for writing linear relations in two forms. This section introduces a third form, slope-point form. Each form is best suited to certain situations.

Materials

- grid paper
- ruler

Investigate Equations in Slope-Point Form

1. Square ABCD in Figure 1 is a composite of four different polygons. The lengths of the sides are shown. What is the area of square ABCD?

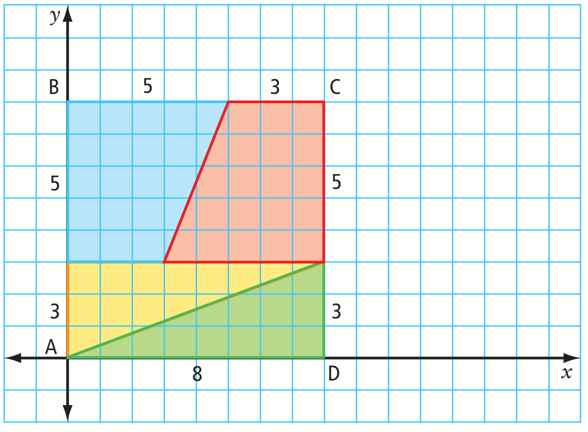


Figure 1

2. Square ABCD is reassembled to form rectangle EFGH, shown in Figure 2. What is the area of rectangle EFGH?

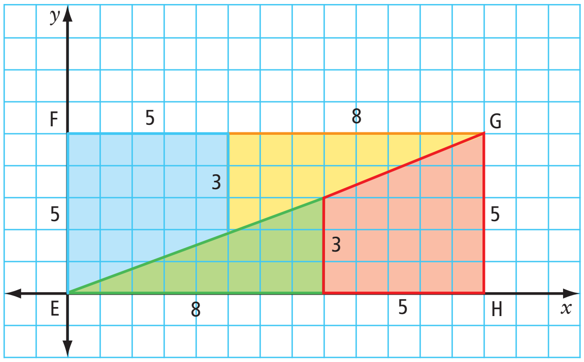


Figure 2

3. There is a discrepancy between the areas of the quadrilaterals shown in Figures 1 and 2. How is Figure 2 deceiving? Justify your answer.

4. On grid paper, draw a line that does not pass through the origin. Label points J, K, and L on the line. Determine the slope of your line.

a) Determine the equation of your line using point J and the slope-intercept form, $y = mx + b$.

b) Use points K and L to determine equations of your line. Compare your equations.

5. Let P(x_1, y_1) represent a point on a line. Develop an equation of the line with slope m using point P.

If you know one point on a line, how can you use the slope to determine a second point?

6. Work with a partner. Have your partner test the equation you developed using his or her line from step 4.

7. Reflect and Respond Describe how to determine the equation of a line using the slope and a point on the line.

8. Show how the **slope-point form** of a linear equation can be developed by using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope-point form

- the equation of a non-vertical line in the form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) are the coordinates of a point on the line

9. What type of line cannot be written in slope-point form? Why?

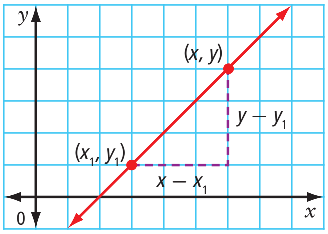
10. Is the following statement always true, sometimes true, or never true? Explain. "To determine the equation of a non-vertical line in slope-point form, you can use the coordinates of any point on the line."

Link the Ideas

The slope of a non-vertical line can be determined using $m = \frac{\Delta y}{\Delta x}$.

If (x_1, y_1) is one point on the line, then (x, y) could represent any other point on the line. Substitute the coordinates of these two points into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of the line could be written as $m = \frac{y - y_1}{x - x_1}$.



Multiplying both sides of the above equation by $(x - x_1)$ gives

$$(x - x_1)m = (x - x_1)\left(\frac{y - y_1}{x - x_1}\right)$$

$$(x - x_1)m = \cancel{(x - x_1)}^{1}\left(\frac{y - y_1}{\cancel{x - x_1}_{1}}\right)$$

$$m(x - x_1) = y - y_1$$

This equation is called the slope-point form of a non-vertical line through point (x_1, y_1) with slope m.

The slope-point form is commonly written as $y - y_1 = m(x - x_1)$.

Example 1 Write the Equation of a Line Using a Point and the Slope

a) Use slope-point form to write an equation of the line through $(-2, 5)$ with slope -3.

b) Express the equation in slope-intercept form, $y = mx + b$.

c) Graph the linear relation using technology.

Solution

a) Substitute -3 for m and the coordinates of the point $(-2, 5)$ for (x_1, y_1).

$$y - y_1 = m(x - x_1)$$
$$y - (5) = -3(x - (-2))$$
$$y - 5 = -3(x + 2)$$

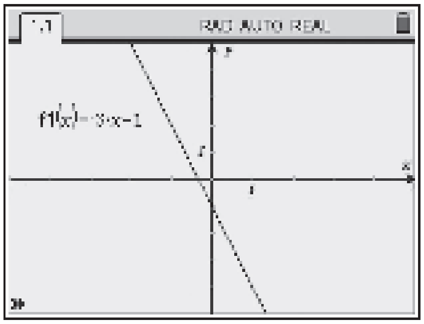
The equation in slope-point form is $y - 5 = -3(x + 2)$.

b) To express the equation in slope-intercept form, isolate y.

$$
\begin{aligned}
y - 5 &= -3(x + 2)\\
y &= -3(x + 2) + 5\\
y &= -3x - 6 + 5\\
y &= -3x - 1
\end{aligned}
$$

In slope-intercept form, the equation is $y = -3x - 1$.

c)



What strategies could you use to sketch the graph?

The equation $y = -3x - 1$ is written in the form $y = mx + b$. So, the slope is -3. This is consistent with the value given in the question.

Your Turn

a) Use slope-point form to write an equation of the line through $(3, -4)$ with slope 2. Sketch a graph of the line.

b) Express the equation in slope-intercept form, $y = mx + b$. Sketch a graph of this line.

c) Compare your graphs.

Example 2 Determine the Equation of a Line Using Two Points

a) Use slope-point form to write an equation of the line through $(3, -4)$ and $(5, -1)$.

b) Sketch a graph of the line.

c) Rewrite the equation in general form, $Ax + By + C = 0$.

Solution

a) Points on the line are given. So, you need to determine the slope. Use the two given points, $(3, -4)$ and $(5, -1)$.

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1}\\
m &= \frac{-1 - (-4)}{5 - 3}\\
m &= \frac{-1 + 4}{5 - 3}\\
m &= \frac{3}{2}
\end{aligned}
$$

In slope-point form, substitute $\frac{3}{2}$ for m and the coordinates of either point (3, −4) or (5, −1) for (x_1, y_1).

Using (3, −4) for (x_1, y_1),

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = \frac{3}{2}(x - 3)$$
$$y + 4 = \frac{3}{2}(x - 3)$$

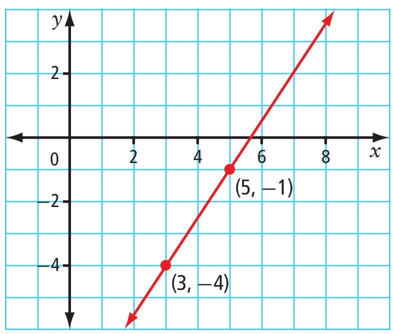
Using (5, −1) for (x_1, y_1),

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{3}{2}(x - 5)$$
$$y + 1 = \frac{3}{2}(x - 5)$$

How can you verify that these equations are equivalent?

Both $y + 4 = \frac{3}{2}(x - 3)$ and $y + 1 = \frac{3}{2}(x - 5)$ are slope-point forms of the equation of the line through (3, −4) and (5, −1).

b)



c) Express $y + 4 = \frac{3}{2}(x - 3)$ in general form.

$$y + 4 = \frac{3}{2}(x - 3)$$
$$2(y + 4) = \cancel{2}^{1}\left(\frac{3}{\cancel{2}_{1}}(x - 3)\right)$$
$$2y + 8 = 3(x - 3)$$
$$2y + 8 = 3x - 9$$
$$0 = 3x - 9 - 2y - 8$$
$$0 = 3x - 2y - 17$$

Express $y + 1 = \frac{3}{2}(x - 5)$ in general form.

$$y + 1 = \frac{3}{2}(x - 5)$$
$$2(y + 1) = \cancel{2}^{1}\left(\frac{3}{\cancel{2}_{1}}(x - 5)\right)$$
$$2(y + 1) = 3(x - 5)$$
$$2y + 2 = 3x - 15$$
$$0 = 3x - 2y - 17$$

The equation, in general form, for the line through (3, −4) and (5, −1) is $3x - 2y - 17 = 0$.

Your Turn

Use slope-point form to write an equation of the line through (−5, 2) and (−2, 1). Explain your steps. Then, write the equation in general form, $Ax + By + C = 0$.

Example 3 Model a Real-Life Situation

Brad Zdanivsky is enthusiastic about mountain climbing. He is a quadriplegic and used custom gear as he climbed the Stawamus Chief in Squamish, BC, on July 31, 2005.

Supposed he moved at a constant rate and climbed the 660-m summit in 11 pitches (sections). Each pitch was approximately 60 m in height. At 5:45 a.m., Brad started his climb 60 m below the top of his first pitch. By 5:55 a.m., he was 40 m below the top of the first pitch.

a) Write an equation that describes Brad's distance, d, in metres, below the top of the first pitch in terms of t minutes past 5:45 a.m. Express the equation in $y = mx + b$ form.

b) How long did it take Brad to reach the top of the first pitch?

c) In total, Brad spent 8.5 h changing ropes between pitches. How long did it take Brad to climb the Stawamus Chief?

Web Link

To learn more about mountain climbing, go to www.mhrmath10.ca and follow the links.

Solution

a) Brad was 60 m below the top of his first pitch at 0 min past 5:45 a.m. After 10 min, he was 40 m below the top of his first pitch. As coordinate pairs (t, d), the data may be represented as (0, 60) and (10, 40). Use these points to determine the slope of the line.

$$m = \frac{d_2 - d_1}{t_2 - t_1}$$

$$m = \frac{60 - 40}{0 - 10}$$

$$m = -2$$

M E

$$m = \frac{60 - 40}{0 - 10}$$

$$m = \frac{20}{-10}$$

$$m = -2$$

Brad's distance to the top of the pitch was decreasing at a rate of 2 m/min.

Substitute the slope, −2, and the coordinates of either point (0, 60) or (10, 40) into the slope-point form of an equation.

Using point (0, 60),

$$d - d_1 = m(t - t_1)$$

$$d - 60 = -2(t - 0)$$

$$d - 60 = -2t$$

$$d = -2t + 60$$

How could you verify your equation?

In slope-intercept form, the equation $d = -2t + 60$ represents Brad's distance below the first pitch.

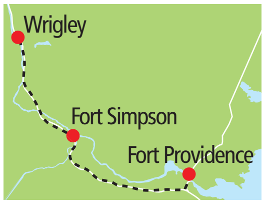
b) At the top of the first pitch, $d = 0$. Determine t.

$$d = -2t + 60$$
$$0 = -2t + 60$$
$$2t = 60$$
$$t = 30$$

It took Brad 30 min or 0.5 h to reach the top of the first pitch.

c) To climb the 11 pitches, it took Brad 11(0.5 h) = 5.5 h. Adding 8.5 h to change ropes, it took Brad 14 h to climb the Stawamus Chief.

Your Turn



A family drives at a constant speed from Wrigley, NT, to visit relatives in Fort Providence, NT. When they start driving at 9:00 a.m., they are 540 km from Fort Providence. At 12:30 p.m., they reach Fort Simpson, located 225 km from Fort Providence.

a) Write an equation that describes their distance, d, in kilometres, from Fort Providence in terms of t hours past 9:00 a.m.

b) What time will the family reach Fort Providence?

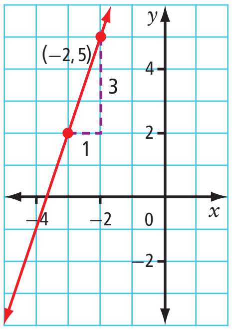
Key Ideas

- For a non-vertical line through the point (x_1, y_1) with slope m, the equation of the line can be written in slope-point form as $y - y_1 = m(x - x_1)$.

  A line through (−2, 5) has a slope of 3.

  The slope-point form of the equation of this line is

  $$y - y_1 = m(x - x_1)$$
  $$y - 5 = 3(x - (-2))$$
  $$y - 5 = 3(x + 2)$$



- An equation written in slope-point form can be converted to either slope-intercept form or general form.
- Any point on a line can be used when determining the equation of the line in slope-point form.

Check Your Understanding

Practise

1. Rewrite each equation from slope-point form to slope-intercept form, $y = mx + b$, and general form, $Ax + By + C = 0$.

a) $y + 3 = x - 5$

b) $y + 4 = 2(x + 3)$

c) $y - 6 = 4(x + 1)$

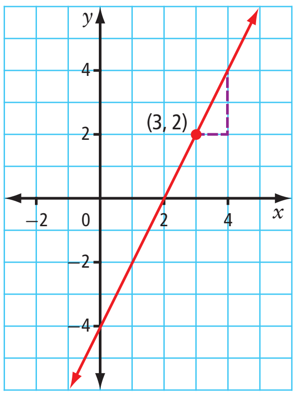
d) $y + 2 = -5(x + 3)$

e) $y - 3 = -\frac{1}{2}(x + 8)$

f) $y + 9 = -\frac{2}{3}(x - 6)$

2. Write an equation in slope-point form, $y - y_1 = m(x - x_1)$, of each line passing through the given point.

a)



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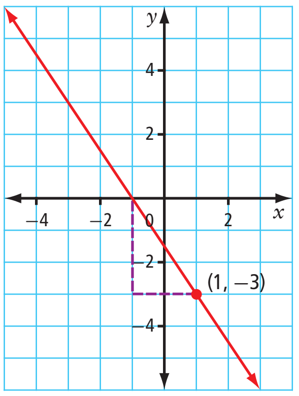
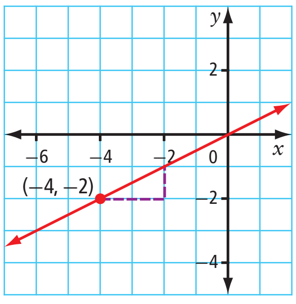
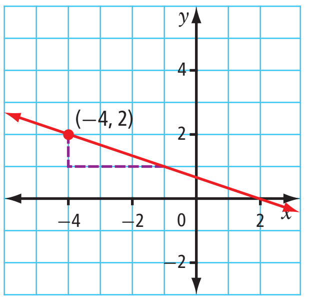


c)



d)



3. Determine the equation of each line using slope-point form. Then, express each equation in slope-intercept form and in general form.

a) $(5, -2)$, $m = 6$

b) $(-3, -5)$, $m = -2$

c) $(-8, 3)$, $m = \frac{1}{2}$

d) $(12, -6)$, $m = -\frac{2}{3}$

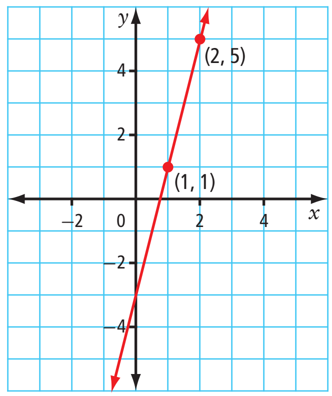
4. Consider the line represented by $y - 1 = \frac{2}{3}(x - 6)$.

a) Identify the slope and a point on the line.

b) Explain how you could sketch the graph of the line using the slope and a point on the line.

5. Write an equation in slope-point form, $y - y_1 = m(x - x_1)$, of the line passing through the given points.

a)



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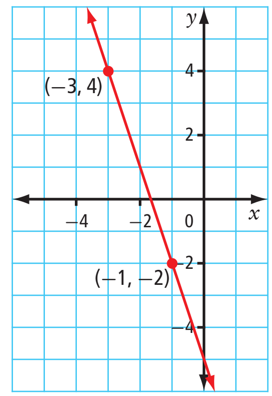
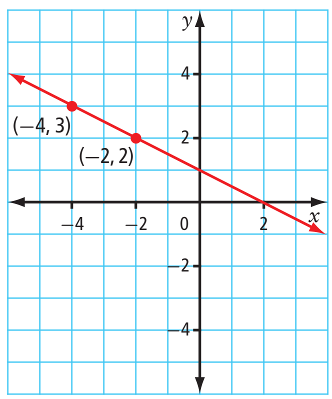
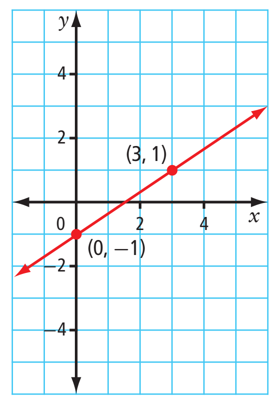


c)



d)



6. Use slope-point form to write an equation of a line through each pair of points. Express each equation in the form $y = mx + b$ and in the form $Ax + By + C = 0$.

a) $(5, 1)$ and $(3, -7)$

b) $(5, -8)$ and $(1, 4)$

c) $(4, 5)$ and $(2, 6)$

d) $(8, -3)$ and $(4, -6)$

e) $(5, -1)$ and $(3, -4)$

f) $(3, 6)$ and $(-1, 0)$

7. Terry's teacher writes the following on the board:

The four equations listed represent only two different lines.
Which equations represent the same line?

① $y - 2 = 3(x + 1)$

② $y - 10 = 3(x - 4)$

③ $y + 5 = 3(x + 1)$

④ $y - 11 = 3(x - 2)$

a) Describe possible strategies students could use to answer the question.

b) Which equations represent the same line? Justify your answers.

Apply

8. Identify the slope and a point on each line. Sketch a graph of each line. Use graphing technology to check your graphs.
   a) $y - 3 = 2(x - 1)$
   b) $y + 2 = -3x$
   c) $y - 4 = \frac{1}{2}(x + 1)$
   d) $y + 6 = -\frac{4}{5}(x - 2)$

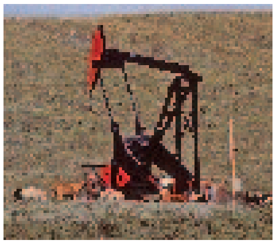
9. Consider the line passing through the points $(-4, 2)$ and $(-2, 6)$.
   a) Work with a partner to develop at least two different strategies for determining the y-intercept of the line.
   b) What is the y-intercept of the line?

10. A line passes through $(0, 1)$ and $(3, 7)$.
    a) Using only slope-intercept form, $y = mx + b$, write the equation of this line.
    b) Determine the equation of the line using only slope-point form.
    c) Compare the two equations graphically.

11. Write the equation of each line using slope-point form. Then, convert to slope-intercept form.
    a) slope of 0 and through $(4, -5)$
    b) same slope as $3x + y = 5$ and through $(-2, 4)$
    c) same slope as the line $x - 2y + 6 = 0$ and the same x-intercept as the line $3x - 2y = 24$
    d) same y-intercept as $x + 4y = 8$ and through $(3, -4)$

12. What is the equation of each line in slope-point form? Convert each equation to general form.
    a) slope of 3 and x-intercept of 4
    b) same slope as $y = -4x + 5$ and through $(2, -1)$
    c) same x-intercept as the line $3x + y = 12$ and through $(0, 2)$
    d) x-intercept of 2 and y-intercept of -6

13. An “iron horse” pumpjack starts to pump crude oil into a tank at a constant rate of 1.2 m^3/h. After 24 h, the tank contains 29 m^3 of oil.

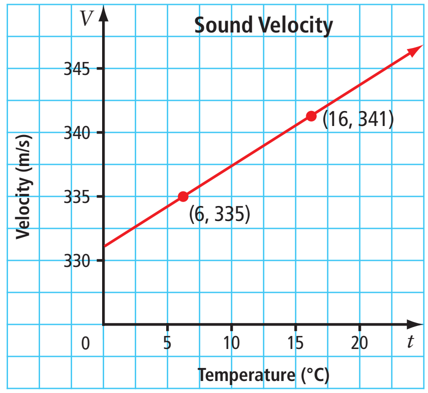


    a) Write an equation that describes the volume, V, in cubic metres, of oil in the tank after t hours.
    b) The tank can hold a maximum of 155 m^3 of oil. How long will it take to fill the tank?
    c) Was the tank empty before it started filling? Explain.

Did You Know?

Oil pumpjacks are common in western Canada. They are a traditional method of oil recovery. The surface deposits of the Athabasca Oil Sands in present day northern Alberta were once used by the Cree and Dene peoples to waterproof their canoes and other items.

14. The graph shows the linear relationship between the velocity of sound, V, in metres per second, and the temperature, t, in degrees Celsius, of dry air. At 6 °C, the velocity of sound is 335 m/s. At 16 °C, it is 341 m/s.



a) What is the slope of the line?

b) What rate of change does the slope represent?

c) What is the equation of the line?

d) Determine the velocity of sound at 35 °C.

e) What is the air temperature when the velocity of sound is 348 m/s?

15. What is the y-intercept of a line with a slope of $\frac{1}{2}$ and an x-intercept of 4?

16. Determine the x-intercept of a line through (3, 4) having a y-intercept of 2.

17. Suppose Canada's population has grown steadily since 2000. In 2001, the population was 30.0 million. In 2009, it was 33.7 million.

a) Let t represent the number of years since 2000. Let p represent the population of Canada in millions. Write the coordinates of two points in the form (t, p).

b) Determine the slope of the line through the points.

c) What rate does the slope represent?

d) Write an equation to represent population growth in Canada since 2000.

e) Predict Canada's population in 2017.

Web Link

To learn more about the components of population growth in parts of Canada, go to www.mhrmath10.ca and follow the links.

18. Suppose your friend's dinner tonight consists of one steak and mini potatoes. The steak has approximately 30 g of protein. The nutrition facts label shows the number of grams of protein per number of mini potatoes.

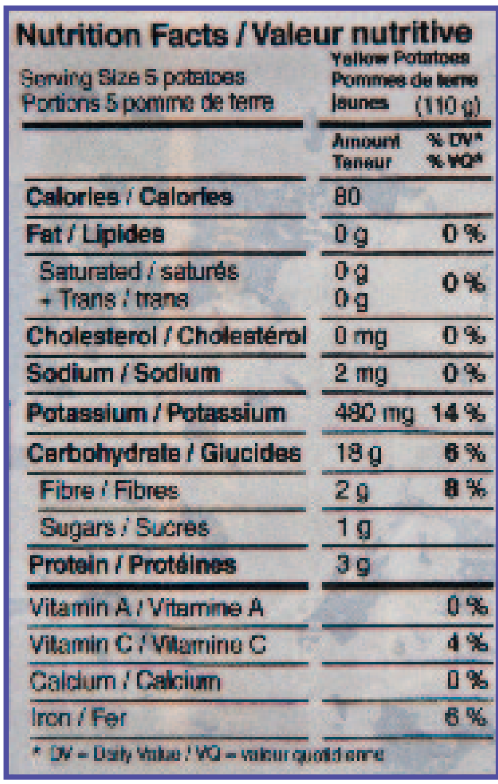
   a) Write an equation relating the protein, p, in the meal to the number of potatoes, n, eaten. Use the data in the nutrition facts label.

   b) What is the slope of the line? What does the slope represent?

   c) What is the p-intercept of the line? Why is the p-intercept not zero?

   d) Suggest a reasonable domain and range for the graph.

> **Did You Know?**
>
> In Canada, potato production is a multi-million-dollar industry. Manitoba produces the second greatest amount of potatoes in Canada.

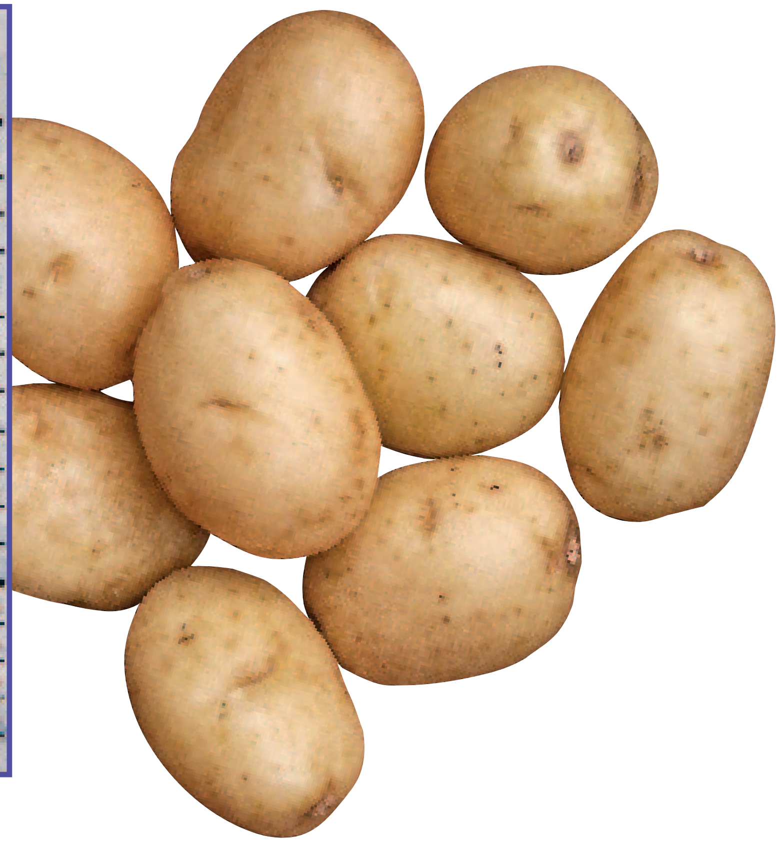


Nutrition Facts / Valeur nutritive

Serving Size 5 potatoes / Portions 5 pomme de terre — Yellow Potatoes / Pommes de terre jaunes (110 g)

	Amount / Teneur	% DV* / % VQ*
Calories / Calories	80	
Fat / Lipides	0 g	0 %
Saturated / saturés + Trans / trans	0 g / 0 g	0 %
Cholesterol / Cholestérol	0 mg	0 %
Sodium / Sodium	2 mg	0 %
Potassium / Potassium	480 mg	14 %
Carbohydrate / Glucides	18 g	6 %
Fibre / Fibres	2 g	8 %
Sugars / Sucres	1 g	
Protein / Protéines	3 g	
Vitamin A / Vitamine A		0 %
Vitamin C / Vitamine C		4 %
Calcium / Calcium		0 %
Iron / Fer		6 %

* DV = Daily Value / VQ = valeur quotidienne



Extend

19. Write the equation of a line with an x-intercept of n and a slope of m.

20. A line passes through the point of intersection of the lines $y = -\frac{1}{2}x - 6$ and $y = 2x + 4$. Determine the equation of the line if it has a slope of $\frac{1}{2}$.

Create Connections

21. How can you develop the slope-intercept form, $y = mx + b$, by substituting a point into $y - y_1 = m(x - x_1)$?

22. To determine the equation of a line in slope-point form, you need to know two pieces of information about the line. List three sets of information that would allow you to determine the equation of a line.

23. To solve a particular problem you may want to write a linear equation in one of the three forms. You may wish to use slope-intercept form, $y = mx + b$; general form, $Ax + By + C = 0$; or slope-point form, $y - y_1 = m(x - x_1)$. Create a visual that helps you decide which form you should start with.

Materials
- SI measuring tape
- grid paper
- ruler

24. MINI LAB **Unit Project** Paleontologists can predict the anatomy of humans and animals based on skeletal remains.

Step 1 Work with a partner of the same gender as yourself. Measure and record the length of each other's humerus bone. It runs from the shoulder to the elbow. Measure and record each other's height without shoes.

Step 2 Collect and share your data with other students of the same gender. Record all data. Use grid paper to plot the data as coordinate pairs. Label the axes and scale used.

Step 3 Draw a straight line that represents the data. What is the equation of this line?

Step 4 Measure the humerus bone of a teacher of the same gender as you. Use your equation to predict the height of the teacher. Compare the teacher's actual height with your predicted height.

7.4 Parallel and Perpendicular Lines

Controlling the movement of your body is important when you play sports or exercise. The words *parallel* and *perpendicular* describe the position of one thing relative to another. Athletes may need to visualize parallel and perpendicular lines to help them improve their performance. For example,

- A football coach may instruct quarterbacks to position their shoulders perpendicular to the target at which they are throwing the ball.
- The gymnast in the photo has trained hard to keep her legs parallel to her arms.

Focus on ...

- identifying whether two lines are parallel, perpendicular, or neither
- writing the equation of a line using the coordinates of a point on the line and the equation of a parallel or perpendicular line
- solving problems involving parallel and perpendicular lines

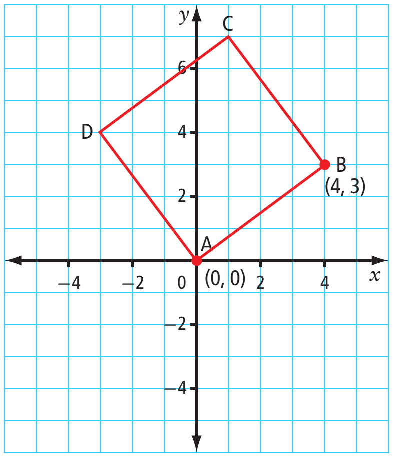
Materials

- two sheets of identical grid paper
- scissors
- ruler

Investigate Slopes of Parallel and Perpendicular Lines

1. On a sheet of grid paper, create a coordinate system by drawing and labelling an x-axis and a y-axis. From another sheet of grid paper, cut out the following three shapes:

- a square with side length of 5 units
- a square with side length of 13 units
- a rectangle with side lengths that are whole units. Draw a diagonal across the rectangle.

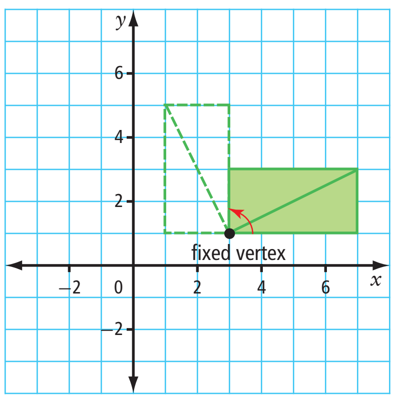


2. For the smaller square, label the vertices A, B, C, and D in a counterclockwise direction. Position the square so that point A is at (0, 0) and point B is at (4, 3). What are the coordinates of vertices C and D? Determine the slope of each side. Compare the slopes of the opposite sides and the adjacent sides.

3. Repeat step 2 using the larger square. Position the square with point A at (0, 0) and B at (12, 5).

4. Position the rectangle on your coordinate system. Determine the slope of each side. Hint: Line up the vertices with the integer coordinates of the grid. Compare the slopes.

5. Compare the slopes of the diagonals formed by rotating the rectangle 90° about one vertex. Discuss your results with a classmate.



6. **Reflect and Respond** How are the slopes of parallel sides related? Explain using an example.

7. How are the slopes of perpendicular sides related? Explain using an example.

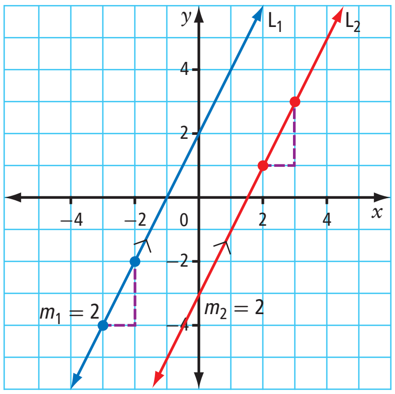
8. a) How are the slopes of vertical sides related?
   b) How are the slopes of a vertical and a horizontal side related?

Link the Ideas

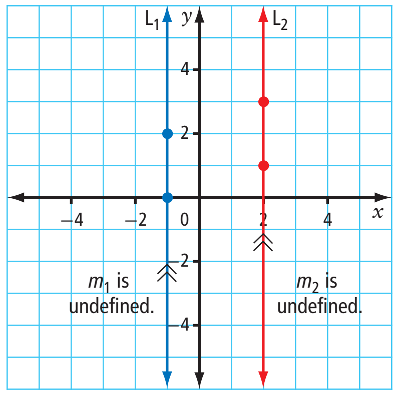
Parallel lines have the same slope but different intercepts. This includes horizontal lines, which have a slope of zero. Vertical lines, which have an undefined slope, are also parallel.

parallel lines

- lines in the same plane that do not intersect
- lines that have the same slope but different intercepts



$m_1 = m_2$

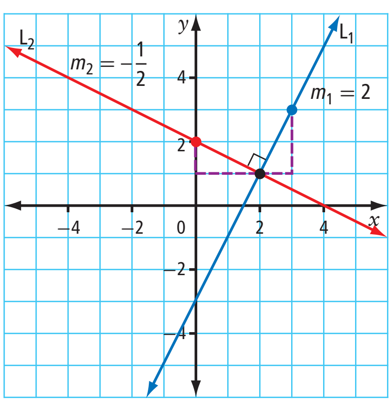


m_1 and m_2 are undefined.

The slopes of **perpendicular lines** are negative reciprocals of each other. The product of negative reciprocals is -1. A vertical line, which has an undefined slope, and a horizontal line, which has a slope of 0, are perpendicular to each other.

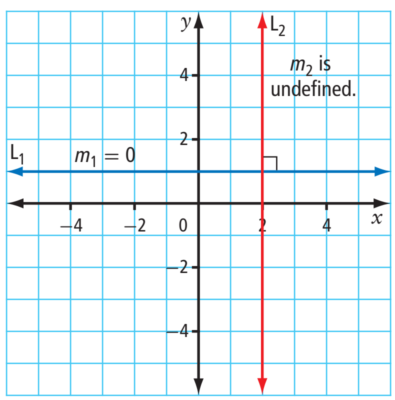
perpendicular lines

- two lines that intersect at right angles (90°)
- lines that have slopes that are negative reciprocals of each other.



$m_1(m_2) = 2\left(-\frac{1}{2}\right)$

$m_1(m_2) = -1$



$m_1 = 0$ and m_2 is undefined.

Example 1 Identify Parallel and Perpendicular Lines

State whether the lines in each pair are parallel, perpendicular, or neither.

a) $y = 3x - 6$
$y = -\frac{1}{3}x + 4$

b) $y = 4x + 3$
$y = 4x - 5$

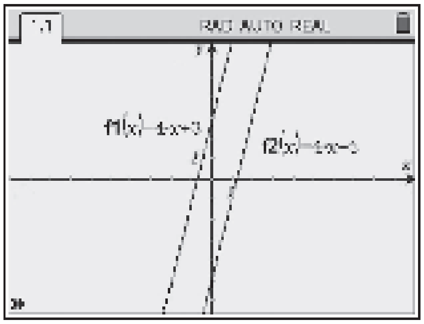
c) $y = 2x + 6$
$6x + 3y + 3 = 0$

Solution

How can you verify that two values are negative reciprocals of each other?

a) The slope of the line $y = 3x - 6$ is 3.
The slope of the line $y = -\frac{1}{3}x + 4$ is $-\frac{1}{3}$.
Since the slopes, 3 and $-\frac{1}{3}$, are negative reciprocals, the lines are perpendicular.

b) The slope of the line $y = 4x + 3$ is 4. The slope of the line $y = 4x - 5$ is also 4. The slopes are equal. So, the lines are parallel.

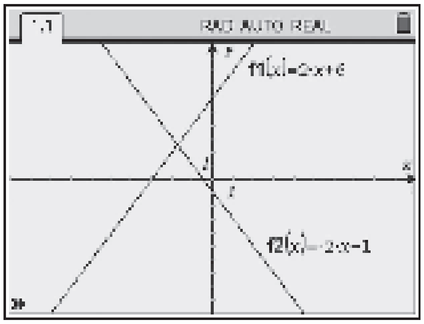


How do you know the lines are not equivalent?

c) The slope of line $y = 2x + 6$ is 2. To determine the slope of the line $6x + 3y + 3 = 0$, rewrite the equation in slope-intercept form, $y = mx + b$.

$$6x + 3y + 3 = 0$$
$$3y + 3 = -6x$$
$$\frac{3y}{3} = \frac{-6x - 3}{3}$$
$$y = -2x - 1$$

The slope of the line $y = -2x - 1$ is -2. The slopes 2 and -2 are not equal and they are not negative reciprocals. Therefore, the two lines are neither parallel nor perpendicular.



Your Turn

Determine whether the lines in each pair are parallel, perpendicular, or neither.

a) $y = \frac{1}{2}x - 7$
$y = 2x - 7$

b) $y = 3x - 4$
$y = 3x + \frac{1}{4}$

c) $y = \frac{2}{5}x - 6$
$5x + 2y = 8$

Example 2 Write an Equation Involving a Parallel Line

a) Write the equation of a line that is parallel to $2x - y + 4 = 0$ and through $(1, -6)$. Express the equation in slope-intercept form.

b) Write the equation in general form.

c) Use technology to verify that the lines are parallel.

Solution

a) The slope of the line will be equal to the slope of $2x - y + 4 = 0$. To find the slope, convert $2x - y + 4 = 0$ to slope-intercept form, $y = mx + b$.

$$2x - y + 4 = 0$$
$$2x - y + 4 + y = 0 + y$$
$$2x + 4 = y \text{ or } y = 2x + 4$$

The slope of the line $y = 2x + 4$ is 2.

Method 1: Use Slope-Point Form

Substitute 2 for m and the coordinates of the point $(1, -6)$ for (x_1, y_1).

$$y - y_1 = m(x - x_1)$$
$$y - (-6) = 2(x - 1)$$
$$y + 6 = 2(x - 1)$$

To convert to slope-intercept form, isolate y.

$$y + 6 = 2(x - 1)$$
$$y + 6 = 2x - 2$$
$$y = 2x - 2 - 6$$
$$y = 2x - 8$$

The equation of the line, in slope-intercept form, is $y = 2x - 8$.

Method 2: Use Slope-Intercept Form

The point (1, −6) lies on the line, so the coordinates must satisfy the equation of the line. Substitute 2 for m and the coordinates (1, −6) for (x, y). Then, determine the y-intercept and rewrite the equation.

$$\begin{aligned} y &= mx + b \\ -6 &= 2(1) + b \\ -6 - 2 &= b \\ -8 &= b \end{aligned}$$

Substitute the values for m and b into $y = mx + b$.

$y = 2x + (-8)$

$y = 2x - 8$

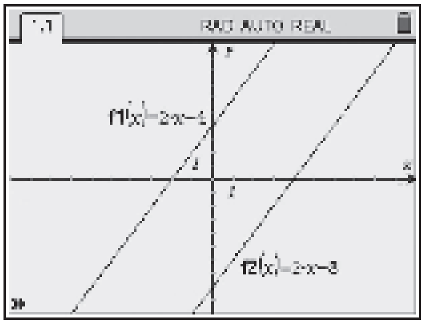
The slope-intercept form of the equation is $y = 2x - 8$.

b) Convert the slope-point equation to general form.

$$\begin{aligned} y + 6 &= 2(x - 1) \\ y + 6 - (y + 6) &= 2(x - 1) - (y + 6) \\ 0 &= 2x - 2 - y - 6 \\ 0 &= 2x - y - 8 \end{aligned}$$

The equation of the line, in general form, is $2x - y - 8 = 0$.

c) The graph to the right shows the original line and the new line, represented by their equations in slope-interecept form.



The equation of original line is $y = 2x + 4$

The equation of original line is $y = 2x - 8$.

The slope-intercept form of the lines and the graph both show that the slopes of the lines are the same, but the y-intercepts are different. Therefore the lines are parallel.

Your Turn

Write the equation of a line that is parallel to $3x + y + 3 = 0$ and passes through (5, −6). Express the equation in slope-intercept form and in general form. Use technology to verify that the lines are parallel.

Example 3 Write an Equation Involving a Perpendicular Line

Write the equation of a line perpendicular to $3x + 2y - 6 = 0$ with an x-intercept of 9. Express the equation in slope-intercept form and in general form.

Solution

To determine the slope of $3x + 2y - 6 = 0$, rewrite the equation in slope-intercept form.

$$
\begin{aligned}
3x + 2y - 6 &= 0 \\
2y &= -3x + 6 \\
y &= -\frac{3}{2}x + 3
\end{aligned}
$$

The slope of the line $y = -\frac{3}{2}x + 3$ is $-\frac{3}{2}$.

The negative reciprocal of $-\frac{3}{2}$ is $\frac{2}{3}$.

Therefore, the slope of a line perpendicular to the given line is $\frac{2}{3}$.

Substitute $\frac{2}{3}$ for m and the coordinates of the point (9, 0) for (x_1, y_1) into the slope-point form of an equation.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 0 &= \frac{2}{3}(x - 9) \\
y &= \frac{2}{3}(x - 9)
\end{aligned}
$$

How else could you find the equation of the line?

For slope-intercept form, $y = mx + b$, expand the slope-point equation.

$$
\begin{aligned}
y &= \frac{2}{3}(x - 9) \\
y &= \frac{2}{3}x - \overset{3}{\cancel{9}}\left(\frac{2}{\underset{1}{\cancel{3}}}\right) \\
y &= \frac{2}{3}x - 6
\end{aligned}
$$

For general form, $Ax + By + C = 0$, rearrange the slope-intercept equation.

$$
\begin{aligned}
y &= \frac{2}{3}x - 6 \\
3y &= 2x - 18 \\
0 &= 2x - 3y - 18
\end{aligned}
$$

What other equation could you use to convert to general form? Why might some people choose to use that equation in this solution?

The equation of a line perpendicular to $3x + 2y - 6 = 0$ with an x-intercept of 9 is $y = \frac{2}{3}x - 6$ in slope-intercept form. Written in general form, the equation is $2x - 3y - 18 = 0$.

The reciprocal of $-\frac{3}{2}$ is $-\frac{2}{3}$. So, the negative reciprocal is $-\left(-\frac{2}{3}\right)$ or $\frac{2}{3}$.

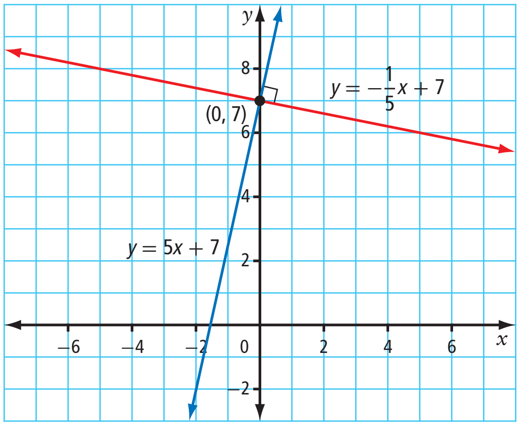
Your Turn

A line is perpendicular to $4x + y - 12 = 0$ and passes through $(8, -6)$. Write the equation of the line in either slope-intercept form or general form.

Key Ideas

- Parallel lines have the same slope and different intercepts. Vertical lines are parallel to each other, as are horizontal lines, if they have different intercepts.
- Perpendicular lines have slopes that are negative reciprocals of each other. A vertical line with an undefined slope and a horizontal line with a slope of zero are also perpendicular.
- The properties of parallel and perpendicular lines can give information about the slopes. Knowing the slopes can help you develop an equation.

  A line perpendicular to $y = 5x + 7$ has the same y-intercept. The line $y = 5x + 7$ has a slope of 5 and a y-intercept of 7. The perpendicular line has a slope of $-\frac{1}{5}$ and a y-intercept of 7. So, the equation of the perpendicular line is $y = -\frac{1}{5}x + 7$.



Check Your Understanding

Practise

1. For a line with each slope, state the slope of a line parallel to it. What is the slope of a line perpendicular to it?

a) $m = 5$
b) $m = -7$
c) $m = -\frac{1}{3}$
d) $m = \frac{6}{7}$
e) $m = 0.5$
f) $m = -0.75$
g) $m = 0$
h) m is undefined.

2. State the slopes of lines that are parallel and lines that are perpendicular to each linear equation.

   a) $y = \frac{3}{7}x + 4$
   b) $y = -x + 9$
   c) $3x + y - 5 = 0$
   d) $2x + y + 11 = 0$
   e) $3x - 2y + 6 = 0$
   f) $5x + 4y - 20 = 0$
   g) $y = 7$
   h) $x + 3 = 0$

3. Consider the line joining points P(−6, 9) and Q(−2, 1).
   a) What is the slope of a line parallel to this line?
   b) What is the slope of a line perpendicular to this one?

4. For each pair of slopes, what is the value of n if the lines are parallel? What is the value of n if the lines are perpendicular?

   a) $\frac{n}{10}, 2$
   b) $\frac{24}{n}, -\frac{1}{3}$
   c) $\frac{3}{2}, \frac{n}{9}$
   d) $\frac{3}{n}, -\frac{7}{2}$

5. Identify whether each pair of lines is parallel, perpendicular, or neither. Explain how you know.

   a) $y = -6x$
      $y = 6x + 1$
   b) $y = \frac{1}{5}x + 3$
      $y = -5x - 4$
   c) $y = -x + 8$
      $x + y = 2$
   d) $y = \frac{3}{4}x + 5$
      $4x + 3y = 6$
   e) $5x + 2y - 10 = 0$
      $2x + 5y + 10 = 0$
   f) $3x - 4y - 24 = 0$
      $4x + 3y - 12 = 0$

6. Write an equation of a line that is parallel to each line and passes through the given point.

   a) $y = 2x + 5$, $(1, -6)$
   b) $y = -3x + 7$, $(-2, 5)$
   c) $5x + y - 1 = 0$, $(3, -8)$
   d) $6x - 2y + 10 = 0$, $(3, -5)$
   e) $y = 8$, $(3, 4)$
   f) $x - 5 = 0$, $(-1, -8)$

7. Write an equation of a line that passes through each point and is perpendicular to each line.

a) $y = 3x + 5$, $(9, 5)$

b) $y = -4x + 7$, $(-12, -7)$

c) $x + 3y + 4 = 0$, $(5, -9)$

d) $4x - 3y - 6 = 0$, $(-2, -1)$

e) $x - 2 = 0$, $(-3, 7)$

f) $y = -5$, $(4, -6)$

Apply

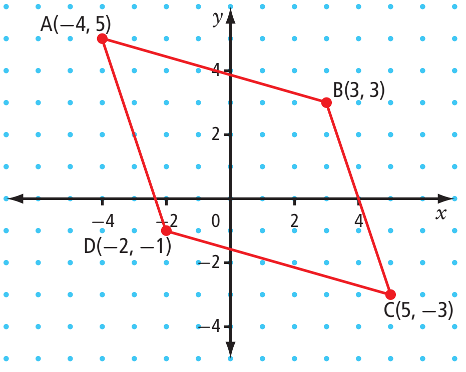
8. Sheldon was asked if line segment AB with A(−9, 2) and B(−3, 4) is parallel to line segment CD with C(−7, −7) and D(1, −3). He sketches a graph of the two line segments and concludes that they appear parallel.

a) Is it correct to assume from a sketch that the two line segments are parallel? Explain.

b) How could you prove that two lines segments are parallel?

c) Is line segment AB parallel to line segment CD? Justify your answer.

9. Is quadrilateral ABCD with vertices A(−4, 5), B(3, 3), C(5, −3), and D(−2, −1) a parallelogram? Justify your answer. Hint: A parallelogram is a quadrilateral with opposite sides parallel.



10. Write the general form equation, $Ax + By + C = 0$, of a line that passes through (7, 5) and is

a) parallel to the x-axis

b) perpendicular to the x-axis

11. Line L_1 passes through points P(n, 4) and Q(1, −2). Line L_2 passes through points R(4, 3) and S(1, 5).

a) What is the value of n if the lines are parallel?

b) If the lines are perpendicular, what is the value of n?

12. Prove that $\triangle ABC$ with vertices $A(-3, 5)$, $B(4, 7)$, and $C(-1, -2)$ is a right triangle.

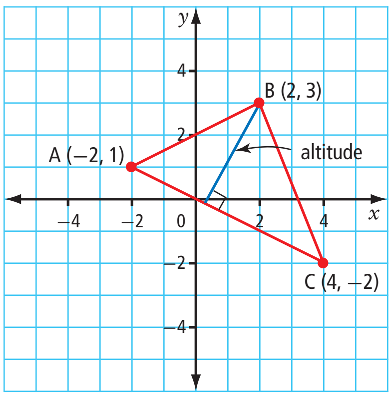
13. Determine an equation representing each line.

a) parallel to $5x + y + 4 = 0$ with a y-intercept of -6

b) perpendicular to $x + 5y - 10 = 0$ with the same y-intercept as $y = 4x - 3$

c) perpendicular to $5x + 4y - 2 = 0$ with the same x-intercept as $3x - 5y = 15$

14. Triangle ABC has vertices $A(-2, 1)$, $B(2, 3)$, and $C(4, -2)$. Write the equation of the line containing the altitude from point B to side AC.

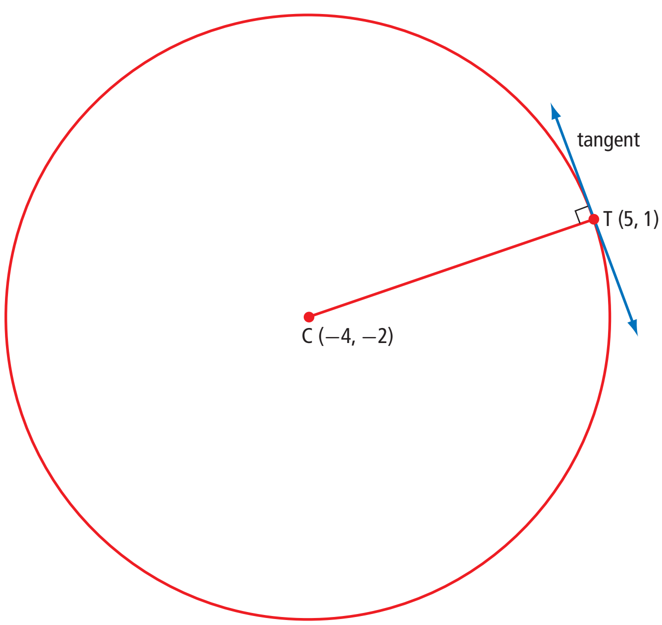


Did You Know?

An altitude is a line segment drawn from a vertex perpendicular to the opposite side.

15. The line through $(5, n)$ and $(1, -2)$ is parallel to the line $3x + 2y - 1 = 0$. What is the value of n?

16. The centre of a circle is located at $C(-4, -2)$ on a coordinate grid. Write the equation of a tangent at point $T(5, 1)$.

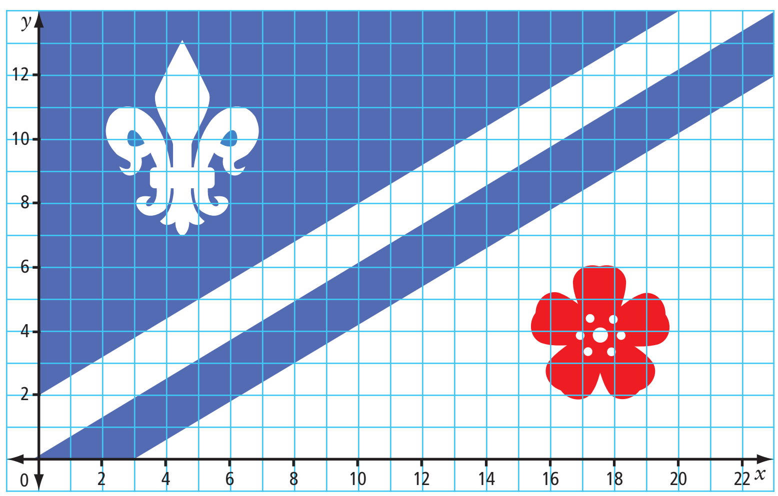


Did You Know?

A tangent is a line that touches a circle at exactly one point. It is perpendicular to the radius at that point.

17. You can monitor your heart rate while you exercise. The Karvonen formula states that the target heart rate during an aerobic workout should be between $H = 0.7(220 - A)$ and $H = 0.8(220 - A)$. In the equations, H represents your target heart rate and A represents your age. Predict whether a graph of these equations would show parallel lines. Justify your answer.

18. In 1982, the French-Canadian Association of Alberta adopted the Franco-Albertan flag. Suppose a coordinate grid is laid over a replica of the flag, with the base along the x-axis and the lower left corner of the flag at the origin. The three parallel lines in the flag pass through the points (20, 14), (23, 14), and (23, 12). Write an equation representing each line.

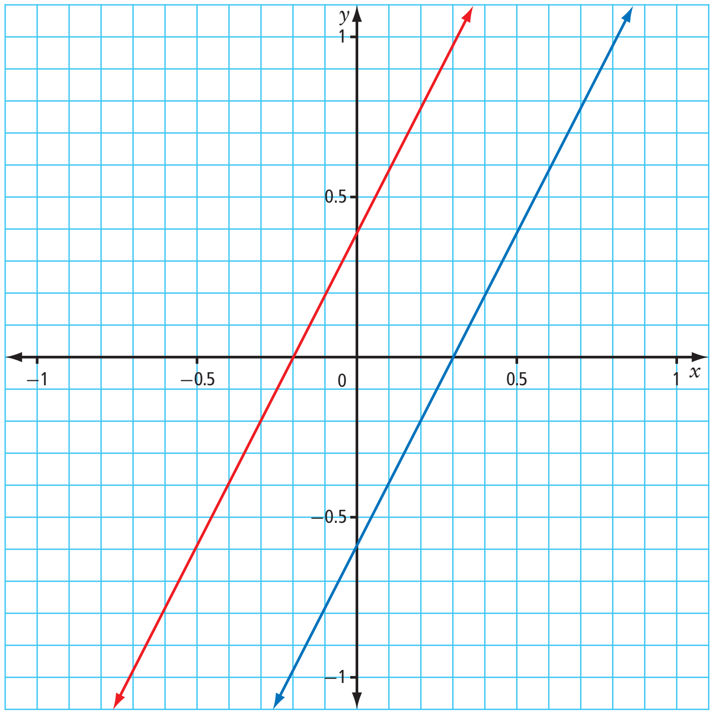


Extend

19. What is the value of n if the graphs of $nx + 4y + 3 = 0$ and $5x - 2y + 6 = 0$ are parallel?

20. The lines $6x - ny + 5 = 0$ and $x + 2y + 4 = 0$ are perpendicular. What is the value of n?

21. What is the shortest distance between the two lines in the graph? Explain your reasoning.



22. Two vertices of right triangle ABC are A(−2, 6) and C(7, 3). If the right angle is at vertex A and vertex B is on the x-axis, identify the coordinates of point B.

23. The lines $nx + 12y - 2 = 0$ and $3x + ny + 6 = 0$ are parallel. What are the possible values of n?

24. Determine the value of n if the lines $nx - 2y + 8 = 0$ and $3x + ny + 6 = 0$ are perpendicular.

Create Connections

25. Is the following statement always true, sometimes true, or never true? "The slopes of perpendicular lines are always negative reciprocals of each other." Explain your reasoning.

26. Suppose you want to determine whether two lines are parallel. Which form of an equation would you prefer to use? Why?

7 Review

7.1 Slope-Intercept Form, pages 340–356

1. What are the slope and y-intercept of each line?

a) $y = -5x + 6$

b) $5x - 6y + 12 = 0$

2. Write the equation of the line with each slope and y-intercept. Explain your steps.

a) slope $= -\frac{4}{5}$, y-intercept $= 6$

b) slope $= 0$, y-intercept $= -8$

3. Kayla is in grade 10. She wants to have longer hair for her graduation. Her hair grows at a constant rate. If it is 24.0 cm long today, in 30 days, it will be 25.2 cm long.

a) Sketch a straight-line graph representing the rate at which Kayla's hair grows.

b) What is the slope of the line? What does it represent?

c) What is the y-intercept? What does the y-intercept represent?

d) Write an equation in slope-intercept form that describes how Kayla's hair length, L, in centimetres, grows each day, d.

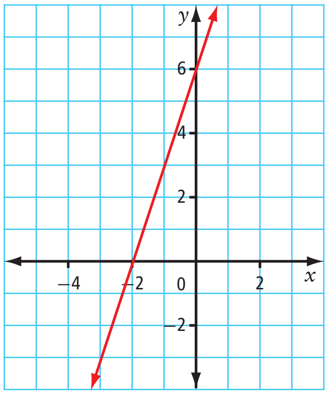
e) Predict the length of Kayla's hair at graduation if she does not get it trimmed over the next two years.

4. Explain how you could use the slope and y-intercept to graph the line $2x + 5y - 20 = 0$.

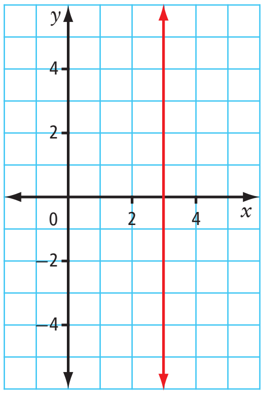
7.2 General Form, pages 357–369

5. State the intercepts of each line as ordered pairs. Then, write the equation of each line in general form.

a)



b)



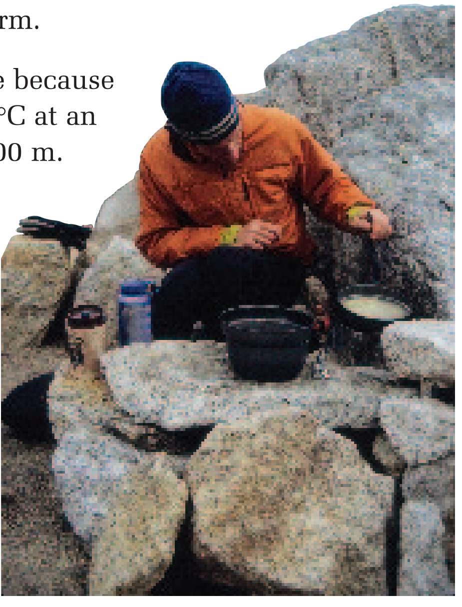
6. What are the x-intercepts and y-intercepts of each line? Use the intercepts to sketch a graph of each line.
   a) $5x - 2y - 20 = 0$
   b) $y = 4$

7. At a community dance, students sell country food as a fundraiser. They sell bannock wheels for \$2 each and buffalo burgers for \$3 each. Sales, at the end of the day, total \$600.
   a) Write an equation in general form that represents the food sales for the day.
   b) What does each intercept represent?
   c) What are the domain and range?
   d) Suppose the students sell 135 bannock wheels. How many buffalo burgers would they sell?

7.3 Slope-Point Form, pages 370–382

8. Use slope-point form to write an equation of a line through each point with the given slope. Express each answer in slope-intercept form and in general form.
   a) $(-2, 1)$ and $m = -4$
   b) $(8, -3)$ and $m = \frac{1}{2}$

9. Explain how you would use slope-point form to write an equation of a line through points (1, 10) and (3, 2). Then, express the equation in slope-intercept form and in general form.



10. At higher altitudes, water boils at a lower temperature because the air pressure is lower. Suppose water boils at 96.5 °C at an altitude of 1000 m, and at 93.0 °C at an altitude of 2000 m.
   a) What is the slope of the line representing the data? What rate does the slope represent?
   b) Write an equation of the line. Express your answer in slope-intercept form or in general form.
   c) Mountain climbers need to adjust their ingredients and cooking techniques when cooking at higher altitudes. What is the boiling temperature of water at an elevation of 4000 m?

7.4 Parallel and Perpendicular Lines, pages 383–395

11. A line joins points R(5, -3) and S(7, 2). What is the slope of a line

a) parallel to line RS?

b) perpendicular to line RS?

12. The slopes of two lines are $-\frac{3}{2}$ and $\frac{18}{n}$. What is the value of n if the two lines are

a) parallel?

b) perpendicular?

13. The equations of five lines are given. Which pairs of lines meet each criterion? Justify your reasoning.

Ⓐ $x + 2y - 6 = 0$

Ⓑ $y = 2x - 3$

Ⓒ $2x + y - 3 = 0$

Ⓓ $y = -\frac{1}{2}x - 3$

Ⓔ $x + 2y + 6 = 0$

a) parallel lines

b) perpendicular lines

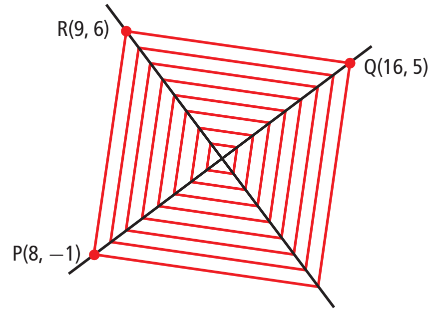
c) equivalent lines

14. Write an equation of a line through (-1, 2) and parallel to $y = -7$. Explain your reasoning.

15. Write an equation of a line through (-6, 7) and perpendicular to $3x + 4y - 12 = 0$. Explain your reasoning.

16. How could you determine the equation of a line perpendicular to $2x + 5y + 10 = 0$ with the same x-intercept as $3x - 2y = 12$?

17. Suppose a spider weaves sticky threads to catch its prey and non-sticky radial lines to allow it to cross its own web. A model of a spider web is shown. Determine an equation of the radial line through R(9, 6) and perpendicular to line PQ.

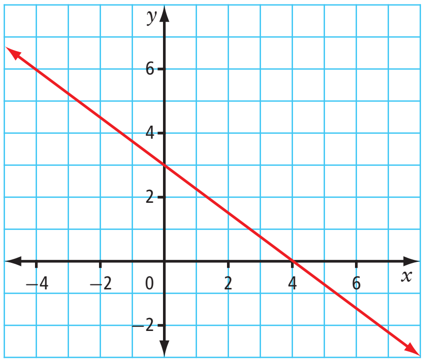


7 Practice Test

Multiple Choice

For #1 to #6, choose the best answer.

1. What are the slope and y-intercept of the graph shown?



A slope: $-\frac{4}{3}$, y-intercept: (0, 4)

B slope: $-\frac{3}{4}$, y-intercept: (0, 4)

C slope: $-\frac{3}{4}$, y-intercept: (0, 3)

D slope: $-\frac{4}{3}$, y-intercept: (0, 3)

2. What are the intercepts of the line $2x - 3y = -6$?

A x-intercept: (3, 0), y-intercept: (0, −2)

B x-intercept: (−3, 0), y-intercept: (0, 2)

C x-intercept: (−3, 0), y-intercept: (0, 3)

D x-intercept: (3, 0), y-intercept: (0, −3)

3. The slope and y-intercept of the line $7x + 2y - 10 = 0$ are

A slope: $\frac{7}{2}$, y-intercept: (0, 5)

B slope: $-\frac{7}{2}$, y-intercept: (0, 5)

C slope: $\frac{7}{2}$, y-intercept: (0, −5)

D slope: $-\frac{7}{2}$, y-intercept: (0, −5)

4. The equation $y = -\frac{3}{4}x + 2$ expressed in general form is

A $3x - 4y - 2 = 0$ **B** $3x + 4y - 2 = 0$

C $3x + 4y - 8 = 0$ **D** $3x - 4y + 8 = 0$

5. A line has a slope of −2 and passes through the point (3, −1). When the equation of the line is written in the form $y = mx + b$, the value of b is

A 5 **B** 1 **C** −1 **D** −5

6. Which line is parallel to the line $2x + 4y - 8 = 0$?

A $y = -\frac{1}{2}x + 5$ **B** $y = \frac{1}{2}x - 1$

C $y = -2x + 3$ **D** $y = 2x - 7$

Short Answer

7. a) If two lines are perpendicular, what is the relationship between their slopes?

   b) If two lines with slopes of $\frac{6}{n}$ and $-\frac{3}{2}$ are perpendicular, what is the value of n?

8. Express the equation of the line passing through the points (2, 4) and (2, −4) in general form.

9. Tickets for a school play were \$10 for adults and \$6 for students. Total ticket sales were \$2900.

   a) Write an equation that represents the ticket sales.

   b) How many adult tickets were sold if 275 students bought tickets?

10. Four equations are listed.

    Ⓐ $y = -3x + 4$
    Ⓑ $y + 10 = -3(x - 2)$
    Ⓒ $6x + 2y - 8 = 0$
    Ⓓ $y + 8 = -3(x - 4)$

    Which equations represent the same line? Justify your answers.

11. Jacob rides his mountain bike at 15 km/h along a 30-km trail. His distance, D, in kilometres, from the end of the trail at time t hours may be modelled by the equation $D = 30 - 15t$.

    a) What does the D-intercept of a graph of the equation represent?

    b) Felicia rides at the same speed as Jacob. She starts 2 km behind Jacob on the same trail. Explain why she will never catch up to Jacob. Provide support for your explanation.

12. What is the equation of a line through (3, −1) and parallel to the line $5x + y - 1 = 0$? Express your answer in general form, $Ax + By + C = 0$. Explain your steps.

Extended Response

13. Explain three different strategies you could use to graph the line $y - 4 = -2(x + 3)$, without creating a table of values.

14. When a railway car is unloaded at a grain elevator, the mass of wheat, W, in tonnes, remaining after t minutes decreases at a constant rate as shown in the table.

Time, t (min)	Mass of Wheat Remaining, W (tonnes)
0	88.0
6	61.6
12	35.2

a) Use the data in the table to sketch a straight-line graph.

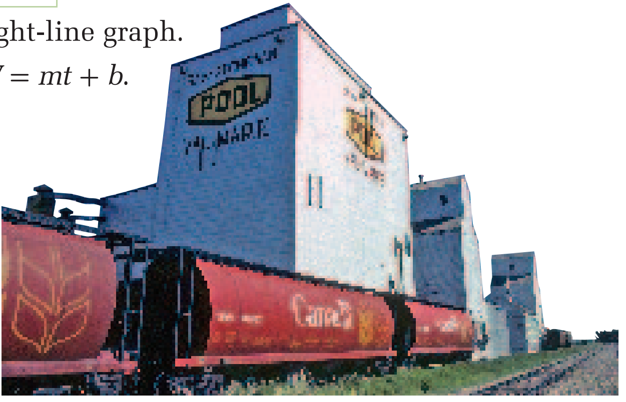
b) Write the equation of the line in the form $W = mt + b$.

c) Identify the intercepts. What does each intercept represent?

d) What is the slope of the line? What does the slope represent?

e) What are the domain and range?

f) How long would it take for half of the wheat to be emptied?

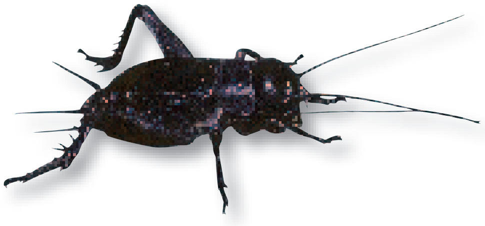


15. The relationship between air temperature and how fast a male cricket chirps is linear. A group of biology students conducted the following experiment. The students counted the number of chirps per minute by a cricket at various locations within the school. In a room where the air temperature was 14 °C, the cricket chirped 70 times per minute. In the cafeteria, the air temperature was 21 °C. The cricket chirped 119 chirps per minute.

a) Write a linear equation relating the number of cricket chirps per minute, n, to the air temperature, T, in degrees Celsius. Express the equation in slope-intercept form.

b) Sketch a graph of the linear equation. Explain your method.

c) In the boiler room, the cricket chirped 168 chirps per minute. What is the temperature in the boiler room?



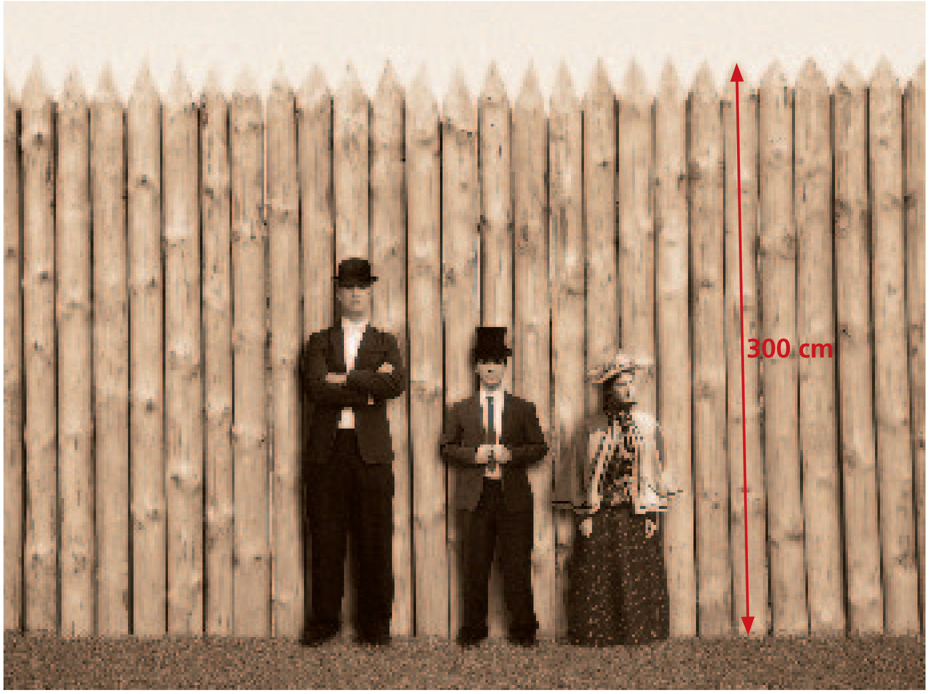
3 Unit Connections

Unit 3 Project

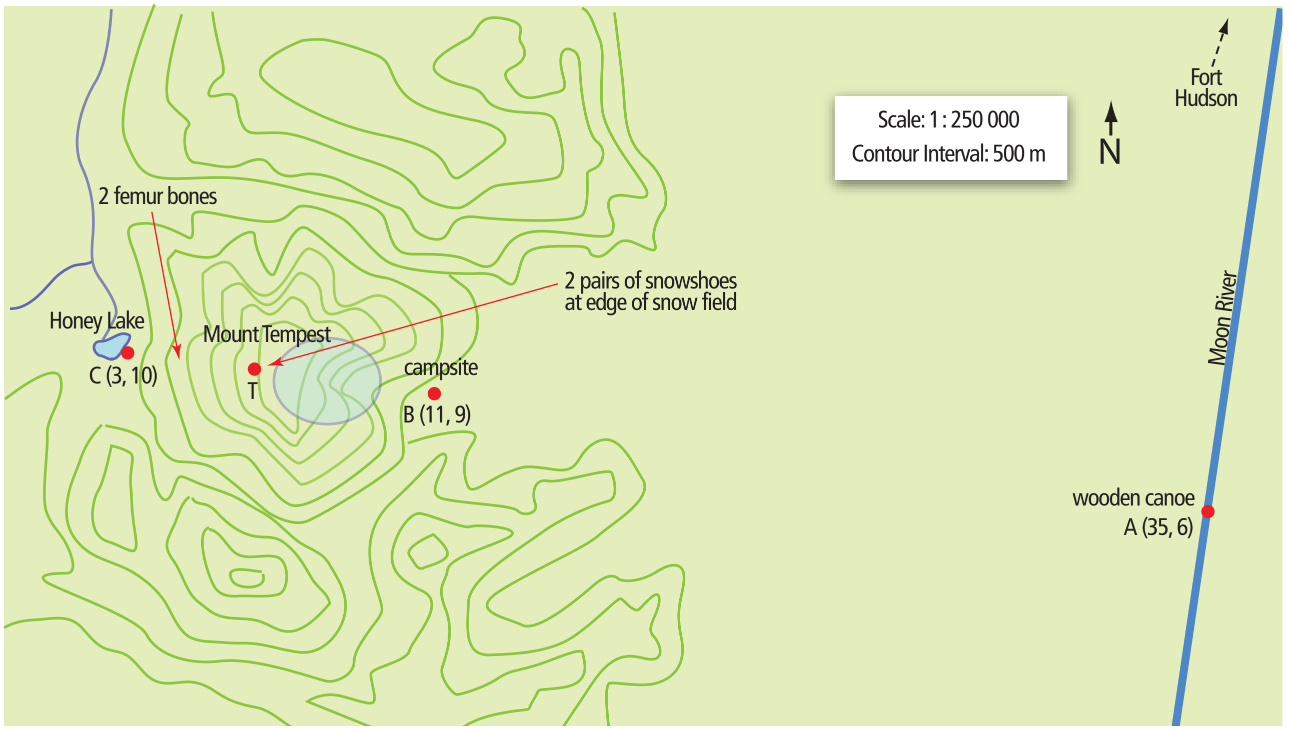
This project is fictitiously set in western Canada. The scenario, places, and people involved are entirely fictitious.

In 1899, Sam Mason, Billy Pratt, and Kittie Carmack left Fort Hudson and paddled by canoe down Moon River to retrieve a stash of gold they had left at Honey Lake. The trio vanished and the gold was never found. You are a forensic archaeologist newly hired to investigate their disappearance.

This photograph of the trio was taken outside Fort Hudson prior to their departure.



Part of a wooden canoe from that period was recently unearthed along the banks of Moon River. This new evidence allows you to retrace the route that Sam, Billy, and Kittie may have taken. A contour map of the region is shown on the following page. The coordinates of three points are labelled.



Part A: Collecting Evidence

The previous archaeologist on the case provided you with the following information.

- The three gold seekers left Fort Hudson on April 15, 1899. They had enough food and camping equipment for ten days.
- The trip by canoe from Fort Hudson to point A takes three days.
- There is evidence that the three gold seekers climbed over Mount Tempest. The east side of Mount Tempest has a snow field that has to be crossed using snowshoes. After climbing the snow field, the gold seekers would have left their snowshoes behind for the return journey. Two pairs of snowshoes are still at the top of Mount Tempest.
- Fort Hudson is at the same elevation as point A. According to fort records, the average daily temperature during April 1899 was 4 °C.
- On the west side of Mount Tempest, two femur (thigh) bones were found. They measure 40.9 cm and 56.1 cm in length.
- When the bones were found, 98.6% of the carbon-14 remained.

Part B: Analysing the Evidence

Use what you have learned in the unit and the information in the Math Toolkit to answer the questions at the bottom of the page.

Math Toolkit

- As altitude increases, air temperature decreases at a rate of 0.0064 °C/m.
- A linear approximation of the age of a bone less than 500 years old is $A(p) = -85.4p + 8541.1$, where $A(p)$ is the age, in years, at p percent of carbon-14 remaining.
- The formula for a male's height, $M(x)$, in centimetres, at a femur bone length x, in centimetres, is $M(x) = 2.38x + 61.41$.
- The formula for a female's height, $H(x)$, in centimetres, at a femur bone length x, in centimetres, is $H(x) = 2.47x + 54.10$.
- The table shows general hiking speeds for several climbing slopes.

Slope of Climb	Average Hiking Speed (km/h)
$0.0 \le m < 0.1$	4.0
$0.1 \le m < 0.2$	2.0
$0.2 \le m < 0.3$	1.5
$0.3 \le m < 0.4$	1.3
$0.4 \le m < 0.5$	1.1
$0.5 \le m < 0.6$	1.0
$0.6 \le m < 0.7$	0.9

- The speed for downhill travel is about the same as the flat ground speed, 4.0 km/h.

1. How long would it take the gold seekers to travel from point A to point B?

2. Is the path from point A passing through point B to point C a straight line? Justify your answer.

3. Is the path from point A to point B level? Explain how you know.

4. a) How many metres above points B and C is the top of Mount Tempest, point T?

b) What was the mountain terrain like? Explain your answer in terms of the slope or angle of inclination.

5. a) How long would it take them to travel from point B to the top of Mount Tempest?
   b) How long would the trip down to Honey Lake, point C, take?

6. At that time of year, what were the approximate temperatures at the base and top of Mount Tempest? How might the temperatures affect the travellers? Explain your reasoning.

7. Use the photograph taken at Fort Hudson to estimate the height of each gold seeker. Blueprints show that the wooden walls of the Fort were 300 cm tall. Express your answers to the nearest tenth of a metre.

8. a) How old were the bones when they were found? Verify that this age is within the range for the relation that you used.
   b) Whose bones do you think they are? Why?

9. The portion of Moon River shown in the contour map is perpendicular to path AB.
   a) What is the equation of the line from point A to point B?
   b) Write an equation of a line in general form that contains this section of Moon River.
   c) An additional clue is found along the same straight section of Moon River. The clue is located at the coordinates $(n, 30)$. What is the value of n?

Part C: Making Conclusions

Create an explanation of what happened to the three gold seekers. Make sure your explanation is consistent with your answers to the questions in Part B. As part of your explanation, include an additional clue of your choice found along the banks of the Moon River. This clue may explain what happened to the gold treasure.

Your presentation may be in the form of a story, a digital presentation, a video, a song, or a play.

Unit Review

Chapter 6 Linear Relations and Functions

1. Sketch and label a graph that could represent each scenario. Describe any restrictions on the domain and state whether the relation is discrete or continuous.

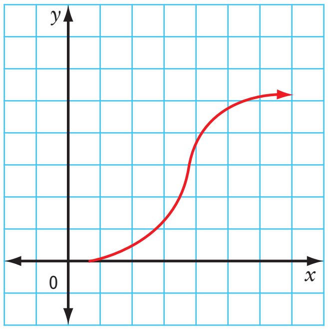
a) The speed versus time of a person riding a bike up a hill and then down the other side.

b) The cost of purchasing bags of peanuts at a baseball game.

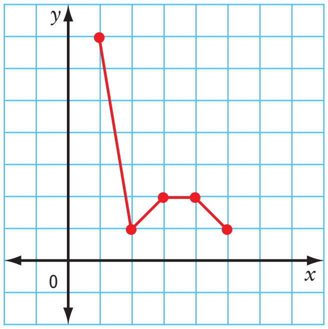
c) The height of the valve on your bicycle tire as you ride down the street.

2. Write a short story that could be represented by each graph.

a)



b)



3. Rachel was hired to work as a tree planter on the south coast of BC. She began with a wage of $125 per day.

a) Create a table of values of her earnings for at least five weeks.

b) Is the relation a function or a non-function? Explain your choice.

c) Is the relation considered discrete or continuous? Explain why.

d) Write an equation that relates the number of weeks worked to Rachel's earnings.

4. Determine whether each set of ordered pairs represents a function.

a) $\{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$

b) $\{(1, 5), (3, 5), (5, 5), (7, 5), (9, 5)\}$

c) $\{(-3, 5), (0, 0), (3, 5), (6, 10), (9, 15)\}$

d) $\{(0, -1), (0, -2), (0, -3), (0, -4), (0, -5)\}$

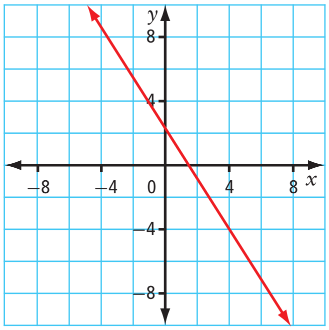
5. A human heart can pump 18 L of blood in 3.6 min and 37.5 L of blood in 7.5 min.

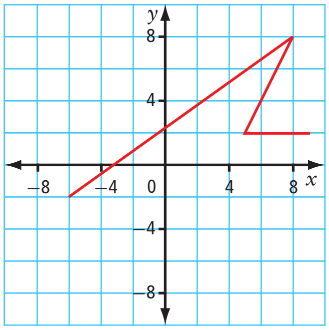
a) Identify the independent and the dependent variables in this situation.

b) Draw a graph, then determine the slope of the resulting line.

c) Explain the meaning of slope in terms of rate of change.

6. Classify each graph as a function or a non-function.

a) 

b) 

7. Determine the slope of each line segment in #6b).

Chapter 7 Linear Equations and Graphs

8. Determine the slope and y-intercept of each line.

a) $y = \frac{1}{2}x + 6$

b) $3x + 5y = 10$

c) $x - 3y = 0$

d) $y + 1 = 0$

9. For each line, write the equation of the line in slope-intercept form. Then, graph and label the line on a grid.

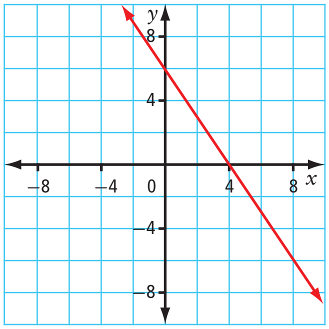
a) slope is -5; line passes through the point $(2, 4)$

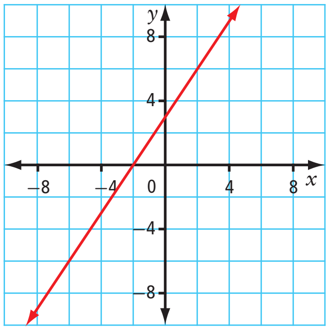
b) $y + 4 = 0$

c) x-intercept is 2; y-intercept is -5

d) slope is $\frac{3}{2}$, line pass through the point $(-4, -3)$

10. Write the intercepts of each line as coordinate pairs. Then, determine the equation of the line using the intercepts. Express the equations in general form.

a) 

b) 

11. Determine the equation, in general form, given the characteristics for each linear relation.

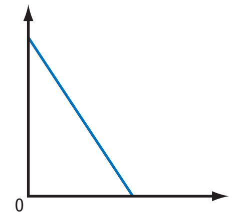
a) passing through the point $(1, 6)$; parallel to the y-axis

b) perpendicular to the line $5x - 3y + 2 = 0$; passing through $(-2, -3)$

3 Unit Test

Multiple Choice

For #1 to #5, choose the best answer.



1. Which of the following is not a possible situation describing the graph.

A Temperature of water in an ice tray after being put in a freezer

B Volume of fuel remaining in the tank of a car driving on the highway

C Speed of a cyclist going down a hill

D Distance from home as you walk home from school

2. Which of the relations listed are functions?

i) $\{(3, 1), (6, 2), (3, 3)\}$

ii) $\{(-2, 7), (3, 5), (2, 5), (1, -7)\}$

iii) $\{(5, 1)\}$

iv) $\{(1, 4), (2, 4), (3, 4), (4, 4)\}$

A i, ii, iii **B** i, ii, iv **C** i, iii, iv **D** ii, iii, iv

3. The slope of a line passing through the points $(-3, 3)$ and $(11, 10)$ can be represented by the expression

A $\frac{1}{7}$ **B** $\frac{1}{2}$ **C** $\frac{7}{8}$ **D** $\frac{11}{8}$

4. If the lines $5x - 2y + 12 = 0$ and $y = \frac{k}{4}x - 3$ are parallel, the value of k is

A -10 **B** -10 **C** 5 **D** 10

5. A line is parallel to $y = 2x + \frac{5}{2}$. It passes through the point $(1, 5)$. The equation of the line is

A $2x - y + 3 = 0$

B $2x + y + 3 = 0$

C $2x - y - 3 = 0$

D $-2x + y - 3 = 0$

Numerical Response

Complete the statements in #6 and #7.

6. The slope of a graph of $6x - 2y + 11 = 0$ would be ■.

7. The equation of the line that passes through $(-3, -1)$ and $(7, -6)$ can be written in the form $Ax + By + C = 0$. The value of B is ■.

Written Response

8. The mark on an exam could be a function of the number of hours you study. Suppose you could achieve a mark of 32% without studying at all and this mark increases by 8% for each hour you study.

a) Identify the domain for this situation.

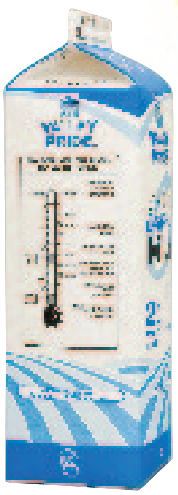
b) Create a table of values representing the hours studied and the predicted mark for the first 4 h of studying.

c) Write the linear equation that represents this function.

d) According to the function, how many hours must you study in order to achieve a mark of 100%?

e) Is a linear model a valid model for this situation? Explain your reasoning.

9. Determine the equation of a line that is perpendicular to $y = \frac{x}{2} + \frac{3}{2}$ and has the same y-intercept as the line $4x + 2y + 12 = 0$. Express the equation in the form $Ax + By + C = 0$.



10. A milk container has the instruction *Keep Refrigerated* printed on it. The function d gives the number of days the milk will last when stored at different temperatures. The temperatures are expressed in degrees Celsius. The output is $d(-7) = 24$, $d(-1) = 10$, $d(5) = 5$, $d(10) = 2$, $d(15) = 1$, $d(20) = 0.5$, and $d(25) = 0.5$.

a) Create a graph relating the number of days the milk will last to the temperature at which it is stored.

b) Estimate the number of days that milk will last if it is stored at 1 °C.

11. Joshua is taking golf lessons. The cost can be modelled by the relation $C = 30t + 25$, where C is the total cost, in dollars, and t is the number of hours of lessons he takes. Assume that lessons can end on the half hour.

a) Explain why this relation is a linear function.

b) Is this function discrete or continuous? Explain why.

c) List five ordered pairs of this function.

d) Suppose Joshua's lessons cost \$130. How long were the lessons?

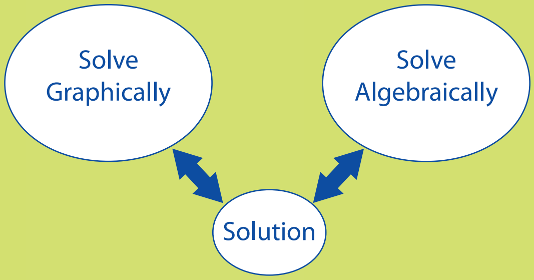
UNIT 4 Systems of Equations

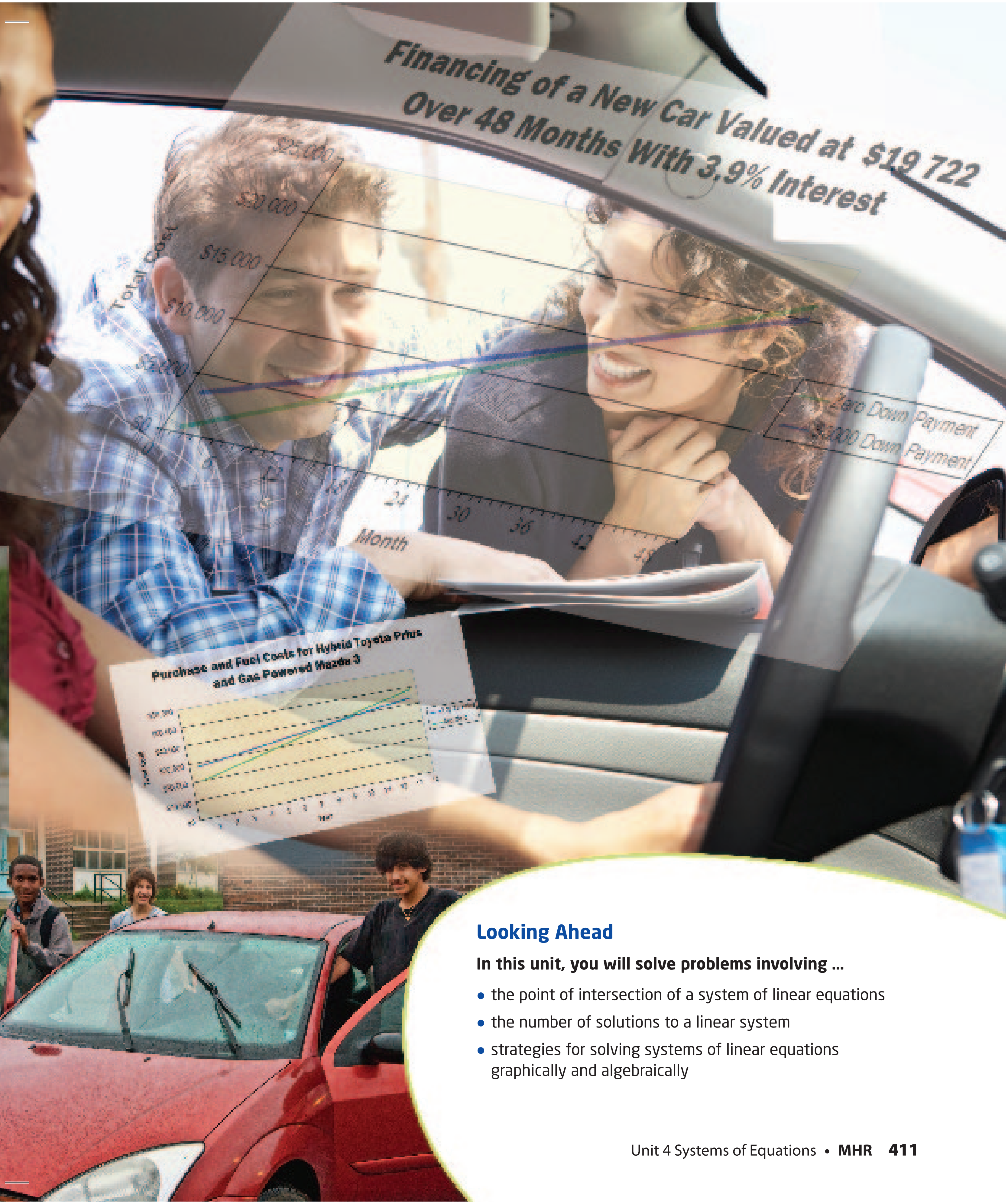
Some day, you and your parents may need to make decisions about buying a new or used car. Should you purchase or lease the car? Should you buy an electric, a hybrid, or a gasoline-powered vehicle? How much money should you borrow at a specific interest rate? Decisions often affect several important aspects of your life.

Many decisions involve two relations where each can be modelled using a linear equation. Analysing a system of two linear equations allows consumers and businesses to make well-informed decisions. In this unit, you will learn how to identify the solution to a system of linear equations shown on a graph. You will learn several algebraic methods for solving systems of linear equations. You will also learn strategies to help you determine when to use each method.

Your Systems of Equations Organizer

You can use this organizer to see how the concepts in this unit are connected. You will see this organizer on the first page of each chapter. The concepts covered in the chapter will be highlighted.





Looking Ahead

In this unit, you will solve problems involving …

- the point of intersection of a system of linear equations
- the number of solutions to a linear system
- strategies for solving systems of linear equations graphically and algebraically

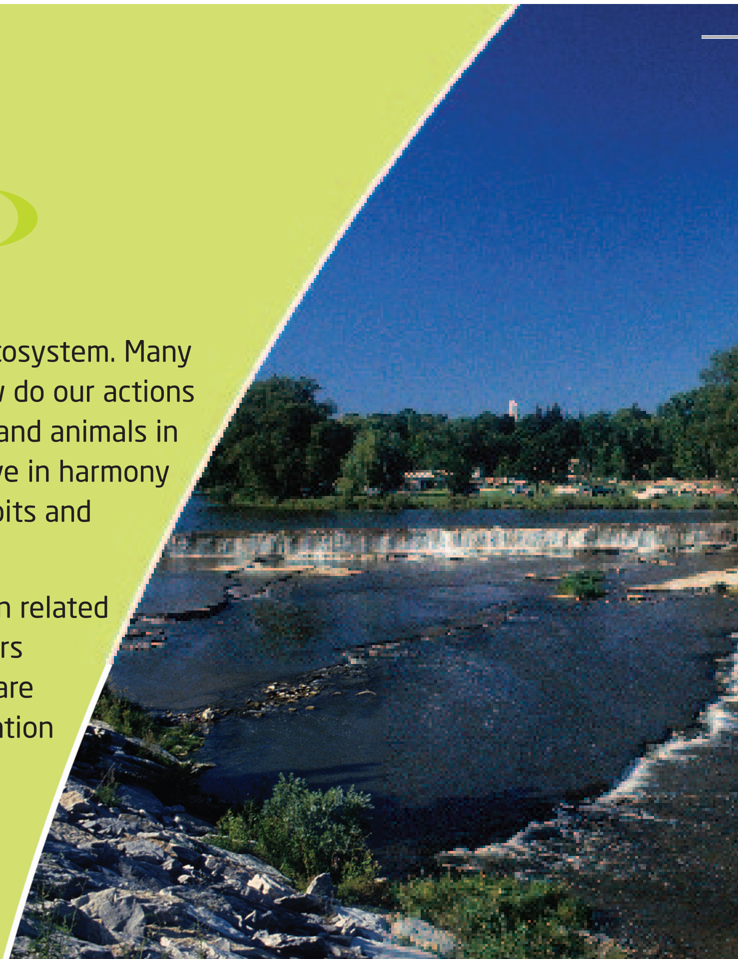
Unit 4 Project

Water Conservation

Water is one of the most precious resources in our ecosystem. Many of us take the water we use every day for granted. How do our actions affect the environment we live in? What about the plants and animals in our ecosystem? Human survival depends on our ability to live in harmony with the environment. What can we do to improve some habits and conserve water?

In the Unit 4 project, you will collect and analyse information related to water use, wildlife, and retrofitting. The term *retrofit* refers to changing or replacing fixtures in a house with ones that are better for the environment. You will then prepare a presentation that convinces people to make changes to help reduce water use.

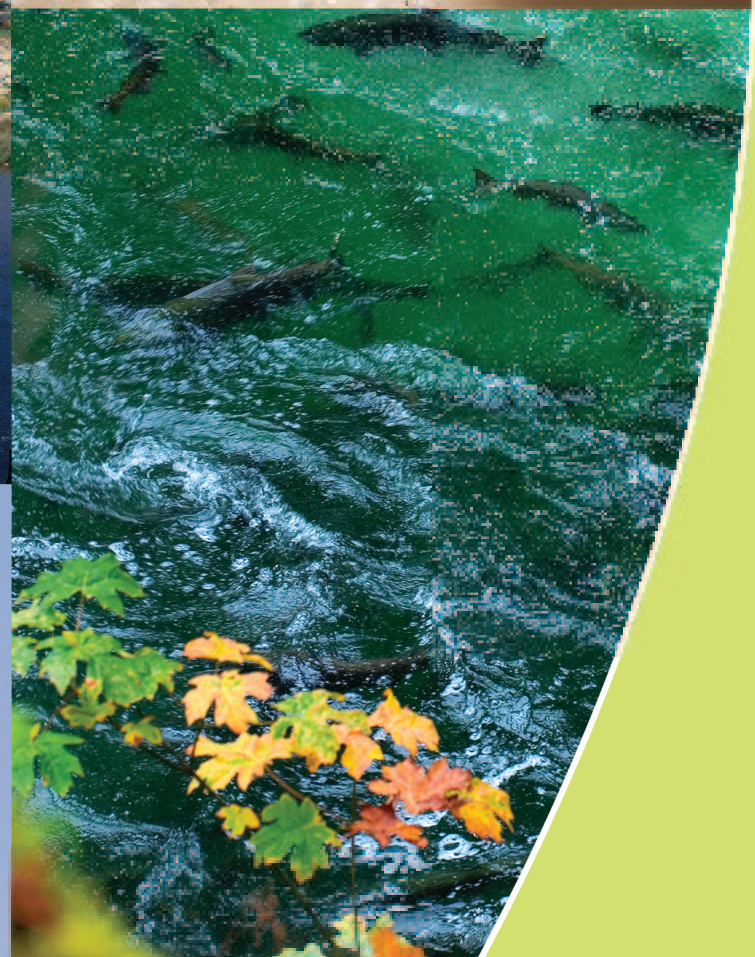
Unit Project questions and activities are included in Chapters 8 and 9. As you move through Chapter 8, you will use graphs to compare information about wildlife and pricing options for retrofitting fixtures in a home. As you move through Chapter 9, you will use several strategies to solve linear systems involving water use and conservation.



While completing your project, you will ...

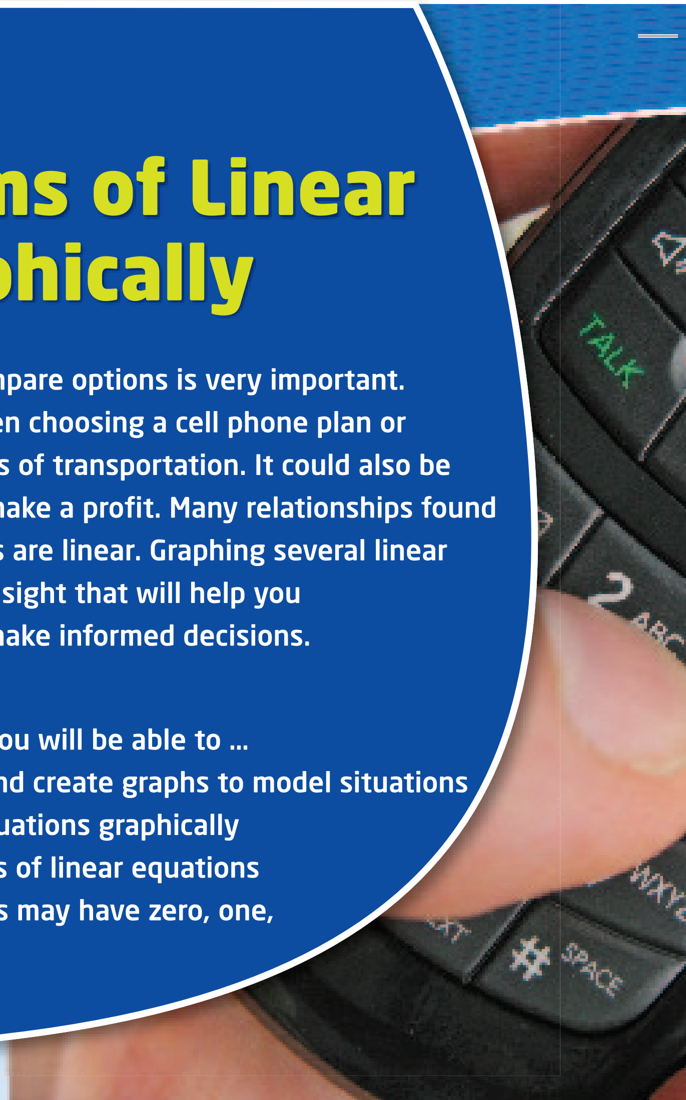
- compare data involving the effect of water contamination on populations of wildlife (Chapter 8)
- use systems of linear equations to analyse the costs of retrofitting (Chapter 8)
- compare the money saved using various water-saving methods in a home (Chapters 8 and 9)
- represent and analyse linear systems involving wildlife that live at a lake, where the lake is slowly being depleted (Chapter 9)






CHAPTER 8

Solving Systems of Linear Equations Graphically



The ability to quantify situations and compare options is very important. This skill could be used, for example, when choosing a cell phone plan or determining travel time for various modes of transportation. It could also be used to predict whether a business will make a profit. Many relationships found in common situations and career contexts are linear. Graphing several linear relations on the same grid may provide insight that will help you analyse situations, solve problems, and make informed decisions.

Big Ideas

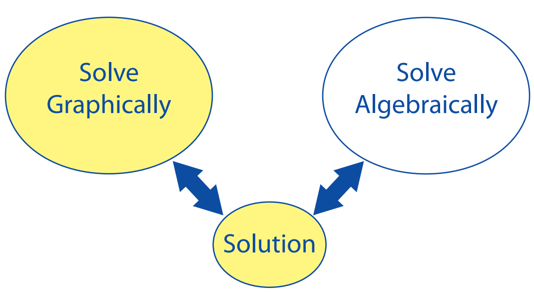
When you have completed this chapter, you will be able to ...

- generate systems of linear equations and create graphs to model situations
- solve two-variable systems of linear equations graphically
- verify solutions to two-variable systems of linear equations
- explain why systems of linear equations may have zero, one, or an infinite number of solutions

Key Terms

point of intersection
system of linear equations
solution (to a system of linear equations)
coincident lines

Your Systems of Equations Organizer



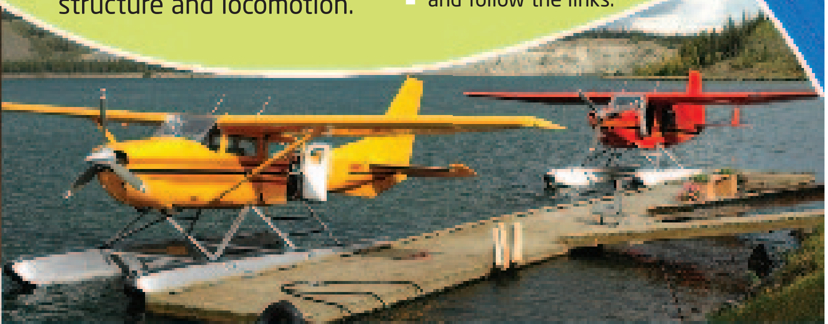
Biologist removing satellite collar.

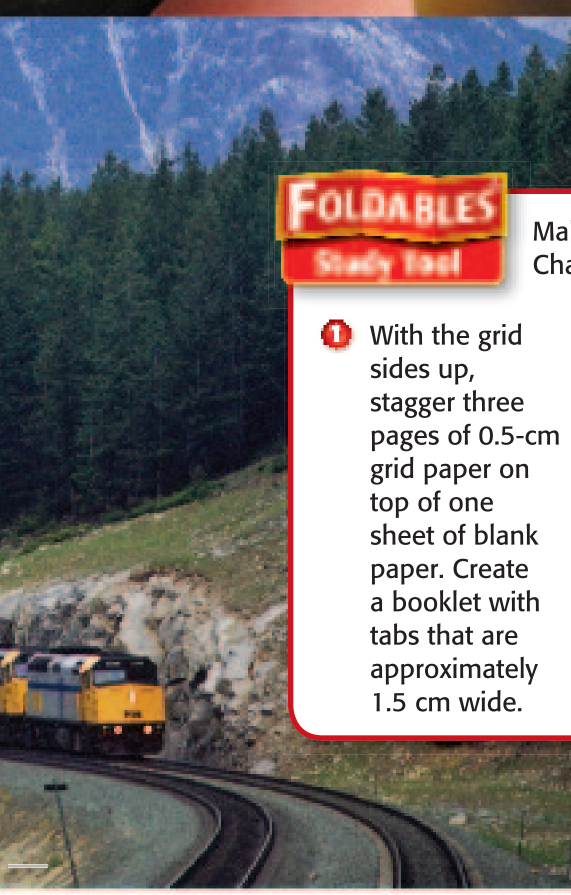
Biologist

Biologists study living things and their environments. For hundreds of years, biologists have used mathematics in many areas. Due to recent advances in technology, biologists have tremendous computing power. The capability to examine vast amounts of data enables biologists to study predator—prey relationships, analyse genetic sequences, and understand animal body structure and locomotion.

Web Link

To learn more about biologists, go to www.mhrmath10.ca and follow the links.

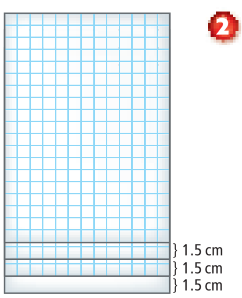




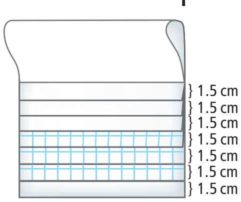
FOLDABLES Study Tool

Make the following Foldable™ to take notes on what you will learn in Chapter 8.

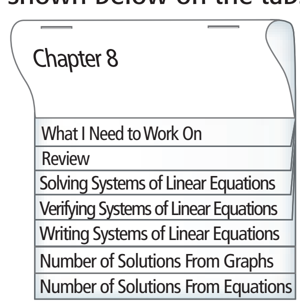
1. With the grid sides up, stagger three pages of 0.5-cm grid paper on top of one sheet of blank paper. Create a booklet with tabs that are approximately 1.5 cm wide.



2. Hold the booklet firmly. Fold the top toward you and line up the tabs. Make a firm crease at the top.



3. Staple the top of the booklet. Write the headings shown below on the tabs.



8.1 Systems of Linear Equations and Graphs

Focus on ...

- explaining the meaning of the point of intersection of two linear equations
- solving systems of linear equations by creating graphs, with and without technology
- verifying solutions to systems of linear equations using substitution

The average cellular phone in North America is used for one and a half years and then replaced. Only 5% of discarded cell phones are recycled. That creates large amounts of waste.

Cell phone communication has increased dramatically in recent years. Many people need to decide whether to buy a cell phone, which type of plan is most beneficial to them, and what to do with a cell phone that they no longer need.

Materials

- 0.5-cm grid paper
- ruler
- coloured pencils

Investigate Ways to Represent Linear Systems

How can you compare and analyse cell phone plan options?

- Plan A costs 30¢ per minute.
- Plan B costs $15 one time plus 10¢ per minute.

1. Create tables of values to show the cost of each option for up to 100 min. Use intervals of 10 min.
2. On the same sheet of 0.5-cm grid paper, graph the data from both tables of values.

Remember to include a scale, labels on the axes, and a title on your graph.

3. **Reflect and Respond** From the graph, explain the cost of each plan as the number of minutes increases.

4. What is the significance of the **point of intersection** of the lines? Explain the connection between this point on the graph and the tables of values you created.

5. Which cell phone plan do you think is a better option? Justify your choice.

point of intersection
- a point at which two lines touch or cross

Link the Ideas

Relations can be represented numerically using a table of values. They can be represented graphically, and verified algebraically. A **system of linear equations** is often referred to as a linear system. It can be represented graphically in order to make comparisons or solve problems. The point of intersection of two lines on a graph represents the **solution** to the system of linear equations.

system of linear equations
- two or more linear equations involving common variables

solution (to a system of linear equations)
- a point of intersection of the lines on a graph
- an ordered pair that satisfies both equations
- a pair of values occurring in the tables of values of both equations

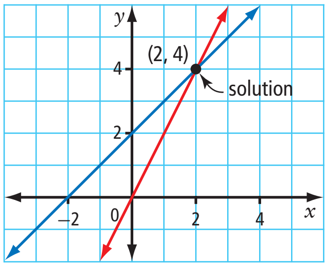
Numerically

x	y
0	0
1	2
2	4
3	6
4	8

x	y
0	2
1	3
2	4
3	5
4	6

There are an infinite number of pairs of values that could be written in each table. The solution, (2, 4), is the only pair that can be written in *both* tables.

Graphically



There are an infinite number of points on each line. The solution point, (2, 4), is the only point that lies on *both* lines.

Algebraic Verification

$y = 2x$
$4 = 2(2)$
$4 = 4$

$y = x + 2$
$4 = 2 + 2$
$4 = 4$

There are an infinite number of ordered pairs that satisfy each equation. Only one ordered pair, (2, 4), satisfies *both* equations.

Example 1 Represent Systems of Linear Equations

Nadia has saved $16, and her sister Lucia has saved $34. They have just started part-time jobs together. Each day that they work, Nadia adds $5 to her savings, while Lucia adds $2. The girls want to know if they will ever have the same amount of money. If so, what will the amount be and on what day?

Solution

The girls use a linear system to model their savings. They represent it numerically and graphically.

Method 1: Use a Table of Values

The amount of money each girl saves is a function of the number of days worked. They each create a table of values to show how their savings will grow:

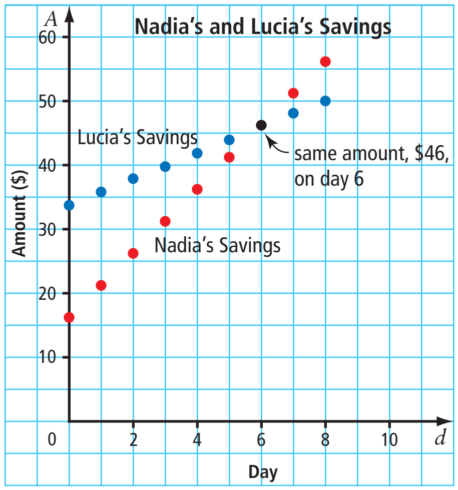
Nadia's Savings	
Day	Amount ($)
0	16
1	21
2	26
3	31
4	36
5	41
6	46
7	51
8	56

Lucia's Savings	
Day	Amount ($)
0	34
1	36
2	38
3	40
4	42
5	44
6	46
7	48
8	50

The tables of values show that both girls will have $46 on day 6. The pair of values, 6 and 46, is the only pair found in *both* tables of values. It represents the only day when the girls will have the same amount of money.

Method 2: Use a Graph

The girls draw graphs on the same grid. This enables them to compare the linear relationships for their savings.

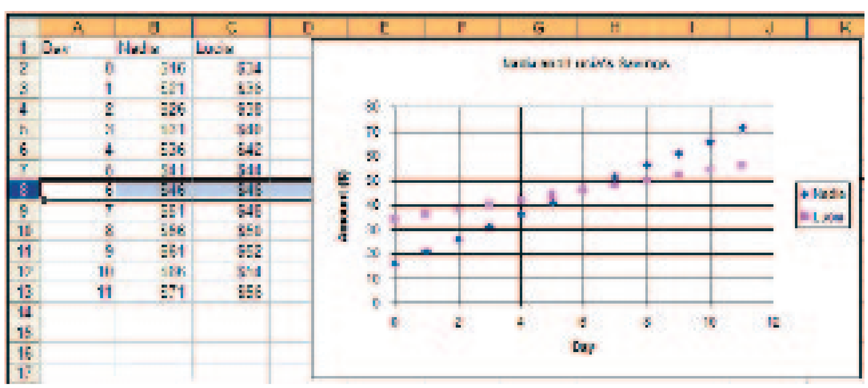


Why are the points not joined on this graph?

The intersection point of the two relationships is (6, 46). This means the girls will have the same amount of money, $46, on day 6.

Method 3: Use a Spreadsheet

In a spreadsheet, the girls enter the headings and amounts shown. Then, they use the spreadsheet's graphing features.



The table of values shows that the girls will have the same amount of money on day 6. The point of intersection on the graph is (6, 46). They will both have $46 on day 6.

Your Turn

Davidee earns $40 plus $10 per hour. Carmen earns $50 plus $8 per hour.

a) Represent the linear system relating the earnings numerically and graphically.

b) Identify the solution to the linear system and explain what it represents.

Example 2 Solve a Linear System Graphically

a) Consider the system of linear equations $2x + y = 2$ and $x - y = 7$. Identify the point of intersection of the lines by graphing.

b) Verify the solution.

Solution

How will the form of each equation help you choose your method of graphing?

a) Graph the equations together.

Method 1: Use Slope-Intercept Form

Rearrange each equation into slope-intercept form by isolating y. Identify the y-intercept and slope to draw the graph.

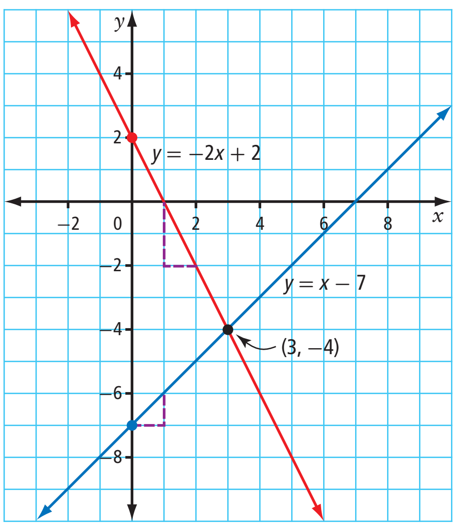
$$\begin{aligned} 2x + y &= 2 \\ 2x + y - 2x &= 2 - 2x \\ y &= 2 - 2x \\ y &= -2x + 2 \end{aligned}$$

$$\begin{aligned} x - y &= 7 \\ x - y + y &= 7 + y \\ x &= 7 + y \\ x - 7 &= 7 + y - 7 \\ x - 7 &= y \\ y &= x - 7 \end{aligned}$$

The y-intercept is 2.
The slope is -2.

The y-intercept is -7.
The slope is 1.

The solution is shown on the graph. It is the point of intersection, $(3, -4)$.



Method 2: Use x-Intercepts and y-Intercepts

Determine the x-intercept and y-intercept of the line $2x + y = 2$.

If given the equation of a line, how can you determine its intercepts?

x-intercept: $y = 0$	y-intercept: $x = 0$
$2x + y = 2$	$2x + y = 2$
$2x + 0 = 2$	$2(0) + y = 2$
$2x = 2$	$y = 2$
$x = 1$	

For the equation $2x + y = 2$, the x-intercept is 1 and the y-intercept is 2.

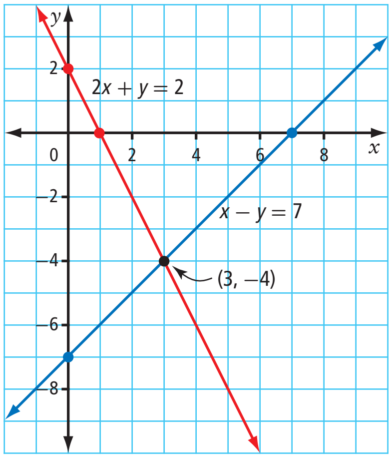
Determine the intercepts of the line $x - y = 7$.

x-intercept: $y = 0$	y-intercept: $x = 0$
$x - y = 7$	$x - y = 7$
$x - 0 = 7$	$0 - y = 7$
$x = 7$	$y = -7$

For the equation $x - y = 7$, the x-intercept is 7 and the y-intercept is -7.

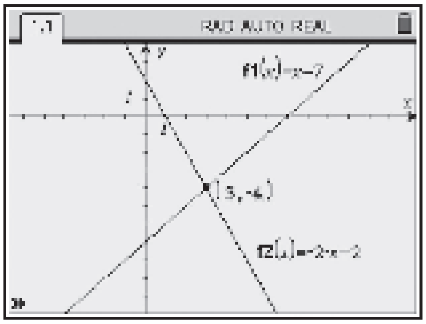
For each equation, plot the x-intercept and y-intercept; then, join the points.

The lines intersect at the point $(3, -4)$. So, the solution to the linear system is $(3, -4)$.



Method 3: Use Technology

Graph each equation using technology. Adjust the dimensions of the graph until you see both intercepts of each line, as well as the point of intersection. Then, use the intersection feature to find the solution.



The intersection point $(3, -4)$ is the solution to the linear system.

b) To verify that (3, −4) is the solution to the linear system $2x + y = 2$ and $x - y = 7$, use a different representation than the method of solving.

Method 1: Substitute Using Paper and Pencil

Verify the solution (3, −4) by substituting the values of x and y into each equation.

In $2x + y = 2$:

Left Side	Right Side
$2x + y$	2
$= 2(3) + (-4)$	
$= 6 - 4$	
$= 2$	

Left Side = Right Side

In $x - y = 7$:

Left Side	Right Side
$x - y$	7
$= 3 - (-4)$	
$= 3 + 4$	
$= 7$	

Left Side = Right Side

Since the ordered pair (3, −4) satisfies both equations, it is the solution to the linear system.

Method 2: Create a Table of Values Using Technology

Enter the equations of the lines and generate a table of values.

x	f1(x):=	f2(x):=
	2·x+2	x 7
0	2	7
-	0	6
2	-2	-5
3	-4	-4
4	6	3
5	8	2

When $x = 3$, both equations have the same value for y of −4. So, the point (3, −4) is the solution to the linear system.

Your Turn

Verify by graphing and one other way that (3, −2) is the solution to the system of linear equations $x - 3y = 9$ and $2x + y = 4$.

Example 3 Connect a Solution and a Graph

Guy solved the linear system $x - 2y = 12$ and $3x - 2y = 4$. His solution is $(2, -5)$. Verify whether Guy's solution is correct. Explain how Guy's results can be illustrated on a graph.

Solution

Substitute $x = 2$ and $y = -5$ into each equation. Evaluate each side to determine whether the values satisfy both equations.

In $x - 2y = 12$:

Left Side	Right Side
$x - 2y$	12
$= 2 - 2(-5)$	
$= 2 + 10$	
$= 12$	

Left Side = Right Side

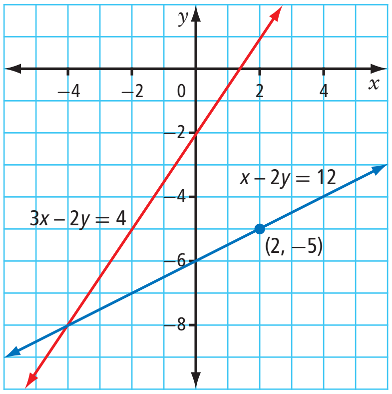
In $3x - 2y = 4$:

Left Side	Right Side
$3x - 2y$	4
$= 3(2) - 2(-5)$	
$= 6 + 10$	
$= 16$	

Left Side $\neq$ Right Side

The given values satisfy the first equation but not the second equation. Since both equations do not result in true statements, the point $(2, -5)$ is not the solution to this linear system.

The point $(2, -5)$ is not a solution to the given linear system. So, a graph of this system will not have a point of intersection at $(2, -5)$. The point $(2, -5)$ is on one of the lines, not *both* lines.



Your Turn

For each system of linear equations, verify whether the given point is a solution. Explain what the results would show on a graph.

a) $3x - y = 2$
$x + 4y = 32$
$(2, 5)$

b) $2x + 3y = -12$
$4x - 3y = -6$
$(-3, -2)$

Example 4 Solve a Problem Involving a Linear System

The Skyride is a red aerial tram that carries passengers up Grouse Mountain in Vancouver, BC. The Skyride travels from an altitude of about 300 m to an altitude of 1100 m. The tram can make the trip up or down in 5 min and can carry 100 passengers.

There is also a blue tram that can carry 45 passengers. This tram takes approximately 8 min to travel up or down the mountain. Each tram travels at a constant speed.

a) Create a graph to represent the altitudes of the trams if the red tram starts at the top and the blue tram starts at the base.

b) Explain the meaning of the point of intersection.

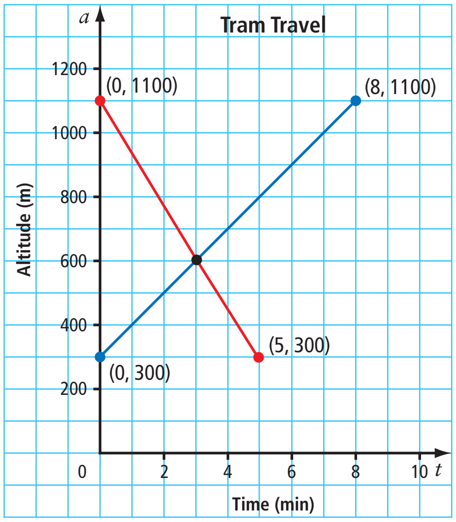
Solution

a) Organize the information before graphing.

Tram	Start		End		Representation on a Graph
	Time	Altitude	Time	Altitude	
Red	0 min	1100 m	5 min	300 m	Line segment joining the points (0, 1100) and (5, 300)
Blue	0 min	300 m	8 min	1100 m	Line segment joining the points (0, 300) and (8, 1100)

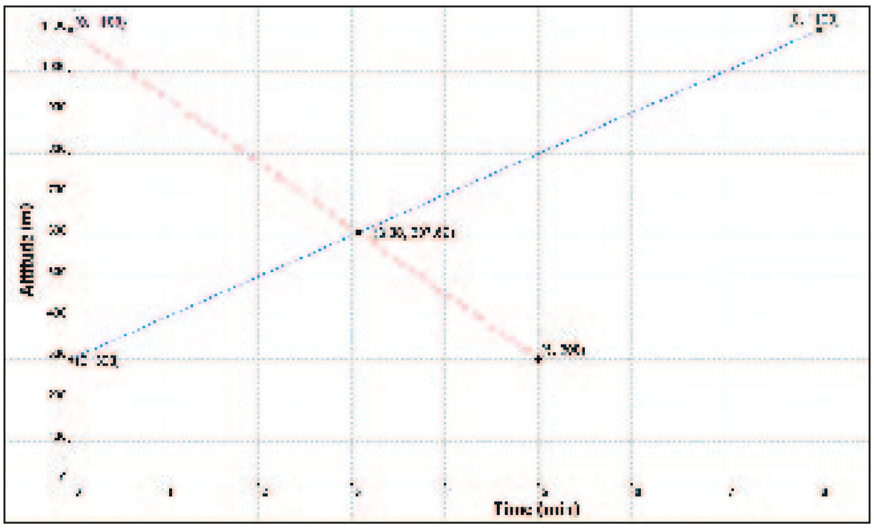
Method 1: Use Paper and Pencil

Label time from 0 min to 10 min on the horizontal axis. Label altitude on the vertical axis, up to 1200 m. Graph a line segment for each tram using the start and end points.



Method 2: Use Technology

Plot the start and end points for the travel of each tram. Use the "segment between two points" or equivalent feature to connect the points to show the continuous travel for each tram.



Use the intersection feature to determine the solution to the linear system.

b) At the point of intersection, the two trams will have the same altitude at the same time. The lines appear to intersect at approximately (3, 600). Therefore, after about 3 min, the two trams will pass each other at about 600 m in altitude.

The red tram travels faster, so it will travel farther than the blue tram in the same time interval. The red tram starts at the top, so the expected solution will be closer to the base of the mountain.

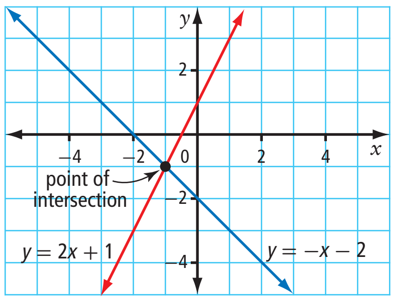
Your Turn

Eric works on the 23rd floor of a building. It takes Eric 90 s to walk down the stairs to the 14th floor. Nathan works on the 14th floor and needs to go up to the 30th floor. He knows it will take 40 s by elevator if the elevator makes no other stops.

Suppose both men leave their offices at the same time. Create a graph to model their travel. What does the point of intersection represent?

Key Ideas

- Systems of linear equations can be modelled numerically, graphically, or algebraically.
- The solution to a linear system is a pair of values that occurs in each table of values, an intersection point of the lines on a graph, or an ordered pair that satisfies each equation.
- One way to solve a system of linear equations is to graph the lines and identify the point of intersection on the graph.

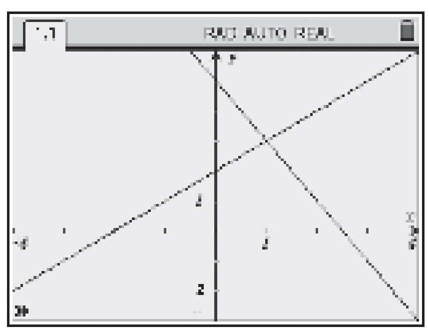


- A solution to a system of linear equations can be verified using several methods:
  - Substitute the value for each variable and evaluate the equations.
  - Create a graph and identify the point of intersection.
  - Create tables of values and identify the pair of values that occurs in each table.

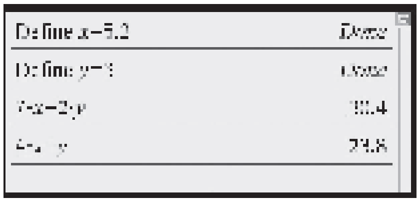
Check Your Understanding

Practise

1. One linear system is shown in the table of values, and another in the graph. Do the two systems have the same solution? Justify your answer.



2. John uses technology to check whether (5.2, 3) is the solution to the linear system $7x - 2y = 30.4$ and $4x + y = 25.1$. The results are shown on the screen. Is John's solution correct? Explain.



3. a) Represent the system of linear equations $y = 2x + 6$ and $y = 3x$ using a table of values and a graph.

b) Explain how you can identify the solution to the linear system from your table of values and your graph.

c) Verify by substitution that the solution you found is correct.

4. Consider the system of linear equations $y = -x + 4$ and $y = \frac{1}{2}x - 5$.

a) Show how a table of values can be used to solve the linear system.

b) Show how the linear system can be solved using a graphical representation.

c) Explain how the solution is related to the original equations.

5. Is each given point a solution to the system of linear equations? Explain.

a) $y = 3x - 5$
$y = 11 - x$
$(4, 7)$

b) $4x + 3y = 5$
$x + 4y = 13$
$(-1, 3)$

c) $2x - 3y = 18$
$x + 2y = -26$
$(-6, -10)$

d) $12x - 3y = 7$
$y = 4.5x - 3$
$(1.2, 2.4)$

6. On grid paper, graph each system of linear equations. What is the solution for each linear system?

a) $y = -2x + 5$
$y = x - 4$

b) $4x - y = -8$
$2x + 3y = -18$

7. Solve each system of linear equations graphically.

a) $y = 2x - 10$
$y = -3x + 8$

b) $y = \frac{1}{2}x - 5$
$y = -\frac{4}{3}x + 1$

c) $2x + y = 24$
$2x + 5y = 50$

d) $x - 2y = -18$
$3x + 4y = -12$

8. Solve each linear system graphically. Then, verify your solution.

a) $y = 0.5x + 4$
$y = 0.8x + 1$

b) $2x - 5y = 40$
$-6x + 5y = -60$

9. Is each given point a solution to the system of linear equations? Explain what the results would show on a graph of the linear system.

a) $3x - y = 2$
$x + 4y = 22$
$(2, 5)$

b) $2x + 3y = -12$
$4x - 3y = -6$
$(-3, -2)$

10. Brad and Sharon are collecting money from family and friends for a local charity. Brad has \$35 and plans to add \$5 each day. Sharon does not have any money yet. She plans to collect \$12 each day.

a) Represent the donations Brad and Sharon are collecting using a table of values and a graph.

b) What is the solution of the linear system? What does it represent?

Apply

11. Maya makes bead necklaces for a craft fair. It costs her \$2.50 to make each necklace. She needs to pay \$49 for a table at the craft fair. She plans to sell each necklace for \$6. Maya models her costs and revenue with the following equations:
Costs: $y = 2.5x + 49$
Revenue: $y = 6x$
In the equations, x represents the number of necklaces and y represents the amount, in dollars.

a) Create a graph of the linear system.

b) How many necklaces must Maya sell in order to break even?

c) Explain how to use the graph to determine the profit Maya will make if she sells 20 necklaces.

12. Consider the system of linear equations $y = \frac{1}{2}x - 5$ and $y = -\frac{1}{3}x + 11$.

a) Sketch a graph of the linear system on grid paper. Estimate the solution.

b) Use technology to solve the linear system by graphing.

c) Discuss the advantages and disadvantages of each method of solving systems of linear equations.

13. Can you solve the linear system involving $f_1(x) = 41 - 2x$ and $f_2(x) = 14 + 4x$ using the table of values shown? Explain.

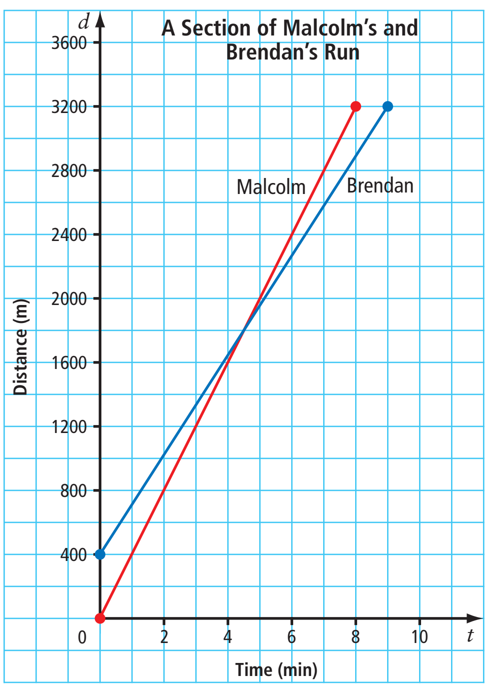
x	f1(x) = 41 − 2x	f2(x) = 14 + 4x
−1	43	10
0	41	14
1	39	18
2	37	22
3	35	26
4	33	30
5	31	34
6	29	38

14. Kayla and Sam sketch a graph to solve the system of linear equations $3x - 2y = -8$ and $4x - 3y = -9$. Sam completes his graph first and tells Kayla, "You need to graph extra carefully to solve this system." Solve the linear system by graphing and verify your solution. Why might Sam be justified in making his comment to Kayla?

15. The Calgary Marathon has been an annual event since 1971. It attracts participants from Canada and the United States. Malcolm and Brendan are training for the marathon. The graph shows a section of one of their long-distance runs.

a) Describe the part of their run represented in the graph.

b) Why can a system of linear equations represent this part of their run?



16. **Unit Project** Two groups of ducks are leaving a field and heading for a water source 50 km away. The green-winged teals leave 25 min before the canvasback ducks. Green-winged teals fly at a speed of 48 km/h.

a) How far do the green-winged teals fly during the 25 min?

b) Canvasback ducks fly at a speed of 115 km/h. The distance, d, in kilometres, travelled by each species is related to time, t, in hours, by the following equations:

Green-winged teals: $d = 48t + 20$
Canvasback ducks: $d = 115t$

What does time, $t = 0$ represent? Justify your answer. Then, sketch a graph of the system of linear equations.

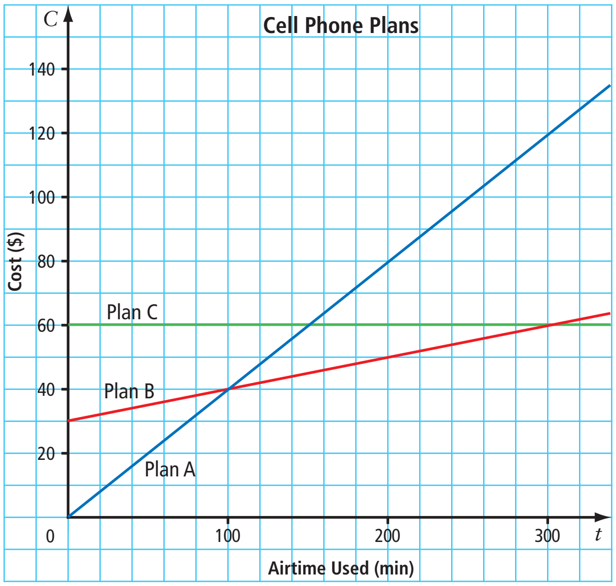
c) Use the graph to describe the trip to the water source for the two groups of ducks. Explain your reasoning.

17. One hot-air balloon is 100 m above ground. It rises at a constant rate and reaches a height of 1000 m in 16 min. Another hot-air balloon is 1600 m above ground. It descends at a constant rate to ground level in 20 min. Create a graph to represent the travel of the balloons. What does the point of intersection represent?

18. Sarah and George are both using the road from Qamani'tauq to the bridge over the Prince River in Nunavut. The road is 35 km long. Sarah drives her ATV at a constant speed from Qamani'tauq to the bridge. She takes 30 min. George drives his snowmobile at a constant speed from the bridge to Qamani'tauq. He takes 23 min. Create a graph to represent each person's distance from the bridge. What does the point of intersection represent?

Extend

19. The graph shows the costs for three different cell phone plans. Describe a situation in which a user would benefit most from each plan as compared with the other plans.



20. A truck travels along a highway. The truck's speed can be modelled with the equation $s = 1.5t + 5$. In the equation, s represents speed, in metres per second, and t represents time, in seconds. As the truck reaches a parked car, the car begins to move ahead. The car's speed can be modelled with the equation $s = 2.3t$. How many seconds does the car travel until its speed is the same as the truck's speed?

Create Connections

21. Describe a situation in your life that could be represented with a system of linear equations. Sketch a graph of the linear system. Explain what the point of intersection would represent.

22. How does *solving* a system of linear equations differ from *verifying* a solution to a system of linear equations? Provide an example.

23. Consider the system of linear equations $Ax + By + C = 0$ and $Dx + Ey + F = 0$. For each set of criteria, describe the lines. Justify your answers.

a) $A = D, B = E, C \neq F$

b) $A = D^{-1}, B = -E^{-1}, C = F$

c) $A \neq D, B = E, C = F$

d) $A = D, B = E, C = F$

8.2 Modelling and Solving Linear Systems

Focus on …

- translating word problems into systems of linear equations
- interpreting information from the graph of a linear system
- solving problems involving systems of linear equations

A big part of travelling involves choosing the mode of transportation. Different modes of transportation usually take different lengths of time, have different costs, and may have different effects on the environment. *The Canadian* is a passenger train that travels across Canada, between Vancouver and Toronto. How might taking *The Canadian* compare with other modes of travel? In what ways could you represent your comparison?

Investigate Creating a System of Linear Equations

Materials

- map of Canada
- ruler
- grid paper or computer with graphing software

1. Work with a partner. Select two cities in Canada that are in different provinces. Make sure each city has passenger rail service. Research the distance between the cities.

2. Suppose both of you need to travel from city A to city B. One person will drive and the other person will take a train. Assign the modes of travel. Then, determine the travel time for your particular mode. Assume the average speed of the car is 90 km/h and the average speed of the train is 70 km/h.

3. Assign a variable to represent each measurement.

a) the distance from city A

b) the length of time that the car has been travelling

4. a) Write an equation to model the travel of the car.

b) Suppose the train leaves 1 h before the car. Write an equation that represents the travel of the train.

5. Graph the system of linear equations you developed. What is the solution to your linear system? What does the solution represent? Discuss your answers with your partner.

6. Suppose the car leaves at 7:30 a.m. and the train leaves at 9:00 a.m.

a) Write a system of linear equations representing the travel from city A to city B. Compare your linear system with your partner's system.

b) Solve your linear system by graphing. What does the solution represent?

7. **Reflect and Respond**

a) Which system of equations did you find easier to write? Explain why. Share your strategies with a classmate.

b) How are the two equations in each linear system similar? How do they differ? Explain.

c) How are the information given in a problem, the equations, and the graph connected? Provide examples to support your explanation.

8. Why can this situation be represented with a system of linear equations? Explain.

Link the Ideas

The ability to translate words or phrases into the language of mathematics is an important skill for solving problems in context. There are a limited number of mathematical operations, but there are many phrases that can be used to describe the operations.

For example, the following situations can all be represented by the expression $7x + 3$.
- 7 km/h for a length of time and then three more kilometres
- \$7 per person plus \$3
- 7 times as many years increased by 3

When it comes to subtraction, the translation requires greater consideration.

$x - 3 \neq 3 - x$ when $x \neq 3$

For example, 3 less than a number means $x - 3$ whereas 3 decreased by a number means $3 - x$.

Situations involving quantities that change at constant rates can be represented algebraically with a system of linear equations.

Example 1 Model a Linear System Algebraically and Graphically

People can rent ski and snowboard equipment from two places at Winterland Resort.
Option A charges a one-time \$30 fee and then \$8 per hour.
Option B charges \$14 per hour.

a) Create a system of linear equations to model the rental charges.
b) Solve the linear system graphically. What does the solution represent?

Solution

a) Both rental options involve a constant rate per hour, so they represent linear relations.
Identify the unknown values and assign variables.
Let C represent the cost, in dollars.
Let t represent the length of time, in hours, of the rental.

Write an equation to model the cost for each rental option.

Option A: The cost is \$30 plus \$8 per hour.
$C = 30 + 8t$

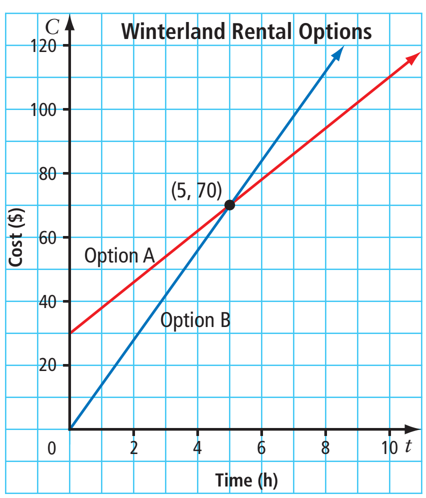
How is this equation related to the slope-intercept form?

Option B: The initial value is \$0 and the rate per hour is \$14.
The cost is \$14 per hour.
$C = 14t$
The equations $C = 30 + 8t$ and $C = 14t$ form a linear system.

b) To solve the linear system $C = 30 + 8t$ and $C = 14t$, graph the equations together and identify the point of intersection.

Method 1: Use Paper and Pencil

Graph the two equations.



What is the length of rental when both options have the same charge? What is that amount? How would you decide which option is better for you?

From the graph, the point of intersection is (5, 70). This is the solution to the linear system. It represents the length of rental when both options have the same charge.

The solution (5, 70) can be verified by substitution.

Option A: $C = 30 + 8t$

Left Side	Right Side
C	$30 + 8t$
$= 70$	$= 30 + 8(5)$
	$= 30 + 40$
	$= 70$

Left Side = Right Side

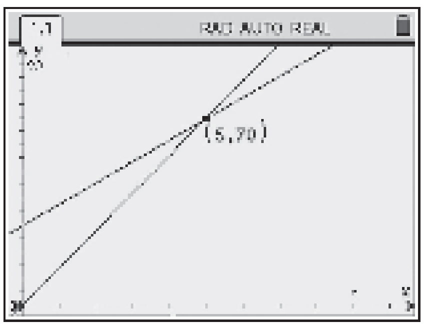
Option B: $C = 14t$

Left Side	Right Side
C	$14t$
$= 70$	$= 14(5)$
	$= 70$

Left Side = Right Side

Method 2: Use Technology

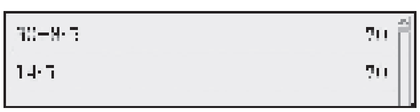
Graph the equations $C = 30 + 8t$ and $C = 14t$. You will likely need to use the equations $y = 30 + 8x$ and $y = 14x$.



How will you determine what range of values to plot?

Use the intersect feature to find the point of intersection, (5, 70). For a 5-h rental, both options cost \$70.

The solution can be verified by substituting using technology.



Your Turn

During a performance by a theatre company, the main act was on stage for 3 min less than twice the time of the opening act. Together, the two acts performed for 132 min.

a) Write a system of linear equations to represent the length of time each act performed.

b) What is the solution to this linear system? What does the solution represent?

Did You Know?

La Troupe du Jour is the only professional francophone theatre company in Saskatchewan. It was founded in 1985 and develops French-language theatre for the community.

Example 2 Interpret Information From the Graph of a Linear System

Two hopper-bottom grain bins are being emptied starting at the same time.

- The larger bin holds 45 m^3 of grain. It is emptied at a rate of 1 m^3 per minute.
- The smaller bin stores 30 m^3 of grain. This bin is emptied at a rate of 0.5 m^3 per minute.

a) Model the volume of grain remaining as a function of time using a system of linear equations.

b) Represent the linear system graphically. Describe how the information shown in the graph relates to the grain bins.

Solution

a) Define the variables.
Let V represent the volume of grain remaining in each bin, in cubic metres.
Let t represent time, in minutes.

Organize the information using a table.

Bin	Starting Volume (m^3)	Volume of Grain Removed (m^3)	Volume of Grain Remaining in Bin, V (m^3)
Larger	45	t	$45 - t$
Smaller	30	$0.5t$	$30 - 0.5t$

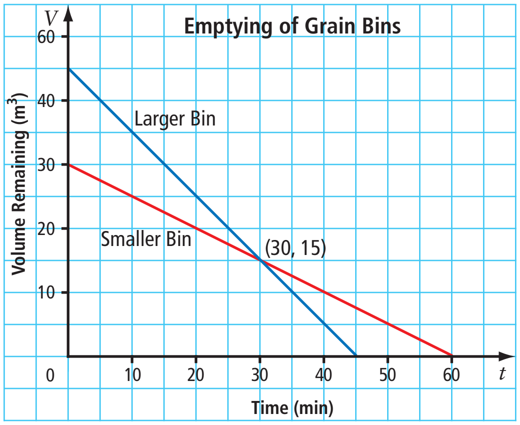
The larger bin starts with 45 m^3. It is being emptied at a rate of 1 m^3/min.
$V = 45 - t$

Why is the rate of change a negative value?

The smaller bin starts with 30 m^3. It is being emptied at a rate of 0.5 m^3/min.
$V = 30 - 0.5t$

A system of linear equations that models this situation is $V = 45 - t$ and $V = 30 - 0.5t$.

b) On the same grid, graph the system of linear equations, $V = 45 - t$ (larger bin) and $V = 30 - 0.5t$ (smaller bin).



The graph shows that the amount of grain remaining in each bin decreases over time. Both lines stop at the horizontal axis because the volume of grain left inside is 0 m^3.

The two lines intersect at (30, 15). This is the only point when the two bins contain the same amount of grain at the same time. At exactly 30 min, each bin has 15 m^3 of grain remaining.

Before 30 min, the line representing the larger bin is *above* the line for the smaller bin. This means that before 30 min, the larger bin has more grain inside. After 30 min, the line for the larger bin is *below* the line for the smaller bin. So, after 30 min, the smaller bin has more grain inside.

Your Turn

Two pools start draining at the same time. The larger pool contains 54 675 L of water and drains at a rate of 25 L/min. The smaller pool contains 35 400 L of water and drains at a rate of 10 L/min.

a) Model the draining of the pools algebraically using a system of linear equations.

b) Represent the linear system graphically. Describe how the information shown in the graph relates to the pools.

Example 3 Model and Solve a Problem Involving a Linear System

A movie theatre charges \$11 for an adult ticket and \$8 for children's or seniors' tickets. Suppose 240 people attended the early movie and ticket sales totalled \$2370.

a) The box office manager wants to know how many adults attended the early movie. What system of linear equations could help the manager determine the answer?

b) How many adults attended the early movie?

Solution

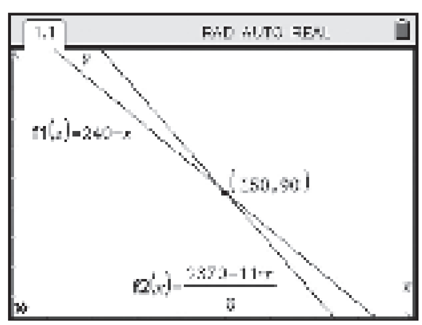
a) Define the variables.
Let a represent the number of adult tickets sold.
Let c represent the number of children's or seniors' tickets sold.

Write an equation to model the number of people at the early movie.
$a + c = 240$

Write an equation to model the ticket sales.
$11a + 8c = 2370$

The manager could use the linear system $a + c = 240$ and $11a + 8c = 2370$ to help determine the number of adults at the movie.

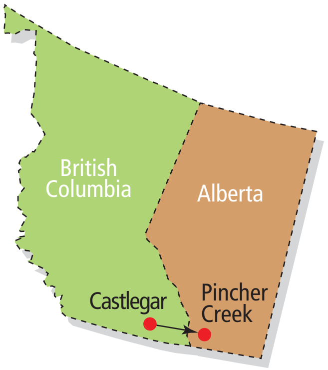
b) Graph both equations and identify the point of intersection.



The coordinates of the point of intersection are (150, 90). This is the only point that satisfies both equations. So, (150, 90) is the solution to the system of linear equations. There were 150 adults and 90 children and seniors at the early movie.

Your Turn

Jamie is travelling with her family from Castlegar, BC, to Pincher Creek, AB. Her dad and cousin do all of the driving. The 440-km trip takes 5.25 h, excluding stops. Jamie's father drives at an average speed of 90 km/h. Her cousin drives at 80 km/h. What system of linear equations could help Jamie determine the length of time each person drove? How many hours does each person drive?



Key Ideas

- When modelling word problems, assign variables that are meaningful to the context of the problem.
- To assist in visualizing or organizing a word problem, you can use a diagram and/or a table of values.
- If a situation involves quantities that change at constant rates, you can represent it using a system of linear equations.

  Two tanks are being filled at constant rates:
  One tank contains 100 L and is filling at 20 L/min.
  The other tank is empty and filling at 25 L/min.
  The situation can be represented by the system
  $V = 100 + 20t$ and $V = 25t$.
  In the equations, V represents volume, in litres, and
  t represents time, in minutes.

- If you know the initial values and rates, you can write the equations directly in slope-intercept form because the initial value is the y-intercept and the rate is the slope. Otherwise, you can determine the rate of change using start and end values.

  An electronics store charges a fee of \$36 plus \$6 per hour to fix a repair.
  The initial value is \$36, so $b = 36$.
  The rate is \$6 per hour, so $m = 6$.
  The equation is $C = 6t + 36$, where C represents cost, in dollars, and t represents time, in hours.

Check Your Understanding

Practise

1. Model each situation using a system of linear equations.

a) One music download option costs \$0.99 per song. Another option costs \$11 plus \$0.79 per song.

b) A helicopter is 800 m above ground and descending at 55 m/min. An airplane is taking off and rising 80 m/min.

c) A recycling plant sorts material at a rate of 20 t per hour. It has sorted 100 t of material so far today. A new plant just opened. It sorts material at a rate of 40 t per hour.

Did You Know?

In 2009, Calgary began the Blue Cart recycling program. In this program, waste is sorted at a local plant owned by Metro Waste Paper Recovery. The plant is the most automated waste recovery facility in North America. It uses high-tech equipment and sorts 40 t of material every hour. In one year, the plant sorts and processes about 120 000 t of waste.

2. Write a system of linear equations to represent each situation.

a) Jamal is three times as old as Maria. In seven years, he will be twice as old as she will be.

b) One day, the temperature in one city drops at a constant rate from 2 °C to −6 °C in 4 h. Meanwhile in another city, the temperature rises at a constant rate from −8 °C to 4 °C in 3 h.

3. Molly has a total of 32 points in her hockey league. One point is earned for an assist or a goal. If she has three times as many assists as goals, write a system of linear equations to represent how many goals and assists she has.

4. A collection of 50 coins contains only dimes and quarters. The value of the collection is \$6.80.

a) Use the table to write a system of linear equations relating the number of dimes to the number of quarters.

Type of Coin	Value of One Coin (\$)	Number of Coins	Value of Coins (\$)
Dime	0.10	d	$0.10d$
Quarter	0.25	q	$0.25q$

b) Rewrite the table, expressing the value of each type of coin in cents. Then, write a system of equations to model this relationship.

Apply

5. Two full water tanks drain at the same time. One tank holds 800 L of water. It drains at a rate of 30 L/min. The other tank holds 300 L of water. It drains at a rate of 12 L/min.

a) Express the draining of the water tanks algebraically using a system of linear equations.

b) Graph the linear system.

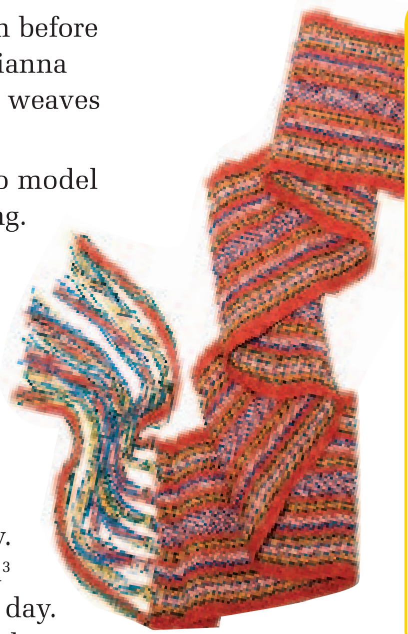
c) Explain how the information shown in the graph relates to the water tanks. What does the point of intersection represent?

6. Kianna weaves 45 cm of her Métis sash before Naomi starts weaving her own sash. Kianna weaves about 15 cm each hour. Naomi weaves about 25 cm every hour.

a) Write a system of linear equations to model the progress of the two girls' weaving.

b) Create a graph of the system of linear equations.

c) Describe how the information shown in the graph relates to the Métis sashes that the girls are weaving.



Did You Know?

Métis sashes are traditionally made with Aboriginal finger-weaving techniques. The weavers use European materials. In the past, the sash had many uses, including as a rope, washcloth, dog harness, baby carrier, and belt. Today, the sash is a symbol of pride. It is common for Prairie Métis organizations to bestow the Order of the Sash. There is no higher honour in the Métis community than receiving a sash as a gift.

7. One oil well has produced 2100 m^3. It produces oil at a rate of 7 m^3 per day. Another oil well has produced 1500 m^3 and produces oil at a rate of 15 m^3 per day. Suppose the two wells continue to produce oil at their current rates. When will both oil wells have produced the same amount? How much oil will they have produced at that time?

8. Megan considers two different car rental options. One option costs \$19 per day plus \$0.12 per kilometre. Another option costs \$42.50 per day for unlimited kilometres.

Compare the two options for a one-day rental. Which option should Megan choose? Why?

Web Link

To learn more about low-flow shower heads and other water-saving fixtures, go to www.mhrmath10.ca and follow the links.

9. **Unit Project** The Benoit family is deciding whether to replace their conventional shower head with a low-flow model. Their current shower head uses 170 L of water per 10-min shower. A typical low-flow shower head costs \$25 to purchase. It uses 85 L per 10-min shower. Heating the water with electricity costs approximately \$0.002 per litre.

a) If n represents the number of 10-min showers, write an expression for the cost of n showers using their current shower head.

b) Write a system of linear equations to represent the cost of showering using each type of shower head.

c) Graph the system of linear equations.

d) What is the solution to your linear system? What does it represent?

e) How would your solution change if each shower was reduced to 8 min? Justify your answer with a graphical analysis.

Did You Know?

All Arctic communities except the very largest are on trucked water systems. So, most showers in the North use low-flow shower heads.

10. Amaruk and Mary are both travelling by boat between Igloolik and Hall Beach, NU. Amaruk is 35 km away from Hall Beach. He is travelling south to Hall Beach at a speed of 40 km/h. Mary is 15 km away from Hall Beach. She is travelling north at a speed of 25 km/h. When they pass each other, how far will they be from Hall Beach?

Did You Know?

The Test of Metal is a mountain bike race that takes place in Squamish, BC, every summer. The 67-km course includes many long, steep climbs and lots of technical off-road riding. More than 800 competitors of all ability levels compete. Finish times range from under 3 h to over 6 h.

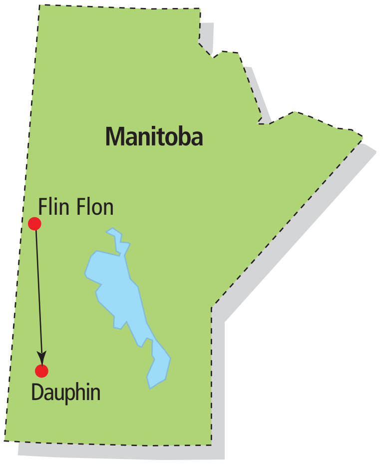
11. Caleb and Mitch compete in the Test of Metal mountain bike race. For a 2-km-long section, Caleb is 400 m up the hill and rides at a rate of 7.5 km/h. Mitch is at the base of the hill and rides at 10 km/h. Use a linear system to model this section of Caleb's and Mitch's race. Will Mitch catch Caleb before they reach the top of the hill? Explain why.

12. **Unit Project** A nearby wetland is estimated to have 100 ducks and 300 fish. A source of pollution seems to have contaminated the water. A local environmental group realizes that the number of ducks is decreasing at an average rate of 5 per year. The number of fish is decreasing at an average rate of 20 per year. Suppose the situation is considered critical if the number of fish equals the number of ducks in the area.

a) Write a system of linear equations to represent the numbers of fish and ducks in the wetland. Create a graph of your system.

b) Will the decreasing rates of fish and ducks become critical? If so, when? Justify your answer.

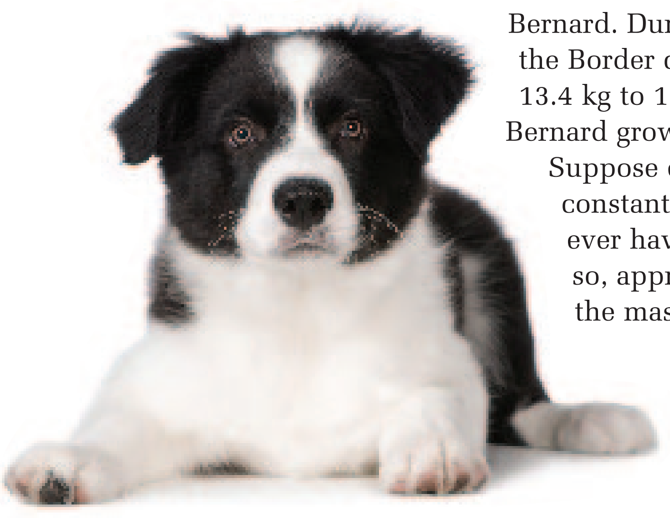
13. While driving from Flin Flon to Dauphin, MB, Kevin and his family had a flat tire. Before the flat tire, Kevin's parents drove at an average speed of 90 km/h. Once the flat tire was replaced with the spare tire, they travelled at an average speed of 75 km/h for the remainder of the trip. The total distance between the two cities is 538 km. The total driving time was 6 h. Write a linear system to model the family's travel. How far did they travel before the flat tire occurred? Include a labelled diagram.



14. Chris paints from one end of a 120-ft-long fence. He paints at a rate of 9 ft/h. Robert paints from the other end at a rate of 12 ft/h. Use a system of linear equations to determine when and where they will finish painting the fence.

15. Trevor is doing a project on tree growth rates. He measures the heights of two trees in early spring and again 20 days later. The younger tree grows from 120 cm to 130 cm tall. The older tree grows from 140 cm to 144 cm tall. Assuming each tree grows at a constant rate, when will the trees be the same height? What will this height be?

16. For part of the first year of a dog's life, its growth can be approximated using a linear function. Emilie has two puppies: a Border collie and a younger Saint Bernard. During a 4-week period, the Border collie grows from 13.4 kg to 17 kg, while the Saint Bernard grows from 6 kg to 12.4 kg. Suppose each dog grows at a constant rate. Will the dogs ever have the same mass? If so, approximately what is the mass?



17. A parachutist descends from a height of 500 m to 300 m above ground in 50 s. During the same time, a balloonist rises from 200 m to 450 m.

a) Write a system of linear equations to model their heights.

b) When are the two people at the same height? What is that height?

18. Andrea has three times as many grapes as Hunter. If she gives Hunter six grapes, she will have twice as many as he has now. Write a system of linear equations to model the number of grapes each person has. How many grapes does each person have before the exchange?

Extend

19. Jesse asks his math teacher her age and her husband's age. Jesse's teacher responds that it is not an appropriate question, but that she would use the opportunity to challenge Jesse with a riddle. She says, "One third of my age is ten less than one half of my husband's age. The sum of our ages is 105." How old is Jesse's math teacher?

20. A man swims 200 m against the current of a stream in 3 min. He swims with the same effort downstream for 150 m in 45 s. Create a system of linear equations to determine the man's swimming speed and the speed of the current of the water, in metres per minute.

21. An alloy is a mixture of a metal with a cheaper metal. A jeweller wishes to make pendants using a 94% silver alloy of sterling silver and pure silver. Sterling silver is 92.5% pure silver. The jeweller wishes to make 100 g of the silver alloy. What linear system could be used to determine how many grams of sterling silver and pure silver must be mixed?

22. Eunji wants to spend time at a local amusement park this summer. She is deciding which option will cost less.

Option A: A season's pass costs \$22, but she will have to pay \$6 for parking each visit.

Option B: A two-visit pass costs \$16.50 and includes parking. She can buy as many passes as she wants.

Represent Eunji's options using linear equations. What decision will she need to make before she can choose an option? Why?

Create Connections

23. Gavin leaves Calgary at 1:30 p.m. and drives to Edmonton. He travels at a speed of 110 km/h. At 2:15 p.m., James leaves Calgary in a helicopter travelling along the same route as Gavin. The helicopter travels at 240 km/h. The distance between Calgary and Edmonton is 300 km.

   a) Draw a diagram of their travel or use a table to organize the information.

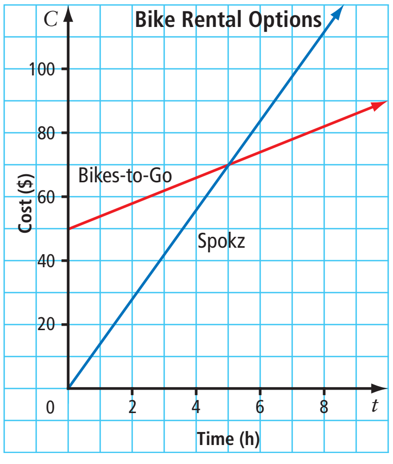
   b) Describe how to determine whether James will catch up to Gavin.

24. Two bike stores, Bikes-to-Go and Spokz, have different rental options. Spokz charges an hourly rate only. For a 5-h-long rental, both stores charge the same price.

   a) How could you use the graph to determine the hourly rate charged by Spokz?

   b) Write a system of linear equations to model the cost of renting from each store.

   c) How could the graph help you decide which option to choose?



8.3 Number of Solutions for Systems of Linear Equations

Focus on ...
- explaining why systems of linear equations can have different numbers of solutions
- identifying how many solutions a system of linear equations has
- solving problems involving linear systems with different numbers of solutions

Web Link

To learn more about the Arctic Winter Games, go to www.mhrmath10.ca and follow the links.

At the Arctic Winter Games, Northern youth share cultural experiences and compete in various events. One of the events is snowshoe racing. Situations involving time and distance, such as a race, can be represented with a system of linear equations. If you solve a linear system involving a race, will you always expect a single solution?

Materials
- stopwatch or other timer or clock showing seconds
- measuring tape
- grid paper and coloured pencils, or computer with graphing software

Investigate Number of Solutions for Systems of Linear Equations

Work in groups of four to act out different race scenarios. Each group needs to assign two people to race, one person to measure time, and one person to record data.

1. As a group, design a data table to record start and end times and distances for each racer.

2. Complete several different race scenarios. Each student should move at a constant speed for the entire race. For each race, record the data. Consider the starting line distance zero.

3. Create a distance-time graph to represent each race. You may wish to use a different colour for each line.

4. Write a system of linear equations to represent each race.

5. **Reflect and Respond** Solve each system of linear equations you graphed. How many solutions were there for each linear system? Explain how each solution relates to details of the race.

6. How could you have predicted the number of solutions for each linear system just by knowing the starting points and speeds of the racers?

Link the Ideas

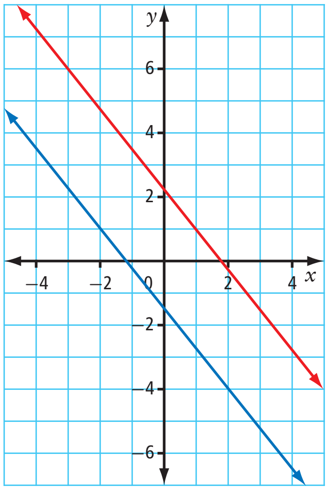
When two lines are graphed on the same grid, they do not always have exactly one point of intersection as seen in sections 8.1 and 8.2. Parallel lines do not intersect at all. So, a system of parallel lines has no solution.

Coincident lines have an infinite number of solutions because the lines are equivalent. They overlap.

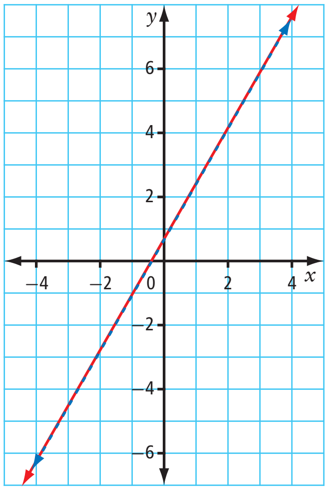
coincident lines

- lines that occupy the same position
- in a graph of two coincident lines, any point of either line lies on the other line

Parallel Lines



Coincident Lines



Reducing the equations to lowest terms may help you identify whether the equations are equivalent. If they are equivalent, then they must have an infinite number of solutions.

For the linear system $x - 2y + 5 = 0$ and $3x - 6y + 15 = 0$, the first equation is in lowest terms, but the second equation is not. Dividing each side of the equation $3x - 6y + 15 = 0$ by 3 gives $x - 2y + 5 = 0$, which is equivalent to the first equation. Therefore, the linear system has an infinite number of solutions.

Example 1 Connect the Number of Solutions to the Situation

A particular dog-mushing race is 13 km long. The distances and speeds for several competitors at a certain time during the race are shown in the table of values.

	Current Distance Travelled (km)	Current Speed (km/h)
Competitor A	6.0	24
Competitor B	5.0	32
Competitor C	4.0	24
Competitor D	4.0	24

Assume the racers continue at their current speeds. For each pair of competitors below,

- write a system of linear equations representing their travel from this point forward
- graph each system of linear equations
- identify and interpret the solution to each linear system

a) competitors A and B
b) competitors A and C
c) competitors C and D

Did You Know?

Many breeds of dogs are used for competitive racing. Teams of Canadian Eskimo Dogs have helped people survive in the harsh climate of northern Canada for centuries. Canadian Eskimo Dogs help move people and materials throughout regions that are covered in snow during most of the winter. The Canadian Eskimo Dog is Nunavut's official mammal in honour of its vital role.

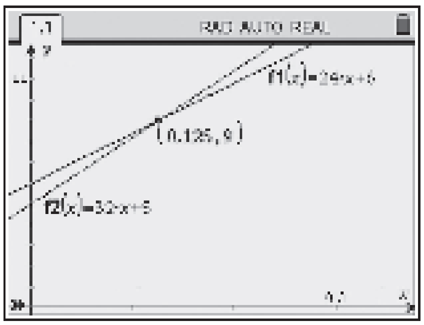
Solution

Let d represent distance from the start of the race. Let t represent time from this point on in the race.

a) Competitor A is travelling at 24 km/h and has travelled 6.0 km. Competitor B is travelling at 32 km/h and has travelled 5.0 km. Their travel can be represented by the following equations:

$d = 24t + 6$

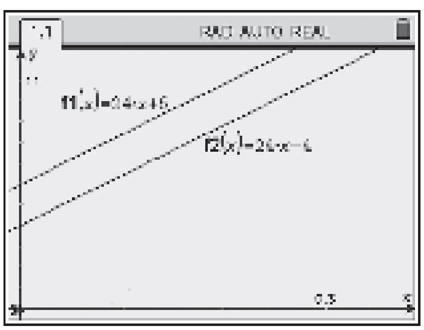
$d = 32t + 5$



The lines intersect at one point. So, the linear system has one solution.

Competitor B will catch up to competitor A after 0.125 h, or 7.5 min, and then proceed past. The point (0.125, 9) is the only point that lies on both lines.

b) Competitors A and C are both travelling at 24 km/h. Competitor A has travelled 6.0 km. Competitor C has travelled 4.0 km. Their travel can be represented by the equations $d = 24t + 6$ and $d = 24t + 4$.

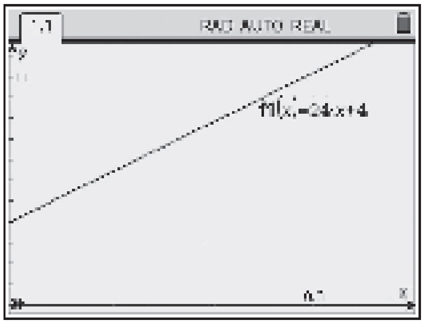


The lines have the same slope but different *y*-intercepts.

The lines are parallel. They have no points in common, so the linear system has no solutions.

Competitors A and C travel at the same speed, but at different distances from the start line. Competitor C will never catch up to competitor A. There is no point where they are at the same distance at the same time.

c) Competitor C is travelling at 24 km/h and has travelled 4.0 km. Competitor D is also travelling at 24 km/h and has also travelled 4 km. They are currently at the same distance and travelling at the same speed. Their travel can be represented by the equations $d = 24t + 4$ and $d = 24t + 4$.



The lines have the same slope and the same *y*-intercept.

The lines are coincident, so they share all the same points. The linear system has an infinite number of solutions.

Competitors C and D are side by side on the course. They will continue this way because they are travelling at the same speed.

Your Turn

Four vehicles travel on a long, straight stretch of the Trans-Canada Highway. Their current distances and speeds are shown in the table of values.

	Current Distance (km)	Current Speed (km/h)
Car	40	90
Minivan	25	90
Truck	30	110
RV	40	90

For each pair of vehicles, represent the distance-time relationship using a system of linear equations. Suppose the vehicles continue at their current speeds. Identify and interpret the solution to each linear system.

a) the car and the minivan

b) the car and the RV

c) the truck and the RV

Example 2 Predict and Confirm the Number of Solutions

Predict the number of solutions for each system of linear equations. Explain your reasoning, and then confirm each answer by graphing the linear system.

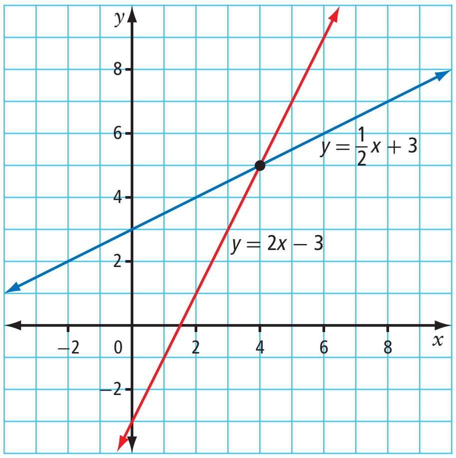
a) $y = 2x - 3$
$y = \frac{1}{2}x + 3$

b) $4x + 10y = 30$
$2x + 5y = 35$

c) $10x - 6y = -12$
$21y = 42 + 35x$

Solution

a) The slope of $y = 2x - 3$ is 2. The slope of $y = \frac{1}{2}x + 3$ is $\frac{1}{2}$.

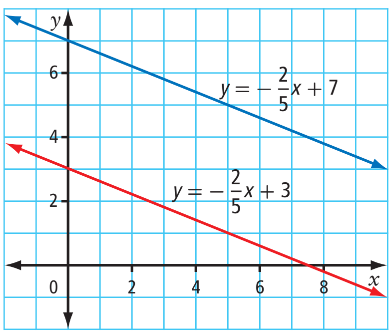


The equations have different slopes. So, the graph will result in two lines that intersect at one point. Therefore, this system has one solution.

b) Rearrange each equation to slope-intercept form by isolating y.

$$\begin{aligned} 4x + 10y &= 30 \\ 4x + 10y - 4x &= 30 - 4x \\ 10y &= -4x + 30 \\ y &= \frac{-2}{5}x + 3 \end{aligned} \qquad \begin{aligned} 2x + 5y &= 35 \\ 2x + 5y - 2x &= 35 - 2x \\ 5y &= -2x + 35 \\ y &= \frac{-2}{5}x + 7 \end{aligned}$$

Since the lines have the same slope and different y-intercepts, the graph will result in parallel lines. The lines will never intersect. Therefore, this linear system has no solutions.



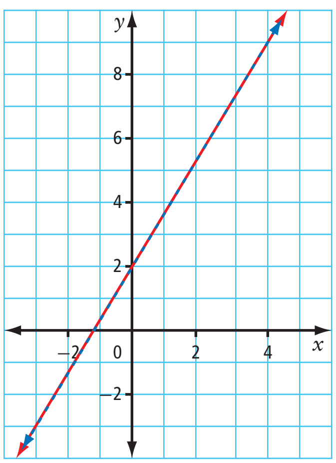
c) For the linear system $10x - 6y = -12$ and $21y = 42 + 35x$, isolate y in each equation to compare the slopes and y-intercepts.

$$\begin{aligned}
10x - 6y &= -12 \\
10x - 6y + 6y + 12 &= -12 + 6y + 12 \\
10x + 12 &= 6y \\
\frac{5}{3}x + 2 &= y \\
y &= \frac{5}{3}x + 2
\end{aligned}$$

$$\begin{aligned}
21y &= 42 + 35x \\
\frac{21y}{21} &= \frac{42}{21} + \frac{35x}{21} \\
y &= 2 + \frac{5}{3}x \\
y &= \frac{5}{3}x + 2
\end{aligned}$$

Both equations have a slope of $\frac{5}{3}$ and a y-intercept of 2.

The graph will result in coincident lines. Therefore, this linear system has an infinite number of solutions.



Your Turn

Predict the number of solutions for each system of linear equations. Justify your answers using a graph.

a) $x + 2y = 4$
$y = -\frac{1}{2}x + 4$

b) $6y - 4x = 6$
$y = \frac{2}{3}x + 1$

c) $y = 3x - 1$
$y = 2x - 1$

Example 3 Identify Zero and Infinite Solutions by Comparing Coefficients

Sabrina's teacher gives her the following systems of linear equations and tells her that each system has either no solution or an infinite number of solutions. How can Sabrina determine each answer by inspecting the equations?

a) $2x + 3y = 12$
$2x + 3y = 20$

b) $2x + 3y = 12$
$4x + 6y = 24$

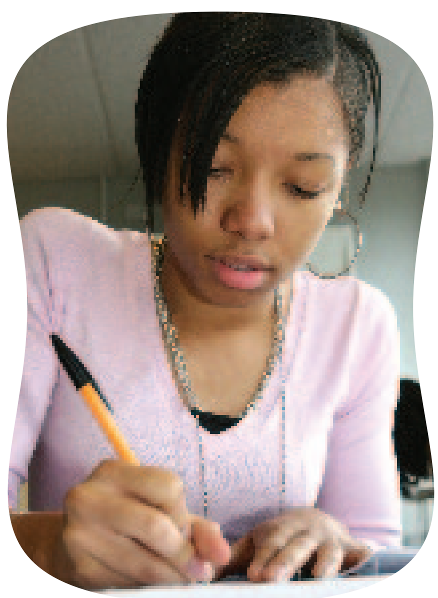
Solution

a) Sabrina notices that the left sides of the equations are identical. So, any ordered pair she substitutes will result in the same value on the left side of each equation. However, the right sides are not equal. There are no ordered pairs that can satisfy both equations, so the lines never intersect. Sabrina concludes that the linear system has no solutions.

How else could you confirm that the lines are parallel?

b) Sabrina notices that the second equation, $4x + 6y = 24$, is not in lowest terms. She divides each term by 2. This results in an equation that is identical to the first equation, $2x + 3y = 12$. Therefore, the equations are equivalent and the graph will be a pair of coincident lines. The linear system has an infinite number of solutions.

$$\frac{4x}{2} + \frac{6y}{2} = \frac{24}{2}$$
$$2x + 3y = 12$$



Your Turn

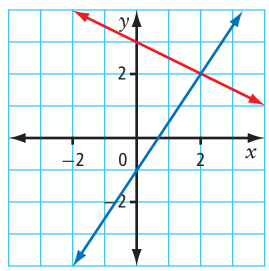
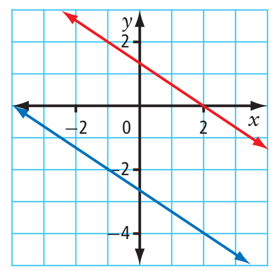
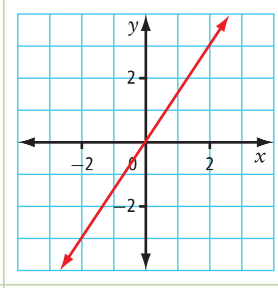
Determine, by inspection, whether each linear system has an infinite number of solutions or no solution. Explain your reasoning.

a) $2x + 10y - 16 = 0$
$x + 5y - 8 = 0$

b) $x + 2y + 4 = 0$
$x + 2y - 6 = 0$

Key Ideas

- A system of linear equations can have one solution, no solution, or an infinite number of solutions.
- Before solving, you can predict the number of solutions for a linear system by comparing the slopes and *y*-intercepts of the equations.

Intersecting Lines	Parallel Lines	Coincident Lines
one solution	no solution	an infinite number of solutions
		
different slopes	same slope	same slope
y-intercepts can be the same or different.	different *y*-intercepts	same *y*-intercept

- For some linear systems, reducing the equations to lowest terms and comparing the coefficients of the *x*-terms, *y*-terms, and constants may help you predict the number of solutions.

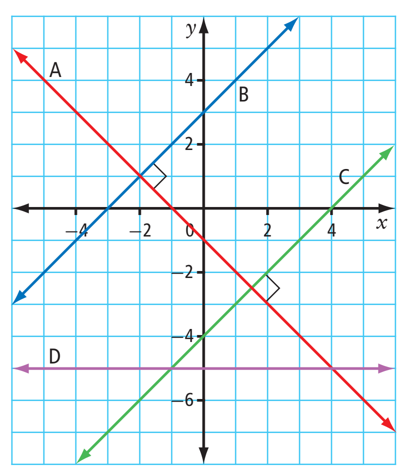
Check Your Understanding

Practise

1. Which pair(s) of lines in the graph form a linear system that has

a) exactly one solution?

b) no solution?



2. Predict the number of solutions for each system of linear equations. Justify your answers.

a) $y = x + 2$
$y = x + 2$

b) $y = 2x - 4$
$y = x + 1$

c) $y = 3x + 2$
$y = 3x - 5$

3. How many solutions does each linear system have? Justify your answers.

a) $x + 3y = 6$
$y = -\frac{1}{3}x + 6$

b) $3x - y = 12$
$4x - y = 12$

c) $x - 4y = 8$
$x + 4y = 20$

4. Describe the graph and the equations of a linear system that has

a) no solution

b) one solution

c) an infinite number of solutions

5. Describe a strategy for predicting how many solutions each system of linear equations has just by looking at it. Test your strategy by graphing each linear system.

a) $2x + 7y = 28$
$2x + 7y = 15$

b) $x + 2y = 12$
$2x + 4y = 24$

Apply

6. One of the equations in a linear system is $2x - y + 5 = 0$. What might the other equation be if the system has

a) no solutions?

b) one solution?

c) infinitely many solutions?

7. A provincial magazine employs sales people who earn money for every subscription they sell. Several employees are comparing their earnings so far this month.

	Current Earnings ($)	Earnings Per Subscription ($)
Alyssa	472	7.00
Brian	360	8.25
Charlie	360	8.25
Dena	413	8.25

Write a system of equations to represent the earnings for each pair of employees. Identify the solution to each system. Explain how the solution relates to the employees' earnings.

a) Brian and Charlie

b) Alyssa and Brian

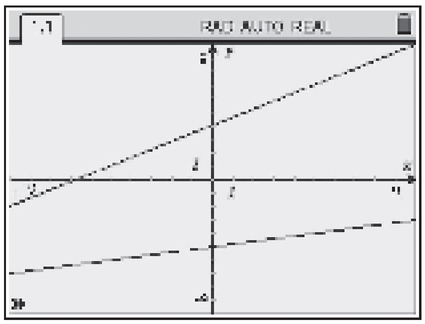
c) Charlie and Dena

8. Sal and Jeff graph the system $y = \frac{5}{8}x + \frac{11}{7}$ and $y = \frac{7}{11}x + \frac{8}{5}$ using technology. They use dimensions of -10 to 10 on the x-axis and -10 to 10 on the y-axis. Sal thinks there is one solution. Jeff claims that there are an infinite number of solutions.

a) Describe how to predict the number of solutions.

b) Use technology to recreate their graph. Explain why each person might have come to his conclusion.

9. Stephanie graphs a system of equations using technology. She obtains the following graph and concludes that the system has no solutions.



a) Explain why Stephanie might have made her conclusion.

b) Is Stephanie correct? Explain.

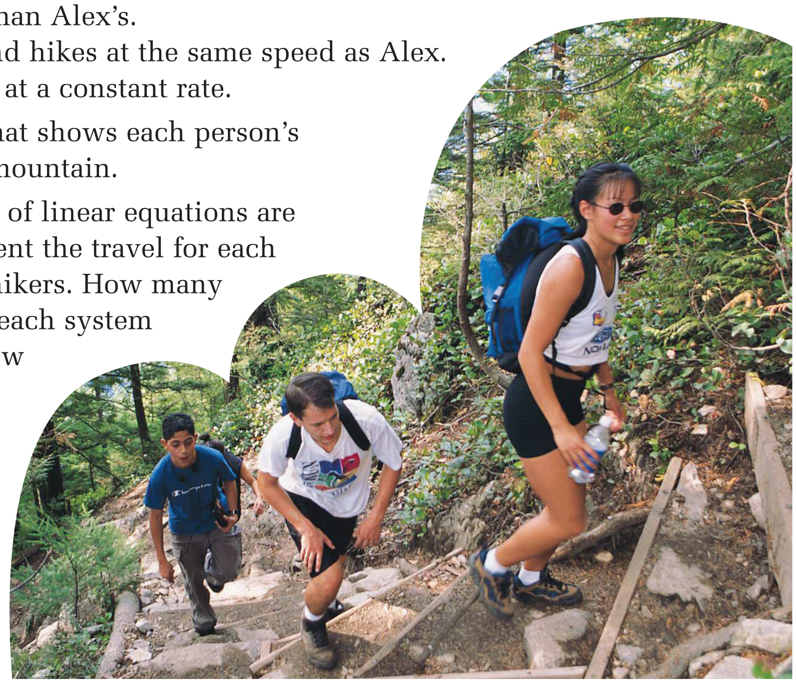
10. Alex, Sandra, Christine, and Jared are hiking the Grouse Grind® near Vancouver, BC.

- Alex starts first.
- Sandra and Christine start a short time later. They hike together at a faster speed than Alex's.
- Jared starts last and hikes at the same speed as Alex.

Each person travels at a constant rate.

a) Sketch a graph that shows each person's progress up the mountain.

b) Suppose systems of linear equations are created to represent the travel for each possible pair of hikers. How many solutions would each system have? Explain how the number of solutions relates to the hike.



Did You Know?

The Grouse Grind® is the most used hiking trail in the Vancouver area. The trail is 2.9 km long and rises 853 m. Over 100 000 people walk or run this steep climb every year. Maintenance workers have built steps up most of the trail to protect against erosion and for the safety of the hikers.

11. Is the following statement true or false? Explain and provide an example to support your answer.

"If a system of linear equations has an infinite number of solutions, then any pair of numbers is a solution to the system."

12. Suppose you are given only the following pieces of information about a system of linear equations. Would you be able to predict the number of solutions to the system? Explain.

a) The slopes of the lines are the same.

b) The y-intercepts of the lines are the same.

c) The x-intercepts are the same, and the y-intercepts are the same.

13. PaperWest has produced 5000 kg of napkins. It continues to manufacture 350 kg of napkins per week. Northern Paper manufactures napkins at a rate of 1400 kg per month and has already produced 28 000 kg. Assume one month has exactly four weeks.

a) Write a system of linear equations to represent the manufacturing of the napkins.

b) Explain how the number of solutions to the system relates to this situation.

Extend

14. For the linear system $2x + 3y = 12$ and $4x + 6y = C$, what value(s) of C will give the system

a) an infinite number of solutions?

b) no solution?

15. Two taxis travel the same route from the airport. One taxi is 6 km from the airport and has a fuel economy of 20 km/L. The other taxi is just leaving the airport and uses 5 L of fuel for every 100 km travelled.

a) Create a system of linear equations relating the distance travelled (y kilometres) to the amount of fuel used (x litres) for each taxi.

b) Explain how the number of solutions to the system relates to the travel of the taxis.

16. Consider the system $y = 56 - 2x$ and $y = 10 + x$.

a) Suppose the domain for the system is restricted to $0 \leq x \leq 8$. How many solutions does the system have? Explain.

b) Suppose there are no restrictions on the domain. How many solutions does the system have?

c) What effects do restrictions on the domain of the equations have when you are predicting the number of solutions? Explain.

17. Consider the system $Wx + 3y = 2W$ and $12x + Wy = 24$.

a) What value(s) of W give the system an infinite number of solutions?

b) How many solutions will the system have for value(s) of W other than those you found in part a)? Justify your answer.

Create Connections

18. Two plants are growing according to the equations $h = 60 + 3t$ and $h = 55 + 2t$. In the equations, h represents height, in centimetres, and t represents time, in weeks. Wendy reasons that one plant is shorter and is growing slower, so it will never catch up to the other plant. Wendy concludes that the system has no solutions. Harriet states that the two lines have different slopes, so there must be an intersection point. Who do you agree with? Create a graph and use it to explain your answer.

19. Can a system of two linear equations have exactly two solutions? Explain your answer, using words and diagrams.

20. Do you think it is always possible to tell how many solutions a system of linear equations has just by looking at the equations? Explain your thinking.

21. Describe a real-life situation that could be modelled by a system of linear equations having each number of solutions. Include examples of possible systems of linear equations.

a) no solution

b) one solution

c) an infinite number of solutions

22. **MINI LAB** Investigate relationships between equations in a linear system and the number of solutions to the system.

Materials
- graphing calculator or computer with graphing software

Step 1 Consider $2x + y = 4$ as equation #1. Create four linear systems (A, B, C, and D) by writing equation #2 according to each of the following instructions.

System A: Multiply (both sides of) equation #1 by a number of your choice.

System B: Add a number of your choice to only the right side of equation #1.

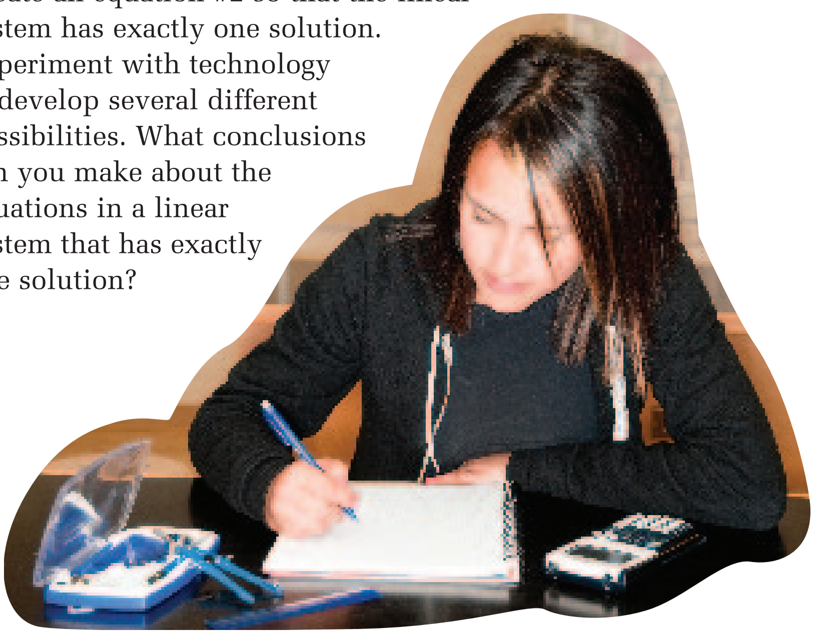
System C: Perform an operation of your choice on both sides of equation #1.

System D: Perform an operation of your choice on only one side of equation #1.

Step 2 Use a graph to analyse the number of solutions for each system of linear equations.

Step 3 What can you conclude about the number of solutions for a system of linear equations if one equation is a multiple of the other? What if two *different* equations in general form have identical coefficients of x-terms and identical coefficients of y-terms?

Step 4 Create an equation #2 so that the linear system has exactly one solution. Experiment with technology to develop several different possibilities. What conclusions can you make about the equations in a linear system that has exactly one solution?



8 Review

8.1 Systems of Linear Equations and Graphs, pages 416–431

1. For each system of linear equations, explain how you could verify whether the given point is a solution. Is the given point a solution?

a) $y = x + 11$
$y = -2x - 10$
$(-7, 4)$

b) $11x + 3y = 18$
$9x - 4y = 7$
$(3, -5)$

2. Solve each system of linear equations by graphing.

a) $y = -2x - 6$
$y = \frac{3}{2}x + 8$

b) $3x + y = 2$
$x + y = 3$

c) $x - y = 1$
$5x - 4y = 12$

3. Use technology to solve each system of linear equations graphically.

a) $y = \frac{2}{5}x - 7$
$y = -\frac{5}{8}x + 2$

b) $6x + 5y = -45$
$2x + 5y = 40$

c) $9x - 8y = -24$
$4x - 3y = -3$

4. The travel of two boats can be represented by the system $d = 4t + 20$ and $d = 5t + 12$. In the equations, t represents time, in seconds, and d represents distance, in metres.

a) Represent the linear system numerically and then graphically.

b) Describe how to identify the solution in each representation.

c) What is the solution to this linear system? What does the solution represent?

8.2 Modelling and Solving Linear Systems, pages 432–445

5. Write a system of linear equations to model each situation.

a) One gym membership is \$85 for the first year plus \$30 per month. Another gym membership is \$35 per month.

b) Two grain bins start filling at the same time. The amount of grain in one bin increases from 5 m^3 to 30 m^3 in 10 min. The other bin is empty and increases to 40 m^3 in 8 min.

c) Two crews begin resurfacing side-by-side roads at the same time. One crew has 3 km left to resurface and covers 100 m every 30 min. The other crew has 4 km to resurface and covers 250 m every 60 min.

6. A flyer advertises two cell phone plans.
   a) Write an equation to represent each plan.
   b) Compare the plans using a graph.
   c) Explain how the graph can help someone make a choice.

7. Atmospheric pressure in two cities is monitored at the same time. The pressure in one city drops from 102.6 kPa to 100.6 kPa in 4 h. In the other city, the atmospheric pressure rises from 99.8 kPa to 101.3 kPa in 6 h.
   a) Write a system of linear equations to model the change in atmospheric pressure.
   b) Graph the system. What does the point of intersection represent?

8. Karen and Andrew ski down the same run at constant speeds. They begin skiing at the same time. Karen skis from an altitude of 2000 m for 3 min and stops at 1700 m. Andrew skis from 2100 m to 1500 m in 4 min. Will Andrew pass Karen before she stops? If so, when and at what altitude? Verify your answer.

8.3 Number of Solutions for Systems of Linear Equations, pages 446–459

9. Describe the graph of a system of linear equations that has
   a) no solution
   b) one solution
   c) an infinite number of solutions

10. Predict the number of solutions for each linear system. Explain your prediction and justify by graphing.
   a) $y = 2x - 3$
      $y = 2x + 1$
   b) $y = 3x + 10$
      $y = 6x + 20$
   c) $2x + 3y - 6 = 0$
      $14x + 21y - 42 = 0$
   d) $2x - y - 10 = 0$
      $4x - 2y - 30 = 0$

11. Pam, Myk, Luisa, and Carl are each holding a helium balloon. Pam and Myk let go of their balloons. The balloons rise into the air. A few seconds later, Luisa lets her balloon go, and it rises more quickly than the first two. After a few more seconds, Carl lets his balloon go. It rises at the same rate as Pam's balloon. Each balloon rises at a constant rate.

    a) Sketch a graph to represent the ascent of the four balloons.

    b) Identify pairs of balloons whose ascent can be represented by a linear system with each of the following numbers of solutions. Explain why you chose each pair.
    - zero
    - one
    - infinite

12. Companies A and B produce sports beverages. Company A has made 150 L and is producing more at a rate of 75 L every 15 min. Company B makes the same beverage at a rate of 300 L per hour and has already made 600 L.

    a) Write a system of linear equations to represent the production of the sports beverages.

    b) How many solutions are there? How does the number of solutions relate to the situation?

13. Suppose two investments earn the same rate of interest.

    a) Which of the following is *not* a possible number of solutions for the linear system representing the investments over time?
    - zero
    - one
    - infinite

    b) Explain your answer.

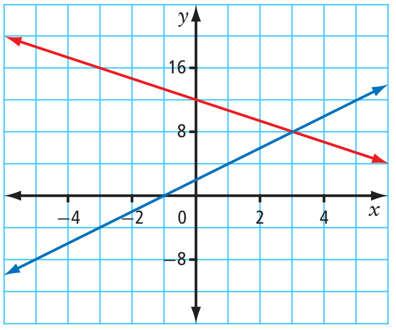
8 Practice Test

Multiple Choice

For #1 to #5, choose the best answer.

1. What is the solution to the linear system shown in the graph?

A $(-1, 12)$ **B** $(0, 0)$
C $(0, 2)$ **D** $(3, 8)$



2. Which number of solutions is *not* possible for a system of two linear equations?

A zero **B** one
C two **D** infinite

3. What is the point of intersection in a system of linear equations?

A the origin
B the ordered pair written as x-intercept, y-intercept of a line
C the point where a line starts
D the ordered pair representing the location where the lines cross

4. How many solutions does the linear system $2x + 3y = 6$ and $3x + 2y = 24$ have?

A zero **B** one
C two **D** infinite

5. The volume of liquid in two different containers begins decreasing at the same time. The volume in one container changes from 22 L to 10 L in 6 s. The volume in the other container changes from 30 L to 15 L in 5 s. Which system of linear equations can be used to model the situation?

A $V = 22 - 10t$
$V = 30 - 15t$

B $V = 10 - 2t$
$V = 15 - 3t$

C $V = 10 - 3t$
$V = 15 - 2t$

D $V = 22 - 2t$
$V = 30 - 3t$

Short Answer

6. For each system of linear equations, verify whether the given point is a solution. Explain what the results would show on a graph of the system.

a) $x + y = 10$
$2x - y = 3$
$(5, 2)$

b) $3x + y = 1$
$7y = 13 - 15x$
$(-1, 4)$

c) $x + 4y + 4 = 0$
$2x - 3y = 27$
$(8, -3)$

7. Consider the linear system $y = 5x + 1$ and $y = \frac{5}{2}x - 4$. Explain how you would solve this system.

8. **a)** Graph the linear system $5x - 4y = 32$ and $3x - 7y = -63$.

b) State the solution to this system of linear equations. Express your answer to the nearest hundredth.

c) Verify your solution. Describe your steps.

9. Explain how the number of solutions for each linear system can be determined without graphing. Briefly describe the relationship between the lines on a graph of each system.

a) $3x + 4y - 12 = 0$
$3x + 4y - 24 = 0$

b) $y = \frac{1}{2}x - 5$
$x - 2y = 10$

10. Consider the system of linear equations $14x + 35y = 7$ and $4x + By = 2$. If a graph of this system shows coincident lines, what is the value of B?

Extended Response

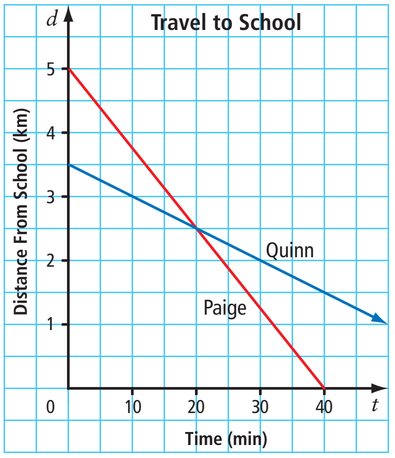
11. The grass on one sod farm is 6 cm high. The grass grows at a rate of 1.5 cm/week. The grass on another farm is 4 cm high. It grows at a rate of 2 cm/week.

a) Write a system of linear equations to model the growth of the grass on each farm.

b) Represent the linear system graphically.

c) Use the graph to describe the growth of the grass on each farm. What does the point of intersection represent?

12. The graph represents the travel of two students walking to school. Write a story that incorporates all the information shown in the graph.



13. Two feathers are released at the same time and begin falling steadily. One feather falls from 5.5 m and comes to rest on a 4-m-high ledge after 8 s. The other feather falls from 6 m to 3 m in 12 s. Are the feathers ever at the same height during their fall? Verify your answer.

14. The Opaskwayak Canoe Classic is a 60-mi canoe race held annually by the Opaskwayak Cree Nation. The race takes place on the Saskatchewan River near The Pas, MB. Information on four racers part way through the race is shown in the table of values.

	Current Distance (mi)	Current Speed (mph)
Taj	12.6	7.0
Donna	11.8	7.2
Marcus	11.8	7.2
Rose	11.2	7.2

Represent the data for each pair of racers using a system of linear equations, assuming each racer continues at his or her current speed. What is the solution to each linear system? What does the solution represent?

a) Donna and Marcus

b) Taj and Donna

c) Marcus and Rose

CHAPTER

9 Solving Systems of Linear Equations Algebraically

Analysing linear relations is a skill that is required in many professions. Air traffic controllers monitor the speed and altitude of airplanes. Electronic engineers determine how to speed up computers. Chemical engineers analyse the flow rates of mixtures. Business owners analyse market data. Consumers, like you, make decisions about the things they buy. What is the cost? What is the environmental impact of their purchases?

Sometimes, a graphical approach to solving a linear system takes too long. At other times, it may not be accurate enough. An algebraic approach may provide a faster and more precise solution.

Big Ideas

When you have completed this chapter, you will be able to ...

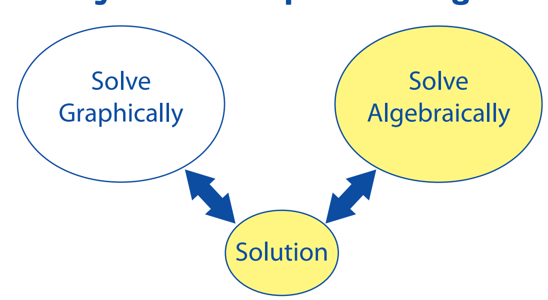
- model mathematical relationships from problems
- relate a system of linear equations to a problem
- analyse linear relationships using different algebraic methods
- determine and verify the solution to a linear system using algebra
- select an appropriate method to solve a problem

Key Terms

substitution method

elimination method

Your Systems of Equations Organizer



Air Traffic Controller

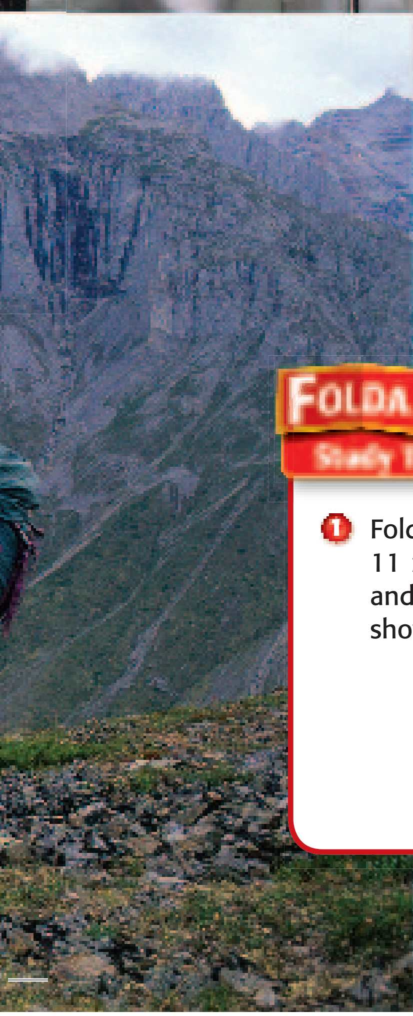
Air traffic controllers (ATCs) are responsible for directing air traffic in the sky and on the ground. They consider such variables as changing weather patterns, airplane types, and time of day. They may work visually from a control tower. They can also work electronically on a computer screen. ATCs analyse numerical relationships to do with wind speeds, airplane specifications, traffic, and rate of speed.

WWW Web Link

To learn more about becoming an air traffic controller, go to www.mhrmath10.ca and follow the links.

Did You Know?

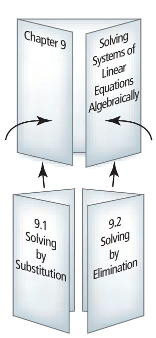
Many planes fly over the Arctic on their way to Europe. For this reason, there are aircraft monitoring sites in small Arctic airports.



FOLDABLES Study Tool

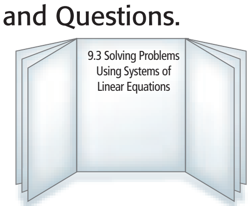
Make the following Foldable™ to take notes on what you will learn in Chapter 9.

1. Fold a sheet of 11 × 17 paper and label as shown.



2. Fold two sheets of 8.5 × 11 paper in half. Label as shown. Attach one sheet inside the left flap and one inside the right flap.

3. Label the inside centre as shown. Label the back What I Need to Work On, and Project Ideas and Questions.



9.1 Solving Systems of Linear Equations by Substitution

Focus on …

- solving systems of linear equations algebraically using substitution

This year, the environmental club is having a fundraiser. Members are selling compact fluorescent light bulbs and 100% organic T-shirts with natural dyes. The price of one T-shirt is three times the price of one light bulb. You purchase two shirts and one light bulb for $42. If you lost your receipt, how could you determine the unit price for a light bulb and for a T-shirt?

Investigate Solving Systems of Linear Equations by Substitution

In the following balance diagrams, each block is identical in mass. Each cone is identical in mass.

Diagram 1

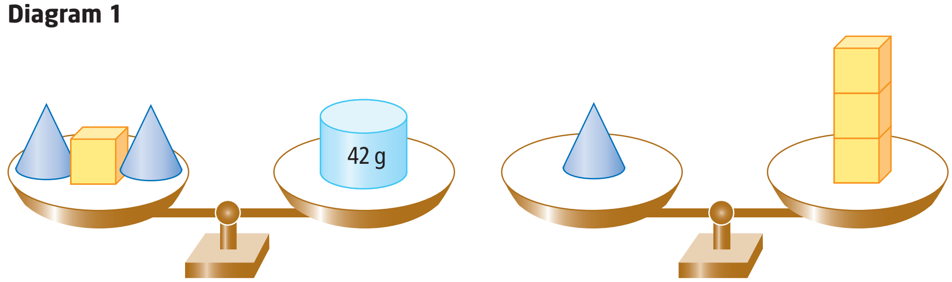
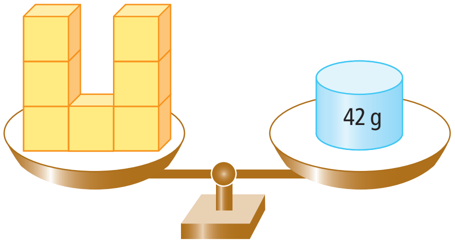


Diagram 2



1. Describe how Diagram 2 relates to Diagram 1.

2. Describe how you could determine the mass of one block from Diagram 2. What is the mass of one block?

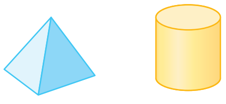
3. What is the mass of one cone? How did you determine your answer?

4. Write an equation for each balance scale in Diagram 1. Remember to state what your variables represent.

5. Write an equation for Diagram 2.

6. Suppose the mass of a block represents the cost of one light bulb, the mass of a cone represents the cost of one T-shirt, and each gram represents one dollar. Use algebra to show how you can determine the cost of one light bulb and the cost of one T-shirt for the scenario on page 468.

7. **Reflect and Respond** Use diagrams to explain how to determine the mass of a single pyramid and the mass of a single cylinder for the following scenario.
   - Five pyramids and three cylinders have a mass of 44 g.
   - Two pyramids have the same mass as one cylinder.



8. Use algebra to determine the mass of one pyramid, p, and the mass of one cylinder, c.

9. Describe a situation where using a diagram is less effective than using algebra.

www Web Link

To practise the algebraic method with a virtual scale, go to www.mhrmath10.ca and follow the links.

Link the Ideas

The skill of substituting algebraic expressions is used regularly in math and science. The **substitution method** can provide a quick solution to a linear system.

substitution method

- an algebraic method of solving a system of equations
- Solve one equation for one variable, substitute that value into the other equation, and solve for the other variable.

Solve the following linear system.

$4x + 5y = 26$
$3x = y - 9$

First, solve for y in $3x = y - 9$.

$$3x + 9 = y - 9 + 9$$
$$3x + 9 = y$$

Substitute $3x + 9$ for y in $4x + 5y = 26$.

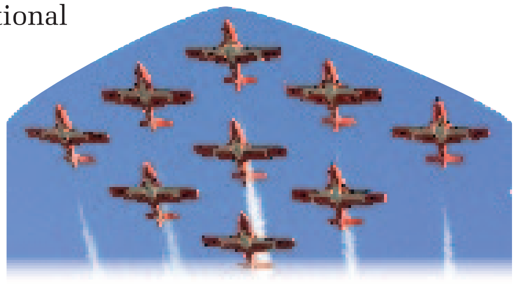
$$\begin{aligned}
4x + 5(3x + 9) &= 26 \\
4x + 15x + 45 &= 26 \\
19x + 45 &= 26 \\
19x + 45 - 45 &= 26 - 45 \\
19x &= -19 \\
\frac{19x}{19} &= \frac{-19}{19} \\
x &= -1
\end{aligned}$$

Substitute -1 for x in $3x = y - 9$.

$$\begin{aligned}
3(-1) &= y - 9 \\
-3 &= y - 9 \\
-3 + 9 &= y - 9 + 9 \\
6 &= y
\end{aligned}$$

Example 1 Solve a System of Linear Equations by Substitution

Admission to the 2009 Abbotsford International Airshow cost \$80 for a car with two adults and three children. Admission for a car with two adults cost \$50. Use algebra to determine the cost for one child and the cost for one adult. There was no charge for the vehicle or parking.



Did You Know?

The Abbotsford Airshow is held every August in Abbotsford, BC. It is one of the largest events of its kind in the world.

Solution

Let C represent the cost for one child, in dollars.
Let A represent the cost for one adult, in dollars.

We often use variables that are capital letters to represent values of money.

For the first car, $2A + 3C = 80$.
For the second car, $2A = 50$.

How do the equations represent the cost of admission for the first and second cars?

Determine the admission prices.

Method 1: Solve for A First

The second equation has only one variable. So, determine the cost for one adult first.

$$2A = 50$$
$$A = 25$$

Solve for C by replacing A with 25.

$$2A + 3C = 80$$

You can also replace $2A$ with 50.

$$2(25) + 3C = 80$$
$$50 + 3C = 80$$
$$3C = 30$$

If $50 + 3C = 80$, how do you know that $3C = 30$?

$$C = 10$$

What does the value 10 represent?

Method 2: Solve for C First

Use substitution.

$$50 + 3C = 80$$

Replace $2A$ with 50 in the equation.

Solve for C.

$$50 + 3C = 80$$
$$3C = 30$$
$$C = 10$$

Solve for A by replacing C with 10.

$$2A + 3(10) = 80$$
$$2A + 30 = 80$$
$$2A = 50$$

If $2A = 50$, how do you know that $A = 25$?

$$A = 25$$

What does the value 25 represent?

Check:

Substitute into the original equations, $2A + 3C = 80$ and $2A = 50$.

Left Side	Right Side
$2A + 3C$	80
$= 2(25) + 3(10)$	
$= 50 + 30$	
$= 80$	

You can also check your solution by graphing.

Left Side = Right Side

Left Side	Right Side
$2A$	50
$= 2(25)$	
$= 50$	

Left Side = Right Side

The admission price is \$10 for a child and \$25 for an adult.

Your Turn

Solve the following linear system algebraically using substitution.

$$3x + 5y = 27$$
$$4x = 16$$

Example 2 Isolate a Variable Before Solving by Substitution

At a dance recital, there were 220 people. Tickets cost \$9 for an adult and \$6 for a child. The dance school collected \$1614 in ticket sales. How many adults and how many children attended the recital?

Solution

Let a be the number of adults at the recital.
Let c be the number of children at the recital.
Write an equation that represents the total number of adults and children.

$a + c = 220$ ①

You can number the equations to make it easier to refer to them throughout the solution.

Write an equation that represents the amount collected by the dance school.

$9a + 6c = 1614$ ②

Isolate a variable in one of the equations.

Method 1: Isolate the Variable *c* in ①

$$a + c = 220$$
$$c = 220 - a$$

You can isolate the variable c in ① easily because the coefficient of the variable is 1.

Substitute for c in ②.

$$9a + 6(220 - a) = 1614$$
$$9a + 1320 - 6a = 1614$$
$$3a + 1320 = 1614$$
$$3a = 294$$
$$a = 98$$

What does the value 98 represent?

Substitute the number of adults into ① to finish solving the system.

$$98 + c = 220$$
$$c = 122$$

What does the value 122 represent?

Method 2: Isolate the Variable *c* in ②

$$9a + 6c = 1614$$
$$6c = 1614 - 9a$$
$$c = 269 - \frac{9}{6}a$$

Compare isolating *c* in Method 2 with isolating *c* in Method 1. Why does Method 2 take more steps?

Substitute for c in ①.

$$a + \left(269 - \frac{9}{6}a\right) = 220$$
$$-\frac{3}{6}a + 269 = 220$$
$$-\frac{1}{2}a + 269 = 220$$
$$-\frac{1}{2}a = -49$$
$$-\frac{1}{2}a(-2) = -49(-2)$$
$$a = 98$$

Substitute a in ① to finish solving the system.

$$98 + c = 220$$
$$c = 122$$

Check:
Substitute into ① and ②.

Left Side	Right Side
$a + c$	220
$= 98 + 122$	
$= 220$	

Left Side = Right Side

Left Side	Right Side
$9a + 6c$	1614
$= 9(98) + 6(122)$	
$= 882 + 732$	
$= 1614$	

Left Side = Right Side

At the dance recital, there were 98 adults and 122 children in attendance.

Which method do you prefer? Why? How might the solution be different if you isolated the variable *a* instead of *c*?

Your Turn

Solve the following linear system algebraically using substitution. Check your solution.

$2x + y = 13$
$x - 0.4y = -16$

Key Ideas

- You can solve systems of linear equations algebraically using substitution.
  - Isolate a single variable in one of the two equations.
  - Where possible, choose a variable with a coefficient of 1.

    Solve the linear system.

    $3x + 2y = -11$ ①

    $-2x + y = 12$ ②

    Isolate the variable y in ② since its coefficient is 1.

    $y = 12 + 2x$

    Substitute the expression for y in ①.

    $$\begin{aligned} 3x + 2(12 + 2x) &= -11 \\ 3x + 24 + 4x &= -11 \\ 7x + 24 &= -11 \\ 7x &= -35 \\ x &= -5 \end{aligned}$$

  - Substitute the solution for the first variable into one of the original equations. Solve for the remaining variable.

    $$\begin{aligned} -2(-5) + y &= 12 \\ 10 + y &= 12 \\ y &= 2 \end{aligned}$$

  - Check your answer by substituting into both original equations.

Check Your Understanding

Practise

1. Solve the following systems of linear equations by first substituting for y.

a) $y = 3x + 2$
$x + y = 14$

b) $y = -3x$
$y - x = 24$

c) $y = x - 7$
$x + y = 17$

2. Solve the following linear systems by substitution.

a) $2x - 3y = 10$
$x + y = 0$

b) $m = 8j$
$-m + 2 = -7j$

c) $2k = 6n + 9$
$n - 2k = -4$

3. Solve each linear system two ways. First, solve by isolating x. Then, solve by isolating y. For each linear system, explain which method you prefer and why.

a) $y = 0.3x - 5$
$1.7x + y = 9$

b) $y = 10 - 2.2x$
$5x + y = 70$

c) $\frac{x}{2} = 5 - y$
$x + y = 7$

4. Solve the following systems of linear equations. Check your answers.

a) $y = \frac{1}{3}x - 5$

$x - \frac{y}{5} = 13$

b) $\frac{y - x}{2} = 5$

$x + \frac{3}{4}y = 4$

c) $3y = \frac{1}{3} - \frac{2x}{3}$

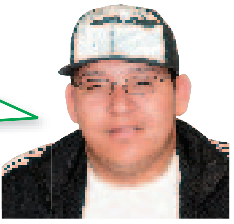
$x + \frac{3y}{2} = 12$

5. Jaret and Helen are going to solve the following system of equations.

$2x = 3y + 6$

$3x + y - 20 = 0$

As an intial step, Jaret decides to isolate x in the first equation.



As an intial step, Helen decides to isolate y in the second equation.

a) Solve this system using Jaret's method. Then, solve it using Helen's method.

b) Explain which method you prefer.

6. The sum of two numbers is 20. Twice one number is four more than four times the other. Write a system of linear equations and determine both numbers.

7. The graph represents the solution to the following linear system.

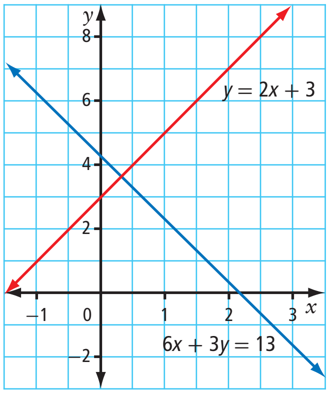
$y = 2x + 3$

$6x + 3y = 13$

a) What are the coordinates of the point of intersection?

b) Solve the linear system using the substitution method.

c) Compare your answers for parts a) and b). What is the advantage of the algebraic approach?



8. a) Solve the following system of linear equations by substitution.
   $0.1y = 0.3x - 1.5$
   $x - 0.2y = 5.6$
   b) Multiply both sides of each equation by 10 first. Then, solve by substitution. How does the multiplication by 10 help you to solve?
   c) Solve the system graphically.
   d) Which of the three methods do you prefer? Explain.

Apply

Solve problems 9 to 20 using the substitution method. Check your answers.

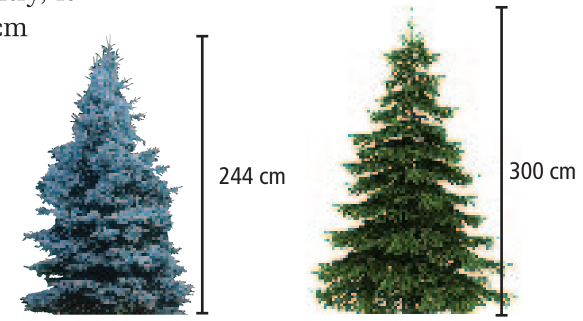
9. An 82-m cable is cut into two pieces. One piece is 18 m longer than the other. What is the length of each piece?

10. Whitehorse, YT, has three times as much snowfall each year as Vancouver, BC. The total combined snowfall for these two cities is approximately 192 cm. What is the snowfall in each city?

11. Alaina has \$72 and earns \$6 each day. Joel has \$48 and earns \$8 each day. In how many days will Joel have as much money as Alaina?

12. In Manitoba, teenagers watch approximately 11 fewer hours of TV each week than adults do. The sum of the hours watched per week for an adult and a teenager is about 37 h. Approximately how many hours per week do teenagers watch?

13. A young Colorado blue spruce tree is growing at a rate of 20 cm per year. Currently, it is 244 cm tall. A 300-cm tall white spruce tree is growing at a rate of 12 cm per year. In how many years will the two trees be the same height?



14. Rory's grandmother is 58 years older than Rory. In 5 years, they plan to have a party to celebrate that their ages have a sum of 100. How old are they now?

15. **Unit Project** A section of a local habitat was damaged during a storm. A local company wishes to preserve the wetland and ensure water quality. The organizers decide to replace some of the bushes and trees. They place two orders with a nursery.

- One order is for 40 bushes and 12 trees. It totals \$1484.
- The other order is for 25 bushes and 18 trees. It totals \$1421.

Create and solve a system of linear equations to determine the cost of one bush and the cost of one tree.

16. In Amir's coin collection, the number of dimes is one more than three times the number of nickels. The total number of nickels and dimes is 69.

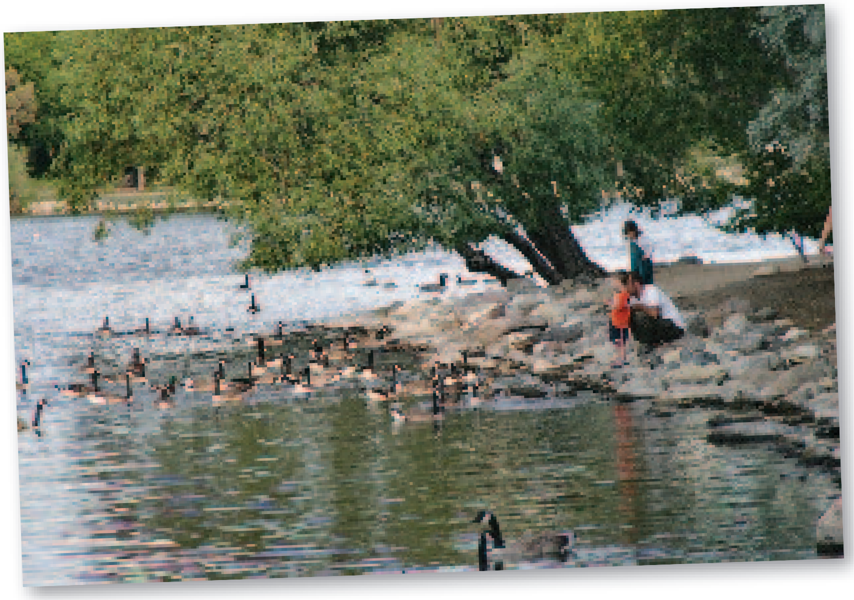
a) Amir also has 40 quarters in his collection. How many more dimes than quarters does he have? How many more quarters than nickels does he have?

b) Explain why you can solve this problem without knowing the value of a nickel, dime, or quarter.

17. Students from two schools went on a trip to the Wascana Waterfowl Park in Regina, SK. They learned about the value of conserving natural resources. School A rented and filled 8 vans and 8 buses with 400 people. School B rented and filled 4 vans and 1 bus with 68 people. Every van had the same number of students in it, as did every bus. Determine the number of students in each van and in each bus.

WWW Web Link

For more information about Wascana Waterfowl Park, go to www.mhrmath10.ca and follow the links.



Did You Know?

Making white bread requires 45% more water than making whole-wheat bread. It takes more flour to make white bread. Also, the flour requires extra processing to remove the brown colour.

18. Water is used during each step of bread manufacturing, from processing the wheat into flour to making the bread itself. Less water is used to produce a slice of whole-wheat bread than a slice of white bread.

- To produce 60 slices of whole-wheat bread and 10 slices of white bread, 2080 L of water are used.
- To produce 20 slices of whole-wheat bread and 50 slices of white bread, 2560 L of water are used.

How many litres of water are used to produce one slice of whole-wheat bread? How many litres of water are used to produce one slice of white bread?

19. Andrew has a collection of 132 coins that consists of quarters and loonies. The value of the collection is \$77.25. He wants to determine the number of quarters and the number of loonies he has without counting.

$q + n = 132$ and $0.25q + 1.00n = 77.25$, where q is the number of quarters and n is the number of loonies.

$0.25q + 1.00(132 - q) = 77.25$	Step 1
$100(0.25q) + (1.00)(132 - q) = (100)77.25$	Step 2
$25q + 1.00(132 - q) = 7725$	Step 3
$25q + 132 - q = 7725$	Step 4
$24q = 7593$	Step 5
$q = \frac{7593}{24}$	Step 6

Andrew stopped when he realized he had made a mistake.

a) How did Andrew know $q = \frac{7593}{24}$ could not be correct?

b) Identify where Andrew made the first error in his work.

c) Solve the linear system correctly.

20. Solve the following linear systems. Express your answers as fractions.

a) $y = \frac{1}{7}x - 2$
$5x = 3y + 1$

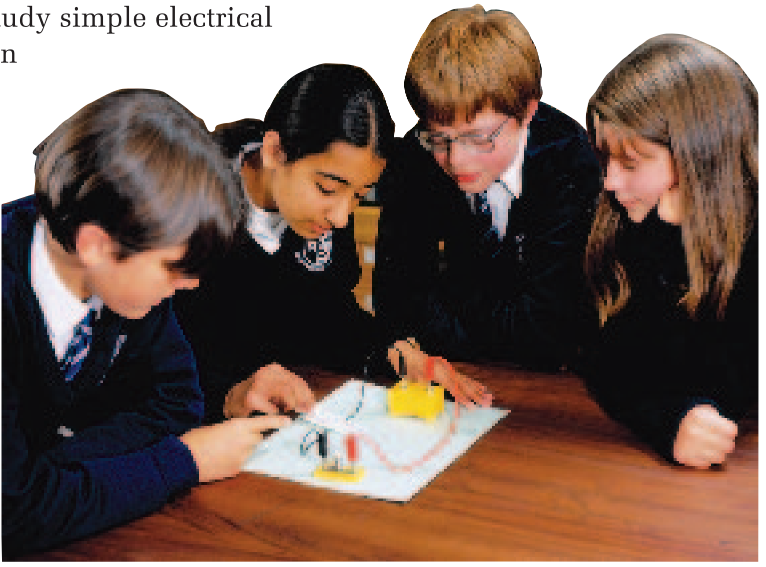
b) $\frac{1}{3}x + 4y = \frac{47}{15}$
$5x + 2y = 3.5$

Extend

21. On a graph, a line with the equation $y = mx + b$ passes through the points (2, 7) and (5, 1). Solve a linear system algebraically to determine the values of m and b. Include a diagram of the coordinate plane with your solution.

22. Arman walks to the train station at 5 km/h. He misses his train by 1 min. If he had run at 10 km/h, he would have had 2 min to spare. How far is it to the station?

23. In science, students frequently study simple electrical circuits. The relationship between the resistance of a circuit, R, the current, I, and the voltage, V, is $V = RI$. The relationship between the power, P, the voltage, and the current is $P = VI$. Use substitution to write a formula that determines the power from the resistance and the current. Show your work.



24. Use substitution to show that the linear system $y = 2x + 5$ and $2y - 4x = -15$ has no solution. How do you know there is no solution?

Create Connections

25. Compare solving a linear system by substitution to solving graphically.

a) How are the methods similar?

b) How are the methods different?

26. Choose a question from section 9.1 that has a system of linear equations.

a) Solve the system using a graphical approach.

b) Compare the graphical solution to the solution using substitution. Which method do you prefer for solving this system of equations? Explain.

9.2 Solving Systems of Linear Equations by Elimination

Focus on ...

- writing equivalent equations to eliminate a variable
- solving systems of linear equations algebraically using elimination

Did You Know?

Fair trade products are goods from developing countries. They are priced to ensure that the producers receive fair payment.

Environmental clubs promote many activities and products that are environmentally friendly. They promote chemical-free foods such as organic coffee. They also promote products that minimize waste. These include reusable shopping bags, and food and drink containers. What other similar products can you think of?

For their fundraiser, an environmental club is selling reusable shopping bags and organic fair-trade coffee. Two recent sales were:

- three bags and two packets of coffee for $17
- one bag and one packet of coffee for $7

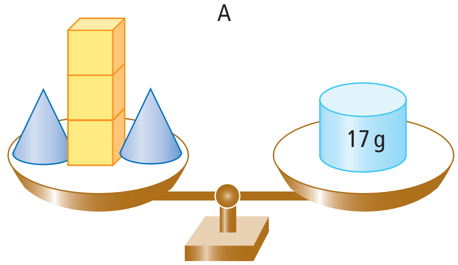
How could you determine the unit price of one bag and one packet of coffee?

Investigate Solving Systems of Equations by Elimination

In the following balance diagrams, each block is identical in mass. Each cone is identical in mass.

Diagram 1

A



B

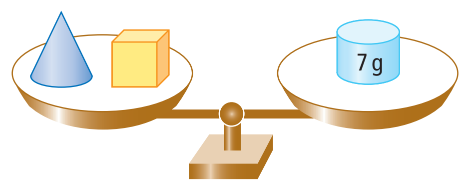
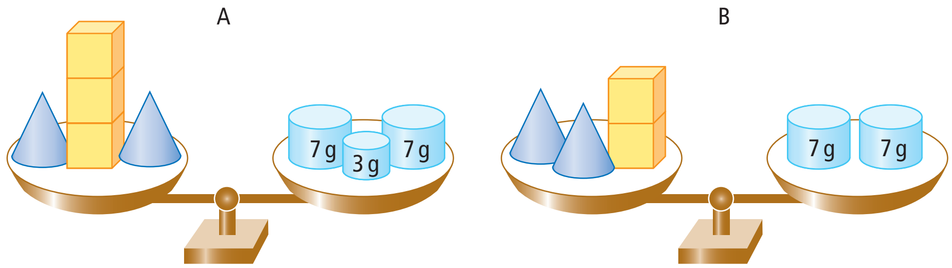


Diagram 2



1. Explain why scale B in Diagram 2 is balanced.

2. Draw a diagram of a scale balance to show how to determine the mass of one block.

3. Explain how you can determine the mass of the cone given the mass of the block.

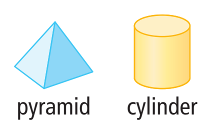
4. Write equations for each scale in Diagram 1.

5. Use algebra to show how you would change scale B in Diagram 1 in order to write the equation for scale B in Diagram 2.

6. Use algebra to show how you can write the equation for the scale in your diagram from the two equations in Diagram 2.

7. Suppose the mass of a block represents the cost of one bag, the mass of a cone represents the cost of one packet of coffee, and each gram represents one dollar. How can you determine the cost of one bag and the cost of one coffee packet using algebra for the scenario on page 480?

8. **Reflect and Respond** Use diagrams to explain how to determine the mass of a single pyramid and the mass of a single cylinder for the following scenario.



   - Four pyramids and three cylinders have a mass of 23 g.
   - Two pyramids and five cylinders have a mass of 29 g.

9. Determine algebraically the mass of one pyramid, p, and the mass of one cylinder, c.

10. Could you use the substitution method to solve this Investigate? Explain.

Web Link

To practise your algebraic skills using a virtual balance, go to www.mhrmath10.ca and follow the links.

Link the Ideas

elimination method
- an algebraic method of solving a system of equations
- Add or subtract the equations to eliminate one variable and solve for the other variable.

You can solve a system of linear equations using the **elimination method**. To do this, a variable in both equations must have the same or opposite coefficient. It is often necessary to multiply one or both equations by a constant.

For example, solve the following linear system:

$$6a + 5b = 24$$
$$4a + 3b = 12$$

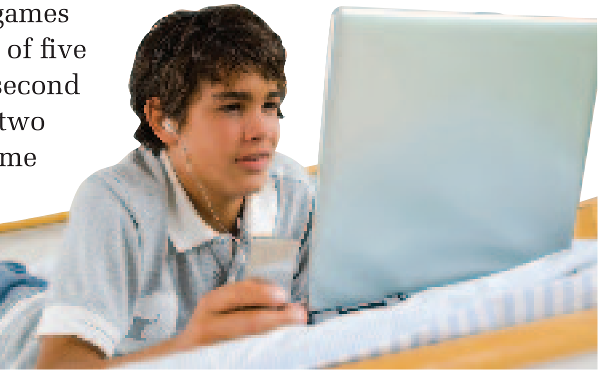
In order to eliminate variable a, you need to multiply the first equation by 2. Multiply the second equation by 3. Now, both equations will contain the term $12a$.

$$2(6a + 5b) = 2(24)$$
$$3(4a + 3b) = 3(12)$$

Why should you choose a constant with the smallest possible value?

Example 1 Solve a System of Linear Equations by Elimination

Connor downloaded two orders of games and songs. The first order consisted of five games and four songs for \$26. The second order consisted of three games and two songs for \$15. All games cost the same amount, and all songs cost the same amount. Write a system of linear equations. Then, determine the cost of one song and the cost of one game.



Solution

Let S represent the cost of one downloaded song, in dollars.
Let G represent the cost of one downloaded game, in dollars.

Write two linear equations. Write an equation to represent the first order.

$5G + 4S = 26$ ①

How does ① represent the cost of five games and four songs?

Write an equation to represent the second order.

$3G + 2S = 15$ ②

Determine which variable to eliminate. One strategy is to examine each variable in both equations. Look for a least common multiple for the coefficients of the G terms or the S terms.

$5G + 4S = 26$

$3G + 2S = 15$

The coefficients of the term $4S$ in ① and the term $2S$ in ② have a least common multiple of 4.

Multiply ② by -2 so that there is an opposite S term to $-4S$ in ①. Then, you can add the equations to eliminate the S term.

You can also multiply by 2. Then, you can subtract the equations.

$$\begin{aligned} -2(3G + 2S) &= -2(15) \\ -6G - 4S &= -30 \quad ③ \end{aligned}$$

Add ③ and ① to eliminate the S terms.

$$\begin{array}{rcr} -6G - 4S &=& -30 \\ +\ (5G + 4S &=& 26) \\ \hline -G \qquad &=& -4 \\ G \qquad &=& 4 \end{array}$$

Solve for G.

What does the value 4 represent?

Solve for the remaining variable, S, by substituting 4 for G in ① or ②.

$$\begin{aligned} 5(4) + 4S &= 26 \\ 20 + 4S &= 26 \\ 4S &= 6 \\ S &= 1.5 \end{aligned}$$

What does the value 1.5 represent?

Check:

Substitute into ① and ②.

Left Side	Right Side
$5G + 4S$	26
$= 5(4) + 4(1.5)$	
$= 20 + 6$	
$= 26$	

Left Side = Right Side

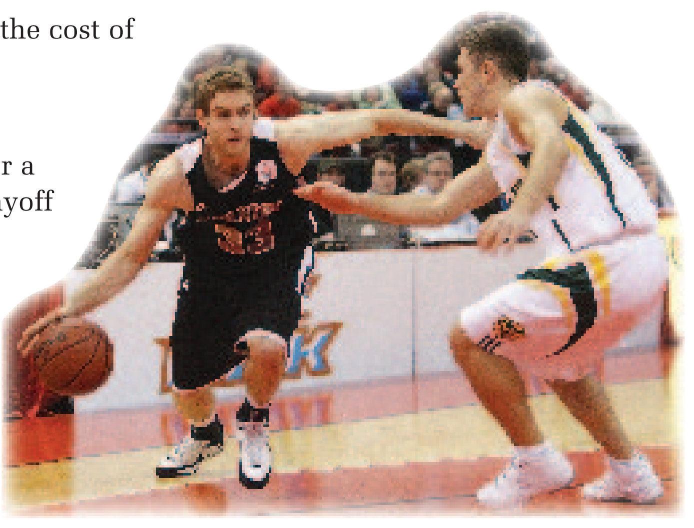
Left Side	Right Side
$3G + 2S$	15
$= 3(4) + 2(1.5)$	
$= 12 + 3$	
$= 15$	

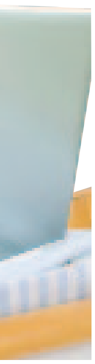
Left Side = Right Side

The cost of one game is \$4.00, and the cost of one song is \$1.50.

Your Turn

A group of people bought tickets for a University of Alberta basketball playoff game. Two student tickets and six adult tickets cost \$102. Eight student tickets and three adult tickets cost \$114. What was the price for a single adult ticket? What was the price for a single student ticket?





Example 2 Solve a System of Linear Equations Using a Table and Elimination

Did You Know?

A carbon sink is the term used for trees and plants that absorb carbon atoms into their roots and leaves. Carbon sinks reduce the amount of carbon dioxide in the atmosphere. Why might this be important?

A crop farmer has contracted with the Pacific Carbon Trust (PCT) to convert some of her cropland into woodland. This will create a carbon sink that is used to offset the production of carbon resulting from her farm activities. The farmer has 500 ha of cropland. She earns approximately \$220/ha from the crops. The PCT will pay her \$60 for every hectare of cropland that she converts. She would like a minimum revenue of \$90 800 that year. Using the elimination method, determine the number of hectares that she needs to convert to woodland. How many hectares of cropland would be left?

Solution

Let c represent the number of hectares of cropland.
Let w represent the number of hectares of woodland.
Organize the information in a table.

Type of Land	Revenue Generated Per Hectare (\$)	Number of Hectares	Revenue Generated (\$)
Cropland	220	c	$220c$
Woodland	60	w	$60w$
Total		500	90 800

Write an equation to show the total number of hectares.
$c + w = 500$ ①

Write an equation to determine the revenue created.
$220c + 60w = 90\ 800$ ②

The farmer wants a minimum revenue of \$90 800.

Determine which variable to eliminate. One strategy is to examine each variable in both equations. Then, identify the coefficient, other than 1, that is closest to zero.

$c + w = 500$

$220c + \boxed{60}w = 90\ 800$

The term $60w$ in ② has the coefficient closest to zero.

Multiply ① by -60 so that there is an opposite w term.

$-60(c + w) = -60(500)$

$-60c - 60w = -30\ 000$ ③

What do you multiply ① by if you want to subtract the equations?

Add ② and ③ to eliminate the w terms.

$$\begin{array}{rrcr} & 220c + 60w & = & 90\ 800 \\ + & (-60c - 60w & = & -30\ 000) \\ \hline & 160c & = & 60\ 800 \\ & c & = & 380 \end{array}$$

What does the value 380 represent?

Solve for the remaining variable, w, by substitution.

$380 + w = 500$

$w = 120$

Why is it more efficient to use ① instead of ② to solve for w?

What does the value 120 represent?

Check:

Substitute into ① and ②.

Left Side	Right Side
$c + w$	500
$= 380 + 120$	
$= 500$	

Left Side = Right Side

Left Side	Right Side
$220c + 60w$	90 800
$= 220(380) + 60(120)$	
$= 83\ 600 + 7200$	
$= 90\ 800$	

Left Side = Right Side

To generate a revenue of \$90 800, the farmer could convert up to 120 ha to woodland. This would leave 380 ha for cropland.

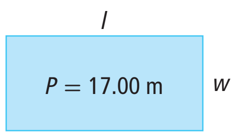
Your Turn

During lunch, the cafeteria sold a total of 160 muffins and individual yogurts. The price of each muffin is \$1.50. Each container of yogurt is \$2.00. The cafeteria collected \$273.50. Set up and solve a linear system in order to determine the number of muffins and the number of yogurts sold.

Example 3 Solve a System of Linear Equations in Different Forms by Elimination

The perimeter of a rectangular garden is 17.00 m. Triple the length is 2.46 m longer than five times the width. Sketch and label a diagram. Create a system of linear equations to determine the dimensions of the rectangle. Solve the system using elimination.

Solution



Let w represent the width of the rectangle, in metres.
Let l represent the length of the rectangle, in metres.

Write an equation to represent the perimeter.
$2w + 2l = 17.00$ ①

Write an equation to represent the difference in the dimensions.
$3l = 5w + 2.46$ ②

Rearrange ② so that it is in the form $ax + by = c$, similar to ①.

$$3l - 5w = 5w + 2.46 - 5w$$
$$-5w + 3l = 2.46 \quad ③$$

Decide whether you need to multiply one or both equations by a constant to eliminate a variable. Multiply ① by 3 and multiply ③ by -2 so that there are opposite l terms.

$$3(2w + 2l) = 3(17.00)$$
$$6w + 6l = 51.00 \quad ④$$

$$-2(-5w + 3l) = -2(2.46)$$
$$10w - 6l = -4.92 \quad ⑤$$

Add ④ and ⑤ to eliminate l.

$$\begin{array}{rrl} & 10w - 6l = & -4.92 \\ + & (6w + 6l = & 51.00) \\ \hline & 16w \quad = & 46.08 \\ & w \quad = & 2.88 \end{array}$$

Solve for w.

Solve for l by substituting into ②.

$3l = 5w + 2.46$
$3l = 5(2.88) + 2.46$
$3l = 14.40 + 2.46$
$3l = 16.86$
$l = 5.62$

Check:
Substitute into ① and ②.

Left Side	Right Side
$2w + 2l$	17.00
$= 2(2.88) + 2(5.62)$	
$= 5.76 + 11.24$	
$= 17.00$	

Left Side = Right Side

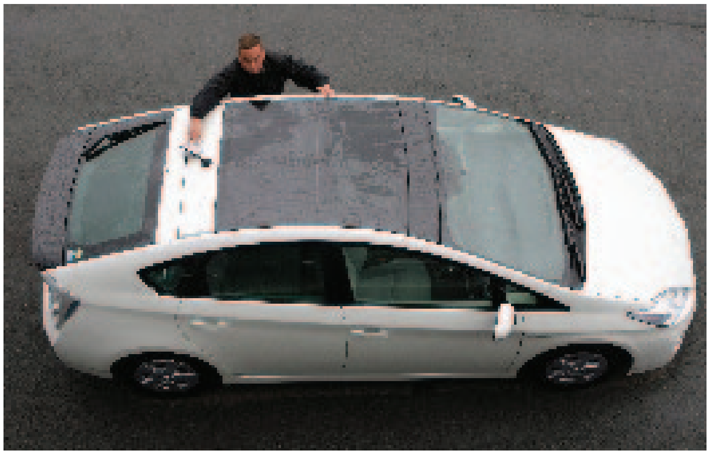
Left Side	Right Side
$3l$	$5w + 2.46$
$= 3(5.62)$	$= 5(2.88) + 2.46$
$= 16.86$	$= 14.40 + 2.46$
	$= 16.86$

Left Side = Right Side

The garden has a width of 2.88 m. Its length is 5.62 m.

Your Turn

A rectangular parking pad for a car has a perimeter of 12.2 m. The width is 0.7 m shorter than the length. Use a linear system to determine the dimensions of the pad.



Key Ideas

- A table can help you organize information in a problem. This can help you to determine the equations in a linear system.
- You can solve a linear system by elimination.

  $3x + 2y + 6 = 0$
  $7y = 5x + 41$

  - If necessary, rearrange the equations so that like variables appear in the same position in both equations. The most common form is $ax + by = c$.

    $3x + 2y + 6 = 0$ $7y = 5x + 41$
    $3x + 2y = -6$ ① $-5x + 7y = 41$ ②

  - Determine which variable to eliminate. If necessary, multiply one or both equations by a constant to eliminate the variable by addition or subtraction.

    Multiply ① by 5 and ② by 3 so that the coefficients of the terms involving x add to zero.

    $5(3x + 2y) = 5(-6)$ $3(-5x + 7y) = 3(41)$
    $15x + 10y = -30$ ③ $-15x + 21y = 123$ ④

    Add ③ and ④ to eliminate x.

    $$\begin{array}{rrcr} & 15x + 10y & = & -30 \\ + & (-15x + 21y & = & 123) \\ \hline & 31y & = & 93 \end{array}$$

    You can also multiply by -3 or by -5 and then subtract.

  - Solve for the remaining variable.

    $31y = 93$
    $y = 3$

  - Solve for the second variable by substituting the value for the first variable into one of the original equations.

    $7(3) = 5x + 41$
    $21 = 5x + 41$
    $-20 = 5x$
    $-4 = x$

- Check your solution by substituting each value into both original equations.

Check Your Understanding

Practise

1. Solve using elimination.

a) $x + y = 10$
$x - y = 4$

b) $x + 2y = 13$
$x - y = 8$

c) $y - 2x = -4$
$y + 3x = 16$

2. Rearrange the equations so that the variables are ordered in the same way for both equations.

a) $y - 3x = 11$
$x - y = -5$

b) $x + 7 = y$
$2x + y = -8$

c) $4 - 3y = x$
$x - y = 16$

3. Use the table to set up a linear system. Use the elimination method to determine the number of tickets sold to:

a) adults b) students

Ticket Type	Price Per Ticket ($)	Number of Tickets Sold by Type	Revenue Collected by Theatre ($)
Students	10	s	$10s$
Adults	13	a	$13a$
Total		430	4804

4. Solve the following systems of linear equations by elimination. Check your answers.

a) $3x + 2y = 7$
$4x + 5y = 14$

b) $7x - 6y = 27$
$2x + 9y = -3$

c) $4y + 29 = 3x$
$8x + 7 = 3y$

5. Solve using the elimination method. Leave your answers in fraction form.

a) $3x + 2y = 10$
$2x - y = 4$

b) $\frac{x}{3} - y = \frac{3}{5}$
$x + 6y = 4$

c) $2 - \frac{y}{2} = \frac{x}{3}$
$\frac{2}{3}(2x - 3y) = 4$

6. Solve the following system. Explain the result.

$3x + 2y = 7$
$9x + 6y = 16$

Apply

Solve problems 7 to 14 using the elimination method. Check your answers.

7. A preschool playground has both bicycles and tricycles. There is a total of 30 seats and 70 wheels. How many bicycles are there? How many tricycles are there?

8. Students at Evergreen High School want to help the community with the Communities in Bloom project. They decide to sell flower bulbs to raise money. Nancy sells 10 bags of tulip bulbs and 12 bags of iris bulbs for $380. Shawn sells 6 bags of tulip bulbs and 8 bags of iris bulbs for $244. What is the cost of one bag of tulip bulbs? What is the cost of one bag of iris bulbs?

Did You Know?

Communities in Bloom is a national non-profit organization dedicated to the creation and maintenance of green spaces in urban settings. On September 20, 2008, Lethbridge, AB, hosted the 14th Communities in Bloom National Awards Ceremony.

9. At the snack bar, five toasted bagels and three cans of juice cost \$12.50. Three toasted bagels and six cans of juice cost \$12.75. What is the price for one bagel? What is the price for one juice?

10. A total of 430 dogs and people attended the Woof Walk fundraiser. Altogether, 1210 legs participated in the walk. How many dogs were there?

11. A ferry is carrying 600 vehicles, including trucks and passenger vehicles. The fees collected total \$29 200. The charge per truck is \$100. The charge per passenger vehicle is \$45. How many trucks and how many passenger vehicles is the ferry carrying?

12. An avalanche rescue team travels 8.55 km along a snow-covered trail. For the first section, the trail is flat. The team averages a speed of 2.7 km/h. Then, the terrain becomes mountainous and their average speed is only 1.2 km/h. The one-way trip takes the team 4.0 h. Determine the distance that the team travels on each type of terrain.

Did You Know?

A Joule (J) is the energy involved when a force of 1 N (newton) acts to move an object through a distance of 1 m.

13. Soo Jin had basketball practice after school. Then, she cycled home. Playing basketball, she expends energy at a rate of 25 kJ per minute. Cycling home, she burns energy at a rate of 21 kJ per minute. She spent a total of 90 min doing both forms of exercise. During this time, she expended a total of 2178 kJ of energy. How much time did she spend doing each activity?

14. **Unit Project** Sharon estimates that she saves 260 L of water per week by washing her car with a bucket and sponge. Her sister Bev washes her car with a hose, which uses more water. Sharon's washing machine uses 225 L of water per load. Bev has upgraded to a washing machine that uses only 95 L of water per load. Both sisters wash the same number of loads of laundry per week. Both wash their car once a week.

a) Develop a system of equations representing their water usage in one week.

b) When their water usage is the same, how many loads of laundry does each sister do in one week?

c) If each sister does eight loads of laundry per week, who uses more water weekly? Explain.

Extend

15. Simplify. Then, solve the following linear systems using elimination.

a) $3(x + 2) + 7y = 11$
$-5(3 - x) + 9y = -12$

b) $5x - 2(y + 4) = y - 3x$
$2(x + 8y) - 4y = 9x$

16. Brittany invested a total of $3000 in two different investments. The safer investment earned 3.5% interest by the end of the year. The riskier investment earned 5.2% interest by the end of the year. Her total interest earned was $126.25. How much did she invest in the safer investment?

17. Milk that has 3.25% milk fat (MF) is mixed with milk that has 1% MF. What volume of each is needed to obtain 60 L of milk that has 3% MF? Express your answers to the nearest tenth of a litre.

Did You Know?

Milk fat, or butterfat, is the fatty portion of milk. Milk and cream are sold according to the amount of milk fat they contain.

18. Determine the value of k so that $6x + 4y = 7$ and $kx + 8y = 7$ do not have a common solution.

19. The solution to the system $10x + 12y = -18$ and $5x + 4y = b$ is $(9, a)$. What can b and a be? Is there more than one possibility? Explain.

Create Connections

20. **a)** Choose a question from section 9.2 and solve it using the substitution method.

b) Explain why you selected the question you did.

c) Are there any questions in section 9.2 that do not suit being solved using the substitution method? Explain.

21. What do you need to consider when choosing whether to use the substitution method or the elimination method to solve a system of linear equations? Provide examples to clarify your explanation.

9.3 Solving Problems Using Systems of Linear Equations

Focus on ...

- choosing a strategy to solve a problem that involves a system of linear equations

Did You Know?

Dog teams are making a comeback in the Arctic. Part of the reason is that dog teams do not need gas or expensive parts.

Many people drive hybrid vehicles because these cars consume less gas. Some people drive hybrids to reduce their ecological footprint. Others like the savings in the cost of gas. How can you determine if a hybrid is cost efficient?

Investigate Solving a Problem Involving a System of Linear Equations

Materials

- graphing calculator or computer with spreadsheet software

A sample price for a hybrid car is \$28 000. The price of a similar car powered by gas is \$21 500. The hybrid vehicle costs \$0.18 per kilometre to operate. The non-hybrid vehicle costs \$0.22 per kilometre to operate.

1. Write a system of linear equations that models the total cost for each vehicle in relation to the distance travelled.

2. Solve the linear system using a graphing calculator or spreadsheet software. After how many kilometres will the hybrid car be more cost efficient?

3. a) Solve the same system of equations from step 1 algebraically. Use either the substitution method or the elimination method.

b) Explain why you chose the method you did.

4. **Reflect and Respond** Compare and contrast the following three methods of solving a system of linear equations:
- graphically
- algebraically by substitution
- algebraically by elimination

Include examples.

5. Compare your response to step 4 with a classmate's. Note any insights that your classmate provides.

Link the Ideas

You can use graphical or algebraic methods to solve systems of linear equations. Each method has its advantages and disadvantages.

Method	Advantages	Disadvantages
Graphical	• provides a visual that can show how two variables relate • can be done with or without a graphing calculator • can result in an accurate and quick solution when using a graphing calculator	• can be time-consuming • may not provide an exact solution
Algebraic	• allows for an exact solution relatively quickly • can be done using more than one method (substitution and elimination)	• does not provide any visual insight into how the two variables relate • can result in an incorrect answer due to a minor arithmetic error

Example 1 Compare Methods of Solving

Jeremy and Shilan participated in their school's Plant-a-thon fundraiser. Jeremy started planting seedlings at 10:00 a.m. He planted at a steady rate of one tree per minute. Shilan started planting at 11:30 a.m. Her planting rate was three trees every 2 min.

a) At what time had they planted the same number of trees? Use a system of linear equations to find out. Solve the system using a graphing calculator and algebraically.

b) Which method do you prefer? Why?

Solution

a) Let n represent the number of trees that were planted after 11:30 a.m. Let m represent the number of minutes that have passed since 11:30 a.m. Determine equations for the number of trees planted by Jeremy and Shilan.

For Jeremy: $n = 90 + 1m$

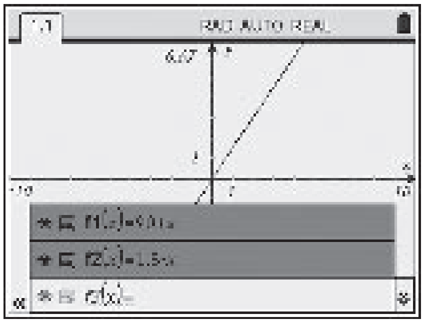
Jeremy started at 10:00 a.m. He planted at a steady rate of one tree per minute. What does 90 represent in the equation?

For Shilan: $n = 1.5m$

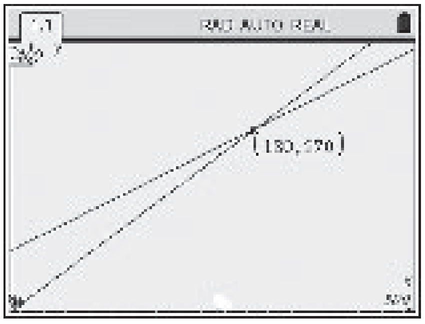
Shilan planted three trees every 2 min. What does 1.5 represent in the equation?

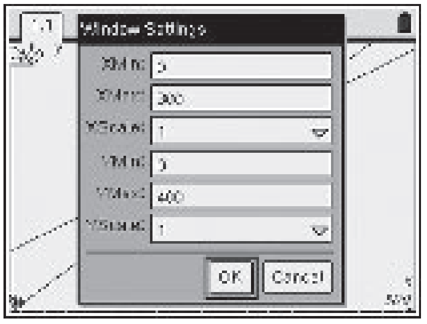
Method 1: Solve Using a Graphing Calculator

Graph the equations using a calculator.



Determine an appropriate window and graph it.





The point of intersection is (180, 270).

Check:
Substitute the intersection point into the original equations.

For Jeremy:

Left Side	Right Side
$n = 270$	$90 + 1m$
	$= 90 + 180$
	$= 270$

Left Side = Right Side

For Shilan:

Left Side	Right Side
$n = 270$	$1.5m$
	$= 1.5(180)$
	$= 270$

Left Side = Right Side

Method 2: Solve Algebraically

In both equations, n is isolated. Therefore, the substitution method is an appropriate algebraic method.

Write Shilan's planting rate to be equal to Jeremy's rate.

$$1.5m = 90 + 1m$$
$$0.5m = 90 \quad \text{Solve for } m.$$
$$m = 180$$

Solve for n by substituting into one of the initial equations.

$$n = 1.5m$$
$$n = 1.5(180)$$
$$n = 270$$

Check:
Substitute into the original equations.

For Jeremy:

Left Side	Right Side
$n = 270$	$90 + 1m$
	$= 90 + 180$
	$= 270$

Left Side = Right Side

For Shilan:

Left Side	Right Side
$n = 270$	$1.5m$
	$= 1.5(180)$
	$= 270$

Left Side = Right Side

Both Jeremy and Shilan had planted 270 trees 180 min after Shilan started. 180 min = 3 h and 11:30 a.m. + 3 h = 2:30 p.m. They had planted the same number of trees at 2:30 p.m.

b) The algebraic method is preferable. For the graphing method, the equations were already in the form $y = mx + b$, so it was easy to graph. However, it took some time to find a reasonable window to determine the intersection point. For the algebraic method, the substitution method was easy to use, since both equations were in the form $y = mx + b$.

Your Turn

Solve the following system of linear equations algebraically and graphically. Which method do you prefer? Explain.

$$y = 0.25x - 200$$
$$3x + 2y = 160$$

Example 2 Compare Algebraic Methods


Folkfest, Saskatoon

At the Métis People Pavilion, visitors can enjoy bannock and buffalo stew. A recent sale of three orders of stew and two orders of bannock cost \$13.50. A second sale of four orders of stew and five orders of bannock cost \$21.50.

a) Use a system of linear equations to determine the price of one order of bannock and the price of one order of stew. Solve the system algebraically using two methods.

b) Compare the two methods.

Did You Know?

There are numerous multicultural festivals across western Canada. People celebrate and learn about the culture, food, and entertainment of various ethnic and indigenous groups. Every August, the Folklorama Festival in Winnipeg and Folkfest in Saskatoon have a Métis pavilion.

Solution

a) Let B represent the cost for one order of bannock, in dollars.
Let S represent the cost for one order of buffalo stew, in dollars.
Write an equation to represent the first sale.

$3S + 2B = 13.50$ ①

Write an equation to represent the second sale.

$4S + 5B = 21.50$ ②

Method 1: Use Substitution

Isolate the variable B in ① since it has the smallest coefficient, 2.

$$\begin{aligned} 3S + 2B &= 13.50 \\ 2B &= 13.50 - 3S \\ B &= 6.75 - 1.5S \end{aligned}$$

You can try isolating the variable S in ① instead.

$$\begin{aligned} 3S + 2B &= 13.50 \\ 3S &= 13.50 - 2B \\ S &= 4.5 - \frac{2}{3}B \end{aligned}$$

Why might it be better to isolate B?

Substitute the expression for B into ②.

$$\begin{aligned} 4S + 5(6.75 - 1.5S) &= 21.50 \\ 4S + 33.75 - 7.5S &= 21.50 \\ -3.5S + 33.75 &= 21.50 \\ -3.5S &= -12.25 \\ S &= 3.50 \end{aligned}$$

What does 3.50 represent?

Substitute the value of S into ① or ② to solve for B.

$$\begin{aligned} 3(3.50) + 2B &= 13.50 \\ 10.50 + 2B &= 13.50 \\ 2B &= 3.00 \\ B &= 1.50 \end{aligned}$$

What does 1.50 represent?

Method 2: Use Elimination

Multiply ① by 5. Multiply ② by −2. Then, eliminate the variable B by addition.

$5(3S + 2B) = 5(13.50)$ $\quad\quad$ $-2(4S + 5B) = -2(21.50)$

$15S + 10B = 67.50$ ③ $\quad\quad$ $-8S - 10B = -43.00$ ④

Add ③ and ④ to eliminate the variable B.

$$\begin{aligned} 15S + 10B &= 67.50 \\ +(-8S - 10B &= -43.00) \\ \hline 7S \quad\quad &= 24.50 \\ S \quad\quad &= 3.50 \end{aligned}$$

Solve for S.

Substitute the value of S into ① or ② to solve for B.

$$\begin{aligned} 3(3.50) + 2B &= 13.50 \\ 10.50 + 2B &= 13.50 \\ 2B &= 3.00 \\ B &= 1.50 \end{aligned}$$

Check:

Substitute into ① and ②.

Left Side	Right Side
$3S + 2B$	13.50
$= 3(3.50) + 2(1.50)$	
$= 10.50 + 3.00$	
$= 13.50$	

Left Side = Right Side

Left Side	Right Side
$4S + 5B$	21.50
$= 4(3.50) + 5(1.50)$	
$= 14.00 + 7.50$	
$= 21.50$	

Left Side = Right Side

The price of one order of stew is \$3.50. The price of one order of bannock is \$1.50.

b) For this question, it took more steps to isolate the first variable using the substitution method than the elimination method. Also, if the variable S had been isolated in ①, the result for S would have made it complicated to solve for B.

Your Turn

Solve the linear system twice, using both algebraic methods. Compare the two methods.

$3x - 4y = 17$

$4x + 5y = 48.5$

Key Ideas

- Systems of linear equations can be solved
  - graphically
  - algebraically by substitution or by elimination
- It may be better to use a graphical approach to solve linear equations when you wish to see how the two variables relate, such as for cost analysis and speed problems.
- It may be better to use an algebraic approach to solve linear equations when
  - you need only the solution (intersection point)
  - it is unclear where to locate the solution on a coordinate plane

Check Your Understanding

Practise

1. Solve each system of linear equations using a method of your choice. Check your answer graphically.

a) $2x - 5y = 12$
$-7x + 5y = 48$

b) $3y = 6 - x$
$5x + 6y = -6$

c) $n = 3k - 2$
$2n - 6k = -4$

2. Solve each system of linear equations using your preferred method.

a) $0.2y + x = 0.7$
$2y + 12x = 11$

b) $\frac{m}{7} + \frac{n}{2} = 7$
$2m + 6 = 3n$

c) $4x - 7y = 6$
$5x = 2y + 3$

Apply

3. In January, the average high temperature for Calgary is 9.9 °C greater than Winnipeg's average high temperature. The sum of these two temperatures is −15.5 °C. What is the average high temperature in January for each of these two cities?

4. In Canada, the percent of workers who drive themselves to work is approximately 11.3 times the percent of workers who walk. The combined percent of Canadian workers who drive themselves and walk is about 78.7%. Approximately what percent of Canadian workers walk to work? Express your answer to the nearest tenth of a percent.

5. A school's multicultural club is selling muffins for a fundraiser. The club spends \$16.00 on advertising. The cost of ingredients for each muffin is \$0.30. The club decides to sell the muffins for \$0.75 each. The following equations model this situation: $C = 0.3m + 16$ and $C = 0.75m$.
   a) Describe in words what each equation represents.
   b) Determine the minimum number of muffins the club will have to sell to cover their total costs.

6. An incandescent 60-W light bulb sells for approximately \$0.75. It costs \$0.0072 to operate per hour. An equivalent compact fluorescent bulb costs \$4.00. It uses 15 W of power and costs \$0.0018 per hour to operate. The following equations model the cost: $C = 0.75 + 0.0072h$ and $C = 4 + 0.0018h$. In these equations, C is the total cost, in dollars, and h is the number of hours. How many hours will it take for the compact fluorescent bulb to be less expensive?

Did You Know?

A compact fluorescent bulb has a lifespan of approximately 10 000 h versus 1000 h for an incandescent bulb.

7. A circus recently had a sold-out performance. There were varying admission prices. The admission for premium seating was \$250 for adults and \$175 for students. The total revenue for premium seating was \$29 125. The receipts showed that 130 premium seats were sold. Determine how many adults and how many students were in premium seats.

Cirque du Soleil

Did You Know?

Cirque du Soleil is a dramatic mix of circus arts and street entertainment. Based in Montréal, Québec, it was founded in 1984. The founders were two former street performers, Guy Laliberté and Daniel Gauthier. Cirque expanded rapidly. It grew from one show to multiple shows around the world.

8. Jason is renting a car for one week. Speed-E-Car Rental offers a compact car for \$379 plus \$0.10 per kilometre. Easy 4 U Auto offers a compact car for \$249 plus \$0.35 per kilometre. Use a system of linear equations to determine when each company would be the better choice for Jason.

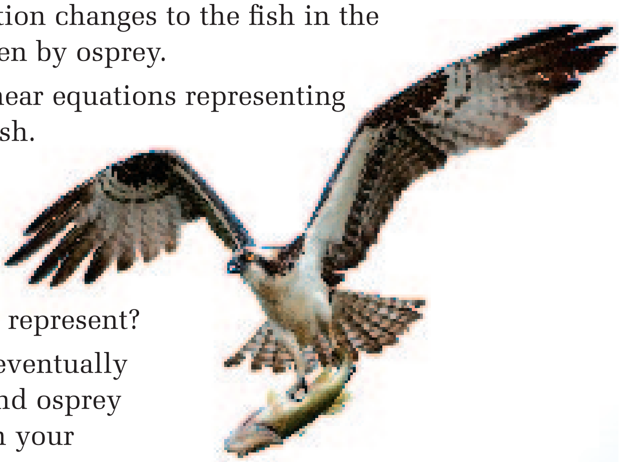
9. **Unit Project** The water level in a lake is decreasing. Wildlife biologists are concerned about the effect on the fish population. They decide to track the number of fish in the lake. The osprey is a fish-hunting bird. As part of their study, the biologists need to estimate the number of fish eaten by osprey.

Year	Fish in Lake	Fish Eaten by Osprey
1	10 000	700
2	9 000	900
3	8 000	1100
4	7 000	1300

a) Describe the population changes to the fish in the lake and the fish eaten by osprey.

b) Write a system of linear equations representing the populations of fish.

c) Solve the system of linear equations graphically. What does the point of intersection represent?



d) Predict what might eventually happen to the fish and osprey populations. Explain your thinking.

Did You Know?

Scuba divers can suffer from a potentially lethal condition called *decompression sickness*, or *the bends*. This occurs if they rise too quickly to the surface of the water. At greater depths, there is extra pressure on the diver. The extra pressure causes nitrogen gas to dissolve in the diver's blood. As the diver slowly rises back to the surface, the nitrogen gas comes out of solution and forms bubbles in the blood. If the diver ascends too quickly, the nitrogen remains dissolved in the blood. The result is the bends, a painful condition that may be fatal.

10. Scuba divers can spend only a limited amount of time at depths between 60 m and 90 m. This amount of time can be represented by a linear relation. A diver can remain for 60 min at a depth of 60 m, and 30 min at a depth of 90 m. Write a system of linear equations to determine the slope intercept form, $y = mx + b$, for this linear relation.

11. Last Saturday, Juan went cross-country skiing in the morning. In the afternoon, he played squash. Cross-country skiers expend 50 kJ of energy per minute. Squash players burn 42 kJ per minute. In total, Juan exercised for 100 min. He used 4850 kJ in energy. How much time did he spend doing each activity?

Extend

12. The lines that enclose a triangle can be represented by graphs of the equations $y = 3$, $y = -x + 7$, and $y = 2x + 16$. Use a system of linear equations to determine the area of the triangle.

13. Answer the following questions using the two systems of linear equations shown.

$$\begin{aligned} 39x + 49y &= 2283 \\ 43x + 54y &= 2516 \end{aligned} \quad \text{and} \quad \begin{aligned} 39x + 49y &= 2283 \\ 43x + 54y &= 2517 \end{aligned}$$

   a) What difference do you see between the two systems of linear equations just by looking at them?
   b) Solve both systems algebraically.
   c) Solve both systems graphically.
   d) Explain why these linear systems are difficult to solve.

Create Connections

14. a) Create a system of linear equations. Solve your system using a method of your choice.
   b) How do you decide on a strategy for solving a system of linear equations? What do you consider? Why?

15. a) Create a system of linear equations with a solution involving a rational number that cannot be expressed exactly on a graphing calculator. Solve your system graphically and algebraically.
   b) Describe your results using the two methods.

9 Review

9.1 Solving Systems of Linear Equations by Substitution, pages 468–479

1. Solve by substitution.

a) $y = 3x - 1$
$x + y = 11$

b) $x - 2y = 4$
$x - 3y = 6$

c) $2 + y = 3x$
$6x - 5y = 8$

2. Determine the intersection point of the two lines $y = 3x - 4$ and $4x + y = 13$. Solve graphically, then algebraically. Which method is a better choice? Why?

3. The table shows data about two vehicles. What distance will both vehicles need to travel for the cost to be the same?

Type of Car	Initial Cost ($)	Cost Per Kilometre ($)
Compact hybrid	31 300	0.27
Compact non-hybrid	26 500	0.42

4. On a web site, the cost to download a game is three times the cost to download a song. The cost for five songs and two games is $15.40. What is the cost of one song and the cost of one game?

9.2 Solving Systems of Linear Equations by Elimination, pages 480–491

5. Solve using elimination.

a) $x - y = 17$
$x + y = -9$

b) $3x + 2y = 10$
$2x - y = 4$

c) $\frac{y}{2} = 2x - 3$
$3x + 2y = \frac{9}{2}$


Vancouver, BC


Yellowknife, NT

6. The number of wet days in a year for Vancouver, BC, is 47 days greater than the number for Yellowknife, NT. The sum of the numbers of wet days for one year in these two cities is 285. How many wet days occur in each city?

7. In an isosceles triangle, the two base angles have a sum that is 6° more than the third angle. Sketch and label a diagram. What is the measure of each of the three angles?

8. The percent of carbohydrates by weight in grapes is 15%. The percent of carbohydrates in an orange is 7%. Danika consumed a total of 325 g of grapes and oranges. The percent of carbohydrates in the mixture she ate was 10%. How many grams of grapes did she eat? How many grams of oranges?

9.3 Solving Problems Using Systems of Linear Equations, pages 492–501

9. The operators of a national park want to be more water efficient. They decide to start with the park's comfort stations. First, they purchase one front-load washing machine and four shower heads for $900. Then, they purchase ten washers and eight shower heads for $8200. What is the cost of each washing machine? What is the cost of each shower head?

10. Michelle rented a car from the same company twice last month. The cost of the first rental was $116.70 for three days. She drove a distance of 98 km. The cost of the second rental was $78.80 for two days of driving a distance of 72 km.

a) What is the daily rental cost? What is the charge per kilometre?

b) What method did you use to solve the problem? Why?

11. A local nature club wants to convert 57 acres of land to campgrounds. Basic campgrounds have a density of 1.5 sites per acre. Developed campgrounds have 4 sites per acre. The amount of land used for basic sites is twice the amount of land used for developed sites.

a) Determine the number of acres to be used for each type of campground if all 57 acres of land are used.

b) How many campsites of each type will there be?

12. Yesterday, Keegan read a book that had 220 pages. He started the book before dinner and read at a speed of 50 pages per hour. At bedtime, he read at a rate of 41 pages per hour and finished the book. He spent a total of 5 h reading. How much time did he spend reading before dinner? Express your answer in hours and minutes.

13. a) Solve the following system of linear equations graphically and algebraically: $y = 4 - x$ and $x + y = 6$.

b) Use the graphs of these two lines to explain your results.

9 Practice Test

Multiple Choice

For #1 to #4, choose the best answer.

1. The ordered pair (3, −2) is the solution for which linear system?

A $5x - y = 7$
$x + y = 12$

B $y = 4x + 11$
$x - y = 1$

C $2x - 7y = 1$
$x + y = -4$

D $x - y = 5$
$3x + 2y = 5$

2. Which of the following is the solution to the system of equations $y = 8 - x$ and $2x + 3y = 14$?

A (−10, 18) **B** (3, 5) **C** (10, −2) **D** (19, −11)

Did You Know?

Recycling in the Arctic is a challenge because of the expense of shipping the materials south to be recycled. Some Northern schools run can-recycling programs. These programs are possible thanks to local airlines. The airlines transport the crushed and packaged cans south to a recycler. They will often do this at a reduced rate or even no charge.

3. A recycling company sells its recycling bins for \$20 each. The fixed expenses to manufacture the bins total \$4000. In addition, there is an expense of \$12 per bin. Let x represent the number of bins. Which linear system allows you to determine when the company's expenses equal their sales?

A $y = 4000$
$y = 12x + 20$

B $y = 4000 - x$
$y = 20x + 12$

C $y = 4000 + 12x$
$y = 20x$

D $y = 4000 + 20x$
$y = 12x$

4. The cost of three compact fluorescent light bulbs (CFBs) and five incandescent light bulbs (ILBs) is \$15.00. The cost of a CFB is five times the cost of an ILB. What is the cost of one CFB? What is the cost of one ILB?

A CFB = \$0.40, ILB = \$2.00 **B** CFB = \$1.25, ILB = \$0.25
C CFB = \$3.75, ILB = \$0.75 **D** CFB = \$7.00, ILB = \$1.40

Short Answer

5. Solve each linear system algebraically. Show your work.

a) $3x - y = 7$
$x + y = 10$

b) $y = 7 - 9x$
$17 - 2y = 16x$

c) $\frac{x}{3} - y = 5$
$5x + 3y = 12$

6. The length of a rectangle is 3 m less than five times the width. The perimeter is 10.8 m. What are its dimensions?

7. At the bulk store, peanuts cost \$1.20 per 100 g and almonds cost \$2.00 per 100 g. They are mixed together to create a 300-g bag of mixed nuts. This mix sells for \$1.50 per 100 g. What amount of each type of nut is used?

8. Use the information in the table to determine the number of nickels and the number of quarters in the coin collection.

Coin Type	Value of One Coin (¢)	Number of Coins	Total Value of Coins (¢)
Nickel	5	n	$5n$
Quarter	25	q	$25q$
Total:		49	885

9. A golf club charges an annual fee. It also charges a green fee for each game played. Tegan played 38 games and paid a total of \$986. Cassandra played 15 games and paid \$480. How much are the annual fee and the green fee?

10. Francofièvre, or French Fever, is an event celebrating francophone culture. Last year, a total of 696 students and teachers from one high school went to the event. On average, every teacher brought 23 students. How many students and how many teachers from the high school attended the event that year?

Did You Know?

Francofièvre is the largest francophone youth rally in western Canada. It is held annually in Saskatchewan each spring. High school students visit to celebrate French music, dance, and artists.

Extended Response

11. Mallory drove 805 km from Edmonton to Regina. From Edmonton to Saskatoon, her average speed was 88 km/h. From Saskatoon to Regina, her average speed was 71 km/h due to road construction. She drove for 9.85 h. Express your answers to the following questions to two decimal places.

   a) How long did each part of her trip take?

   b) What is the distance between Edmonton and Saskatoon? Explain the method you used.

12. The three lines that enclose a triangle can be represented by graphs of the equations $x = 3$, $x + y = 10$, and $5x - 2y = 15$.

   a) Graph the three lines.

   b) Determine the intersection points between each pair of lines. Solve algebraically and graphically.

4 Unit Connections

Unit 4 Project

For this project, you will do an analysis of the effect of our water use on wildlife, reducing water use in homes, and retrofitting plumbing fixtures. Use your answers to the unit project questions throughout Chapters 8 and 9, as well as your own research. Your analysis should include the following:

- data involving the effect of our water use on populations of wildlife
- information about costs and flow rates of various low-flow plumbing fixtures
- a comparison using linear systems (represented multiple ways) of the cost of keeping conventional fixtures and the cost of retrofitting

WWW Web Link

For information and suggestions about reducing water use in the home, go to www.mhrmath10.ca and follow the links.

To complete your project, prepare a presentation that outlines the environmental and economic benefits of retrofitting and reducing water use. Keep in mind that your presentation is intended for community residents or local governments. Address the following questions in your presentation:

- What impact does reducing water use have on the environment?
- In what ways can water use in homes be reduced?
- What should be considered when making decisions about water use in homes?
- What costs and savings are associated with retrofitting?
- Do your local governments offer any incentives to encourage people to retrofit their houses or reduce water use? If so, what are they? If not, what suggestions do you have?

Unit Review

Chapter 8 Solving Systems of Linear Equations Graphically

1. Sketch and label a graph of two linear equations that could be used to represent each scenario.

a) The cost of two different cell phone plans over a number of months

b) The height of two hot-air balloons descending at the same rate from different initial heights

c) The constant speed of a racehorse running around a track and the constant speed of a racehorse after it has crossed the finish line

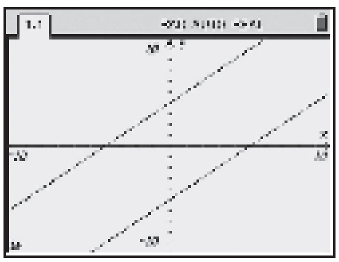
2. Match each system of equations to the correct graph.

A $y = -x + 4$
$y = x - 5$

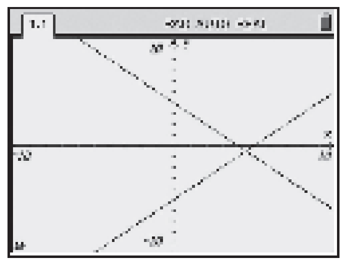
B $y = x + 4$
$y = x - 5$

C $y = -x + 4$
$y = -x - 5$

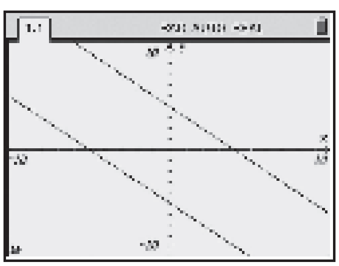
i)



ii)



iii)



3. Explain the meaning of the given point in relation to the system of linear equations.

a) $(1, -1)$, $y = -3x + 2$ and $y = x - 2$

b) $(0, 3)$, $y = \frac{3}{2}x + 2$ and $y = -x + 3$

4. Solve each of the linear systems graphically. Verify the solution.

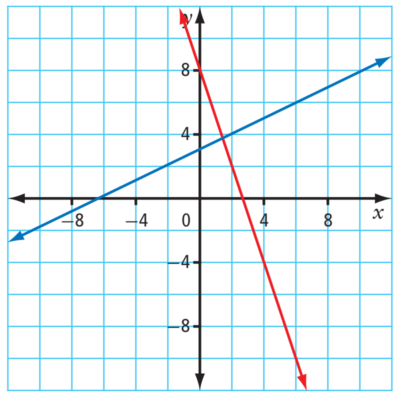
a) $y = -5x - 3$
$y = 4x - 3$

b) $y = -3x - 6$
$y = x + 2$

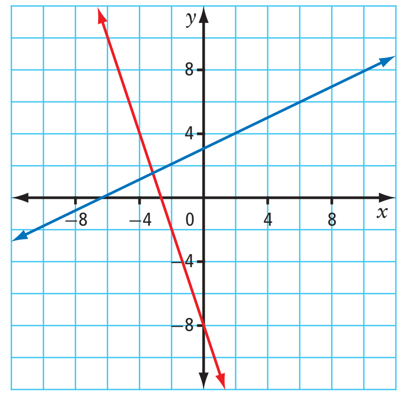
c) $y = 2x - 3$
$y = -x - 9$

5. Which of the following graphs shows the solution to the linear system of equations $2x + 4y - 12 = 0$ and $3x + y - 8 = 0$?

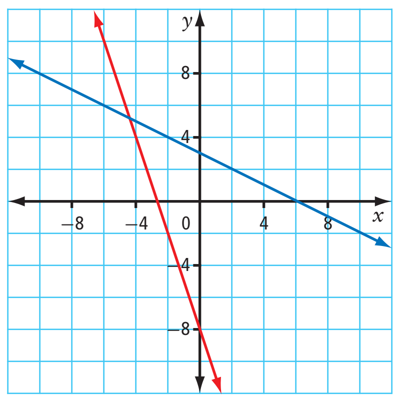
A



B



C



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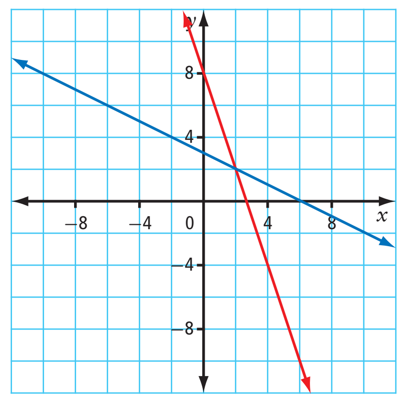


6. Use technology to solve each linear system graphically. Express solutions to the nearest tenth, if necessary.

a) $y = -5x + 2$
$y = 6x - 9$

b) $y = 7x - 2$
$y = x + 5$

c) $y = 4x - 5$
$y = -5x + 1$

7. Mei compared the cost of renting a car from two car rental companies. She graphed the number of days versus cost for each. She discovered that the point of intersection was (7, 580). Explain the meaning of this point.

8. Daniel had 20 coins in his pocket, consisting of dimes and quarters. The total value of the coins was \$2.75.

a) Model the situation using a system of linear equations.

b) What would be the domain and range of each function?

c) Solve graphically to determine the number of each type of coin Daniel has.

9. Determine whether each linear system has no solution, one solution, or an infinite number of solutions.

a) $y = \frac{5}{3}x - 4$
$y = \frac{3}{5}x - 4$

b) $3x + y - 11 = 0$
$3x + y + 11 = 0$

c) $2x - 6y + 5 = 0$
$4x - 12y + 10 = 0$

Chapter 9 Solving Systems of Linear Equations Algebraically

10. Colin's team is part of a lacrosse association in Manitoba. One point is awarded for an assist and two points for a goal. Colin has 23 points so far this season. Twice the number of goals is one more than the number of assists Colin has. Write a system of linear equations to model this situation.

11. Identify which method you would use to solve each linear system algebraically: the substitution method or the elimination method. Explain each choice.

a) $y = -x + 5$
$2x + 3y - 7 = 0$

b) $5y - 3x = 9$
$2y + 3x = 12$

c) $3x - 4y + 6 = 0$
$2x + 8y - 5 = 0$

d) $x + 8y = 20$
$3x + 6y = 24$

12. Solve each linear system. Verify your solutions.

a) $3x + y = 2$
$2x + 5y = 23$

b) $3x + 2y = 8$
$x - 12y = -10$

c) $3x + 5y = 1$
$7x + 9y = 5$

13. Solve each linear system. For each one, explain why you chose the method you did.

a) $x + 2y = 11$
$3x - 2y = 9$

b) $2x - 6y - 12 = 0$
$3x - 2y - 4 = 0$

c) $8x + 5y = -11$
$3x + 2y = -4$

14. The Thelon River stretches across 900 km of Northern Canada. Cheng and Tammy took a ten-day canoe trip along part of the river. They left camp going with the current of the river. After 20 min, they discovered they forgot something and had to return. It took 36 min to paddle the 3 km back to camp against the current.

a) Write a system of linear equations to represent the situation.

b) What was their paddling speed? What was the speed of the current?

4 Unit Test

Multiple Choice

For #1 to #4, choose the best answer.

1. The interpretive centre at Batoche, SK, depicts Métis life between 1860 and 1900. The site charges \$7.80 for each adult and \$3.90 for each youth. Imagine that on one day the total for adult and youth admissions was \$214.50. There were five fewer youths than adults. Which linear system could be used to solve for the number of adults, a, and the number of youths, y?

A $7.80a + 3.90y = 214.50$
$a - y = 5$

B $7.80a + 3.90y = 214.50$
$y + a = 5$

C $7.80a + 3.90y = 214.50$
$a = y - 5$

D $7.80a + 3.90y = 214.50$
$y = 5 - a$

2. James graphs two lines. Under what condition(s) must there be a point of intersection?

i) Both lines have the same y-intercept and the same slope.

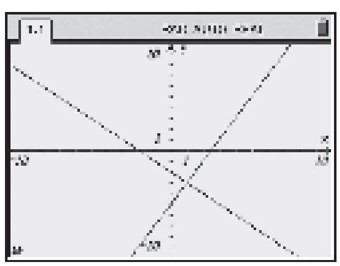
ii) Both lines have the same y-intercept but different slopes.

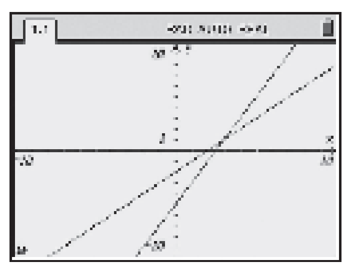
iii) Both lines have the same slopes and the same x-intercept.

iv) Both lines have different slopes but the same x-intercept.

A II only **B** I and III **C** II and IV **D** IV only

3. Which of the following graphs could represent the following linear system: $x + y = -2$ and $2x - y = 5$?

A


B


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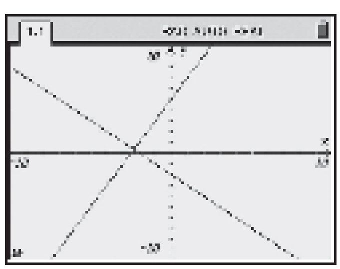

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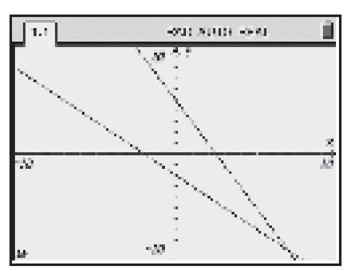


4. Carla correctly described the number of solutions to the system of equations $7x - 5y = 2$ and $7x - 5y = 4$ as

A one solution
B two solutions
C an infinite number of solutions
D no solution

Numerical Response

Complete the statements in #5 and #6.

5. You solve the system of equations $x + 3y = 0$ and $x + 6y - 30 = 0$ graphically. The y-coordinate of the solution is ■.

6. Mary Kuutsiq Mariq of Baker Lake, NU, created the wall hanging shown. It has a perimeter of 120 in. Its length is 6 in. shorter than twice its width. The length of the wall hanging is ■ in.

Hungry for Wildlife by Inuk artist Mary Kuutsiq Mariq

Written Response

7. Solve the following system of linear equations: $\frac{1}{2}x - \frac{2}{3}y = 6$ and $\frac{1}{4}x + \frac{1}{3}y = -1$. Verify the solution. Then, explain why you chose the method you did.

8. Dakota is going to make a bead necklace. The cost of 25 red beads and 15 green beads is \$2.75. The cost of 7 red beads and 13 green beads is \$1.65.
   a) Write a linear system of equations to represent the situation.
   b) Determine the cost of a red bead and the cost of a green bead.
   c) How much would ten red beads and five green beads cost in total?

9. American bison and North American elk are surprisingly fast. The sum of their top speeds is 104 km/h. The difference is 8 km/h. If the bison is the faster animal, what is its top speed?

Answers

Chapter 1

1.1 SI Measurement, pages 15 to 21

1. a) Example: Use a fingernail width as a referent for 1 cm.
   i) 18 fingernail widths (18 cm)
   ii) 23 fingernail widths (23 cm)
b) Example:
   i) 18.6 cm
   ii) 25 cm. It is not necessary to measure all sides. Opposite sides are equal, and subtraction can be used to calculate some of the smaller sides.

2. a) i) Example: Using a fingernail width as a referent for 10 mm, the curve of the S could be about $2\frac{1}{2}$ fingernail widths.
   ii) Example: Using a hand width as a referent for 10 cm, the curve of the S could be about 2 hand widths.
b) Use a piece of string and lay it down along the shape of the letter, and then measure the string.

3. a) 72 mm; 7.2 cm **b)** 18.4 mm; 1.84 cm
c) 34.4 mm; 3.44 cm

4. a) 105 m **b)** 300 cm or 3 m

5. a) No. Mountain heights are usually reported in metres. 5959 m
b) No. Centimetres are commonly used for distances about as wide as a hand. 6.4 cm
c) No. Metres would be more appropriate for something about twice the height of a person. 4.2 m
d) No. Metres or centimetres are commonly used for distances about the height of a person. 1.95 m or 195 cm

6. a) 384 cm **b)** 611 mm
c) Example: 643 mm

7. Examples: Measuring tape, ruler, laser ruler, car odometer, metre stick, trundle wheel, caliper, transit. To use a measuring tape for shorter distances, place the "0" end at one end of the object you want to measure and then read the length at the other end; for longer distances, count several convenient lengths (for example, 25 m) and then measure the last portion as explained for shorter distances.

8. Example: 159 mm

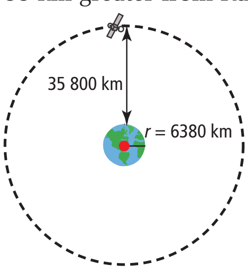
9. No. The ratio of the lengths is 8 cm : 5 cm or 8 : 5. The ratio of the heights is 5 cm : 2.5 cm or 10 : 5. The ratios are not the same, so the reduction would not be proportional.

10. 1.4 m

11. a) dime: 17.79 mm
quarter: 23.59 mm
b) 17.79 : 23.59 ≈ 1 : 1.3
c) Example: 31.28 mm. The actual diameter of a loonie is 26.5 mm. The ratio does not apply. Also, the penny and nickel are bigger in size than the dime yet worth less. The size of the coin does not indicate its value.

12. a) 1 : 6 250 000
b) 475 km
c) Example: from Virginia Falls: 250 km; from Rabbitkettle Lake: 338 km. The distance is 88 km greater from Rabbitkettle Lake.

13. a)



b) 40 086.7 km
c) 265 024.8 km **d)** 9372.4 km/h faster

14. a) Example: Use a fingernail width as a referent for 1 cm: 10 cm by 1.5 cm by 5 cm
b) front face: 22.4 cm; side face: 12.2 cm; top face: 28.6 cm. Three perimeters were calculated because opposite faces are the same. There are six faces in total.
c) 293 mm

15. 230 m

16. a) 442.5 m
b) Example: The route affects the total distance because if you finish cutting the last portion of grass far away from your starting point, you will then have to walk across grass you have already cut to get back to A. By using a route that allows you to finish the last part of cutting near your starting point, you minimize the amount you walk over any part twice.

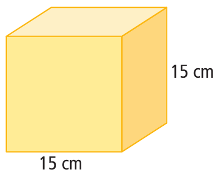
17. 15.9 cm

18. a) about 1 m or 100 cm

c) Example: The bandage will stretch more if pulled tight, and as well, there will be some overlapping of the bandage.

19. a) Example: Estimate that the tubes are 10 mm in diameter; about $(15)(15) = 225$ tubes would fit inside. But tubes are $\frac{4}{5}$ of 10 mm. So, about $\left(\frac{4}{5}\right)(225) \approx 280$ tubes would fit inside.

b)



c) Example: 360 tubes, if each tube settles between the two tubes below it. To check, you could draw a 15-cm by 15-cm square on a piece of paper and cut an 8-mm circular hole in another piece of paper. Then, trace as many circles as possible inside the square.

20. a) Example: Measuring down the inside of a pan, across the bottom, and up the other side gives 36.8 cm. Inside surface area is 338.56π cm²; measuring according to the formula gives $d = 30$ cm and $h = 6.0$ cm. $S = 405\pi$ cm²; the factory formula yields a surface area that is too large, since the formula does not take into account the curved nature of most frying pans.

b) No. Since d is the diameter of the entire pan, including the distance the sloping side adds to the flat bottom dimension, the formula will produce an area that is too large unless the side of the pan is perfectly vertical.

21. Step 2: Examples: A student may cut the corners. A student may slow down for the corners. A student may take smaller steps in order to make the corner.

1.2 Imperial Measurement, pages 29 to 35

1. a) $\frac{1}{16}$ in. **b)** $\frac{1}{40}$ in.; 0.025 in.

c) $\frac{1}{1000}$ in.; 0.001 in.

2. a) $13\frac{1}{2}$ in. **b)** $\frac{3}{4}$ yd

c) 76 mi **d)** 15 840 ft

3. Example:

a) $1\frac{7}{8}$ in.; length of an eraser

b) 3.154 in.; width of a calculator

c) 0.593 in.; diameter of a penny

4. Example:

a) a caliper; $\frac{5}{16}$ in. **b)** a string; 1 in.

c) a ruler; $5\frac{7}{8}$ in.

5. Example:

a) $4\frac{3}{4}$ small paper clips; $5\frac{1}{4}$ in.

b) $6\frac{3}{4}$ paper clips; 7 in.

6. a) Example: Use one large step as a referent for 3 ft.

b) Example: Use the length of your calculator as a referent for 6 in.

7. a) 6.25 mph **b)** 9.6 min

8. 183.4 in.

9. a) Determine the circumference of the drive wheels and the caster wheels. Then, divide the first value by the second. 1:8

b) about 119 **c)** about 1261

10. a) 1:2 **b)** about $3\frac{7}{8}$ in.

11. a) Example: No. Gail calculated $(7.5)(5) = 37.5$ ft², but 0.5 ft² ≠ 5 in². 1 ft² = (12 in.)(12 in.) = 144 in.², so 0.5 ft² = 72 in.².

b) 75 tiles

12. a) $4\frac{11}{16}$ in. **b)** $\left(4\frac{1}{4}\text{ in.}\right)\left(\frac{5}{8}\text{ in.}\right)\left(2\frac{3}{4}\text{ in.}\right)$

c) Example: 10 in.

13. a) Example: about 1000 yd. The direct distance on the map from the captain to the cache is about $2\frac{1}{2}$ in. The distance on the map along the route taken is about 5 in.

b) 950 yd or 2850 ft. The actual distance is farther since the paths between the red dots are not straight.

14. a) 19 ft 1 in.

b) 62 ft: 19 ft 9 in.; 65 ft: 20 ft 8 in.; 70 ft: 22 ft 3 in.

c) The 62 ft length would add approximately 4 in. all around the circumference of the pool.

15. a) Comet Hyakutake: 9 462 909 mi; Comet Hale-Bopp: 122 236 992 mi

b) 112 774 083 mi

16. a) Any pairs of corresponding sides.

$AC:DF = \frac{3}{4}:1\frac{3}{8} = 1:1.8;$

$BC:EF = \frac{3}{8}:\frac{11}{16} = 1:1.8;$

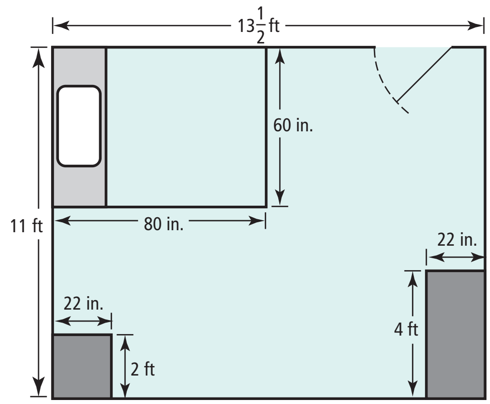
$AB:DE = \frac{9}{16}:1 = 1:1.8$

b) Similar triangles have the same shape but are larger or smaller by a scale factor.

c) Look for a figure enlarged by a factor of 3. Check that the lines drawn from the point to the enlargement are three times as long as the ones drawn from the point to the original figure.

17. a) double bed: 54 in. by 75 in.; queen-size bed: 60 in. by 80 in.

b) Example:



c) Example: Sam could buy either bed, as each size fits with his current furniture. The queen-size bed is larger and would be the better purchase.

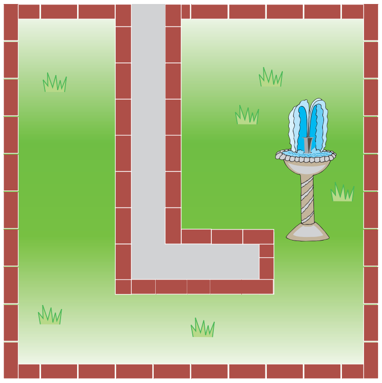
18. a) 14 ft

b) small turbine: 41.77 mph; large turbine: 46.77 mph

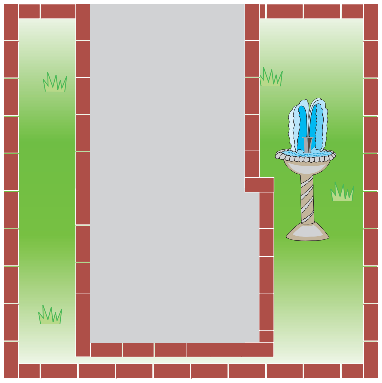
19. a) Example: 15 in. Each side looks to be about 2.5 in. and the border around the pathway looks to be about the same as the length of two sides.

b) 11.75 in.; Estimate overstated perimeter by 3.25 in.

c) Perimeter is 11.75 in.



d) Example: As the width of the pathway increases, the perimeter of the border remains the same.



1.3 Converting Between SI and Imperial Systems, pages 42 to 47

1. a) $2\frac{3}{8}$ in. **b)** 83 mm

c) $\frac{7}{8}$ in. or 22 mm

2. a) 0.03 mm **b)** 16 ft

c) 42.19 km **d)** 9.84 cm

3. a) Example: 8 in. or 20 cm

b) Example: 5 hand spans

c) Example: 24 in. or 60 cm

d) Example: about 2650 paces to walk a mile; about 1700 paces to walk a kilometre

e) Example: SI is easier because the units are multiples of powers of 10.

4. a) about 1.4 m **b)** 1:150

c) 1.5 m **d)** 9.2 ft by 12.8 ft

5. Maximum depth is 399 yd; Mount Columbia is 12 287 ft (2.33 mi) high; Mount Athabasca is 11 453 ft (2.17 mi) high; Average snow fall is 3.28 ft (39.4 in.).

6. a) 63 mm; $2\frac{1}{2}$ in. b) $2\frac{5}{16}$ in.; 5.87 cm
   c) 1.536 in.; 3.901 cm d) 4.78 cm; 1.9 in.
7. a) $45\frac{1}{2}$ ft or 45 ft 6 in.
   b) 13.87 m. Usually, measurements greater than 200 or 300 cm are reported using metres.
8. 175 km
9. a) metres to kilometres: divide by 1000; metres to centimetres: multiply by 100; yards to miles: divide by 1760
   b) In a length conversion, if the unit gets larger, the number must get smaller. Therefore, divide by the conversion factor to get a smaller number of the larger units. This works in either system, because a smaller number of larger units are required for the equivalent length.
   c) If the units get smaller, the number must get larger. Therefore, multiply by the conversion factor to get more of the smaller units.
10. Multiply the number of kilometres by $\frac{5}{8}$. Since 164 km ≈ 103 mi, Penny travelled a greater distance.
11. Lake Baikal: 1636 m; Great Slave Lake: 614 m; Great Slave Lake is 108 m deeper than Quesnel Lake
12. about 3951 mi
13. a) $41\frac{3}{4}$ in.
    b) Example: Using the SI measurements given, calculate the perimeter taking advantage of equal distances, then convert to inches. This way, you only have to do one conversion instead of many, which would make your answer less accurate.
14. a) 1667 LPs. Example: $\frac{20\,000}{10} = 2000$ LPs
    b) 185.2 in. or 15 ft $5\frac{3}{16}$ in.; 9400:183 or 51.4:1
15. a) 24 blocks
    b) 63 in. by $47\frac{1}{4}$ in. by $15\frac{3}{4}$ in.
    c) 32 extra blocks. Example: Each length and each width will be increased by 2 units. So, 1 extra block is needed on each side. $(2)(4) = 8$ more blocks than for the inside wall. $24 + 8 = 32$
16. a) Example: 234 m^2
    b) This formula will work for imperial units, as long as the imperial units are converted to decimals. For example, 6 ft 6 in. = 6.5 ft.
17. a) Example: SI: distances between towns, distances in Olympic events, thickness of plastic sheeting. Imperial: dimensions of building materials, distances between towns in the United States, dimensions of paper
    b) Approximate: distances between towns, distances between towns in the United States. Exact: distances in Olympic events, thickness of plastic sheeting, dimensions of building materials, dimensions of paper stock
18. Example: 144 in.: 144 in.$\left(\frac{1 \text{ yd}}{36 \text{ in.}}\right) = 4$ yd;
    4 yd$\left(\frac{0.9144 \text{ m}}{1 \text{ yd}}\right) = 3.6576$ m

Chapter 1 Review, pages 48 to 50

1. Example: Use a string to follow the curve of the object or, if possible, roll the object across a surface one rotation and measure the distance travelled.
2. 5.7 cm
3. 2.46 cm
4. 62.8 cm; 285.8 cm^2
5. Example: 46.7 cm
6. b) Use a string to trace out the S shape, then measure the length of the string.
7. D = $6\frac{3}{4}$ in.; distance from C to D is $2\frac{9}{16}$ in. You could count by $\frac{1}{16}$ or subtract the reading at C from the reading at D.
8. a) 5.356 in.
   b) 5.121 in. The perimeter of the triangle is smaller because the triangle can be drawn inside the quarter circle. The shortest distance between two points is a straight line, not a curve.
9. a) $2\frac{1}{4}$ in. by $4\frac{1}{2}$ in.
   b) Scale factor is 1.78 to fit the height. The width will need to be cropped, as the scale factor for the width is smaller than that of the height.
10. a) 256 mi b) 83 ft
11. Approximation, because 8 ft $11\frac{1}{10}$ in. = 2.72034 m. The stated height of 2.7 m is shorter by a little more than 2 cm.
12. Example: About 2.6 times as tall with head down. About 63.75 in. tall with head up. This could be stated as 5 ft $3\frac{3}{4}$ in. Most heights are given this way.

13. a) 1 cm = 8.8 km b) 1 in. ≈ 13.9 mi
c) about 30 mi d) Faust

Chapter 1 Practice Test, pages 51 to 53

1. D
2. B
3. C
4. C
5. C
6. a) Example: Use a natural step as a referent for 2 ft. Use an exaggerated step as a referent for 1 m; feet, yards, metres
b) inches and centimetres; 1 in. = 2.54 cm
7. 7 cm by 15 cm
8. 7, 8, and 9 mm
9. Example: 29 ft 10 in.
10. a) Assuming their first line was 5 yd from an end wall, they drew 8 lines.
b) 3.76 yd
c) 404 yd. They will run twice the distance from the wall to each line and then the length of the gym.
d) 320 m (about 350 yd, 54 yd less). Yes. Using yards, they had to make an extra round trip almost the full length of the gym.

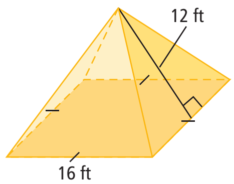
Chapter 2

2.1 Units of Area and Volume, pages 61 to 65

1. a) 1 500 000 m^2 b) 82.8 km^2
c) 2258.1 cm^2 d) 97.5 m^2
2. a) 75 000 mm^2 b) 0.005 16 m^2
c) 27 870.9 cm^2
3. 194 yd^2
4. a) 3540 cm^3 b) 39 329 cm^3
5. 1.3 m^3
6. 58.8 ft^2, 5.46 m^2
7. a) The bedroom in the new house is bigger by 3.9%.
b) $177.23
8. 51 km^2
9. a) 2604 in.^{2} b) 180 tiles
10. a) The locker from the double stack has more volume.
b) It has 0.158 m^3 more space.
11. Example: When converting from a smaller unit to a larger unit, divide by a power of 10. The conversion factor for SI units of volume is found by raising the conversion factor from the smaller unit to the larger unit, to the power of 3. This result is the power of 10 to be used when dividing. The same method is used when converting from larger to smaller units, except for multiplying by the power of 10.
a) 1 000 000 cm^3
b) 0.000 355 m^3
c) 2 500 000 000 mm^3
12. a) 80 586 ft^2
b) 7487 m^2
c) 0.7487 ha
14. Example: architect, draftsperson, mechanic, carpenter, electrician, grocer, tile layer, plumber, engineer
16. Example: There may be a need for conversions in the meat, deli, and produce department. Some customers may still think in imperial measurements when purchasing a mass of meat, cheese, vegetables, or fruit.

2.2 Surface Area, pages 74 to 79

1. a) 13.6 m^2 b) 4775.2 in.^{2}
c) 275.7 cm^2 d) 237 in.^{2}
e) 162.9 cm^2
2. 640 ft^2



3. a) 43.1 cm b) 11.0 m c) 2.7 m
4. 6 in.
5. 734.3 cm^2
6. Example: There are no computational errors in Austin's work. But, he does not need to paint the ends of the pillars if they will be standing upright. I would just paint the lateral surface area which is 50.24 ft^2 for each pillar.
7. 182.46 m^2
8. a) 664 cm^2 b) 414.34 cm^2
9. I assumed there were two bases. 96.9 cm
10. 58 307 960 mi^2
11. 314 in^2
12. 1841 m^2
13. 380 mm^2
14. 5 m^2

15. a) Minimum: 4902 mm^2;
Maximum: 5153 mm^2

b) Minimum: 39.5 mm by 39.5 mm by 39.5 mm;
Maximum: 40.5 mm by 40.5 mm by 40.5 mm

16. 192.3 cm^2

17. 1353.2 mm^2

18. a)

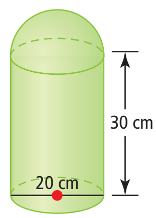
Investigating Changes in Dimensions of a Sphere			
Stretch Ratio	Radius	Surface Area	Ratio of New SA to Original SA
1	2	50.265	1
2	4	201.06	4
3	6	452.385	9
4	8	804.24	16
5	10	1256.625	25
6	12	1809.54	36

b) It is the square of the stretch ratio.

c) 1809.54 square units

d) Example: The stretch ratio causes the area to change by the square of the stretch ratio.

19. Example: The surface area is 2826 cm^2. To convert to square inches, divide 2826 by 6.45. So, the surface area in imperial units is 438.14 in.^{2}



20. Example: Surface area is the sum of the areas of the faces of a three-dimensional object. Area is measured in square units, so surface area is also measured in square units.

2.3 Volume, pages 86 to 91

1. a) 192 666.7 cm^3 or 0.2 m^3

b) 578 000 cm^3 or 0.6 m^3

c) 1005.3 ft^3

d) 335.1 ft^3

e) 2 226 094.9 mm^3

2. a) 48 in.^{3} **b)** 754 cm^3

3. 0.1 m^3

4. a) Erin is correct. Example: Janine is incorrect because she divided the volume of the entire object by 3.

b) Example: Erin's method.

5. a) 8.4 cm **b)** 12 in.

c) 6.0 cm **d)** 1.8 yd

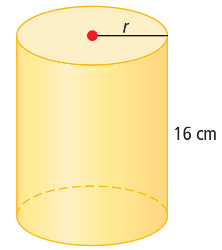
6. 801 599.64 m^3

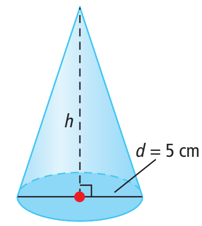
7. 6 in. by 6 in. by 6 in.

8. 18 cm^3

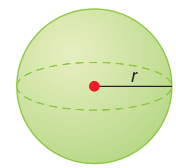
9. 160 cm^3

10. a) $r = 3.2$ cm **b)** $h = 3.1$ cm





c) $r = 2.3$ cm



11. 5.1 m^3

12. 12 ft

13. a) 45 ft^3 **b)** Example: 35 ft^3

c) 33.75 ft^3; The volume decreased by $\frac{1}{4}$.

14. 81.5 cm^3

15. Example: 70 in.^{3}

16. 621.12 songs/cm^3 on the MP3;
0.06 songs/cm^3 on the vinyl record

17. Example: If a cone and a sphere have the same radius and the height of the cone is the same measurement as the radius of the cone, then the volume of the sphere is 4 times the volume of the cone.

18. 11.2 cm^3

19. a)

Investigating Changes in Radius of a Sphere			
Stretch Ratio	Radius	Volume	Ratio of New Volume to Original Volume
1	3	113.1	1
2	6	904.8	8
3	9	3053.7	27
4	12	7238.4	64
5	15	14137.5	125
6	18	24429.6	216

b) Example: The stretch ratio is cubed.

c) Example: The ratio will be 6 cubed or 216.

d) Example: The volume is increased by the cube of the stretch ratio or the radius.

20. Example:

Investigating Changes in Dimensions of a Square Pyramid				Ratio of New Volume to Original Volume
Stretch Ratio	Side	Height	Volume	
1	2	2	2.67	1
2	4	4	21.33	8
3	6	6	72.00	27
4	8	8	170.67	64
5	10	10	333.33	125
6	12	12	576.00	216

21. Example: stereo cabinet

a) Estimate the volume to be 0.5 m³ in SI units and 0.5 yd³ in imperial units.

b) In imperial units it is actually 0.4703 yd³ and in SI units it is 0.35958 m³.

c) The estimate was closer to the imperial units. My personal referent for imperial units is more accurate.

Chapter 2 Review, pages 92 to 94

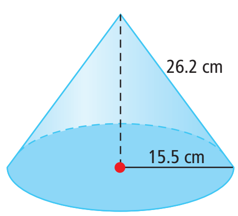
1. a) 8100 cm² **b)** 0.54 in.²

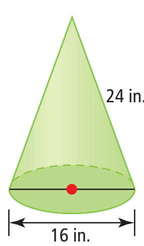
2. a) 423.84 ft³ **b)** 1029.63 cm³

3. Example: For 1 cm², my referent is the area of my little finger nail. For 1 in.², my referent is the area of a 25¢ coin. For 1 m², my referent is the front of the square bookshelf in my bedroom.

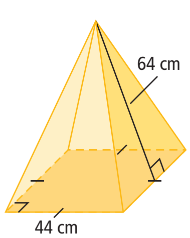
4. 25.84 lb

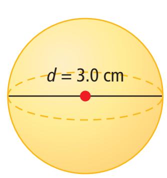
5. a) 2030.57 cm² **b)** 804.25 in²





c) 7568 cm² **d)** 28.27 cm²





6. a) $s = 1.64$ m **b)** $s = 40$ cm

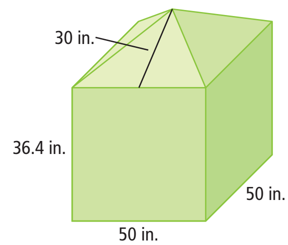
c) $r = 5.54$ cm

7. 2 m², assuming the tire is a cylinder and the cover will enclose the total surface

8. a) 50 in. by 50 in. by 36.4 in.

b) 28 in. by 28 in. The dog should be able to walk in without crouching down.

c)



d) 12 780 in.²

9. 1.77%

10. a) 21 205.75 ft³ **b)** 74.22 cm³

c) 1008 ft³ **d)** 288.70 cm³

11. a) 0.73 m **b)** 4.66 cm

c) 4.20 m **d)** 8.38 cm

12. a) 4300 m³ **b)** 239 truckloads

13. a) 6 283 185 mm³ **b)** 5 497 787 mm³

Chapter 2 Practice Test, pages 95 to 97

1. A

2. A

3. D

4. A

5. B

6. $SA = 2463$ cm²; $V = 11\ 494$ cm³

7. 24 572.5 mm³

8. 58.24 m²

9. The small size should be the container with radius 7 cm and height 18 cm since its volume is 2770.88 cm³. The large size should be the container with the radius of 8 cm and height of 16 cm since its volume is 3216.99 cm³.

10. a) 760.7 cm³ **b)** 1051.3 cm³

11. $SA = 72.2$ in²; $V = 39.65$ in³

12. a) 100 000 000 m³, assuming the kimberlite is a cylindrical shape with a height of 2 km and base area of 50 000 m²

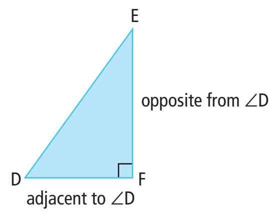
13. a) 3.9 ft

b) The prism has a surface area of 94 ft². The cylinder has a surface area of 85.15 ft². Therefore, the cylinder's surface area is the least.

Chapter 3

3.1 The Tangent Ratio, pages 107 to 113

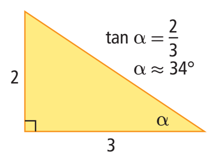
1. a) hypotenuse: XZ; opposite: ZY; adjacent: XY
b) hypotenuse: ST; opposite: SR; adjacent: RT
c) hypotenuse: LM; opposite: MN; adjacent: LN

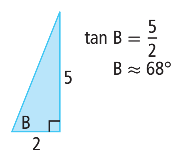
2. a)


b) $\tan D = \frac{EF}{DF}$

3. a) 3.4874 **b)** 1 **c)** 1.7321
d) 57.2900 **e)** 0.7536 **f)** 0.3249

4. a) 35° **b)** 60° **c)** 29°
d) 49°

5. a)


b)


6. a) $x \approx 19.8$ m **b)** $\theta \approx 3.6°$

7. a) 45° **b)** 17 ft

8. approximately 33.8° and 56.2°

9. a) 29 ft
b) The ratio 1:12 is the tangent of the angle of the ramp. For a safe ramp, the angle of inclination would have to be less than or equal to 4.8°. The ramp shown is not safe.

10. T1: 11.7 km; T2: 37.0 km; T3: 49.2 km

11. approximately 0.5°

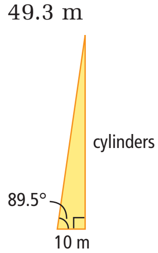
12. 1592 ft

13. a) approximately 35°
b) approximately 260.19 m

14. a) approximately 304.3 m
b) approximately 783.24 m

15. a) approximately 6.11 ft
b) approximately 16.75 ft

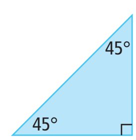
16. a) 49.3 m **b)** 50.0 m

17. a)


b) 1145.89 m **c)** 11 459 cylinders

18. Example: Ratio: A ratio is the proportion of one number to another. The tangent ratio is the proportion of the length of the side opposite the subject angle in relation to the length of the side (that is not the hypotenuse) adjacent to the subject angle.
$\theta = 63°$: The symbol θ is generally used to indicate the size of an angle. If $\theta = 63°$, the proportion of the lengths of the opposite and adjacent sides is approximately 1.96.
tan 42°: The value of tan 42° is approximately 0.9, so the length of the side opposite the angle measuring 42° is approximately 0.9 times the length of the side that is not the hypotenuse, but that is adjacent to the angle measuring 42°.
$\tan \theta = 1.428$: The value of θ is approximately 55°, so the proportion of the length of a side in a right triangle opposite an angle measuring 55° compared to the length of the side that is not the hypotenuse, but that is adjacent to the angle measuring 55° is approximately 1.428.
$\tan \theta = \frac{3}{4}$: The value of θ is approximately 36.87°.

Opposite Side: You can determine the length of the side opposite the subject angle in a right triangle by using the tangent ratio if the length of the side that is not the hypotenuse, but that is adjacent to the subject angle is known, as well as the size of the subject angle.
Adjacent Side: You can determine the length of the side that is not the hypotenuse, but that is adjacent to the subject angle in a right triangle by using the tangent ratio if the length of the side opposite the subject angle is known, as well as the size of the subject angle.

19.


The triangle is isosceles with both acute angles measuring 45°.

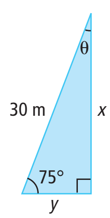
20. In the small triangle, both the lengths of the opposite side of the angle and the side adjacent to the angle are known, so Devin could input $\tan^{-1}(2 \div 1.1)$ into his calculator to find that the angle is approximately 61.2°.

21. **Step 2:** Sight through the straw on the transit to the object. Record the angle the straw makes with the protractor on the transit. Use the tangent ratio with this angle and the baseline distance, AB, to determine the distance from point B to the object.
Step 3: Example:

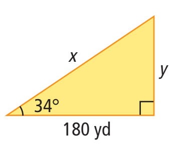
Object	Length of Baseline AB	Measure of ∠A	Distance to the Object (to the nearest tenth of a metre)
Goal posts	30 m	60°	52.0 m
Back stop	25 m	35°	17.5 m

3.2 The Sine and Cosine Ratios, pages 120 to 124

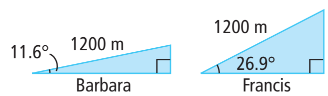
1. a) 0.8290 b) 0.5534 c) 0.8902
   d) 0.3355 e) 1 f) 0.1736
2. a) $\frac{12}{13}$ b) $\frac{5}{13}$ c) $\frac{12}{13}$
   d) $\frac{21}{29}$ e) $\frac{21}{29}$ f) $\frac{20}{29}$
3. a) 62° b) 47°
   c) 34° d) No such angle exists.
   e) 30° f) 41°
4. a) 15.3 b) 22.7
5. a) 33.2° b) 36.9°
6. a) $\theta \approx 44.4°$ b) $x \approx 14.2$ ft
   c) $\theta \approx 40.7°$ d) $x \approx 226.5$ m
7. 10.1 m
8. a)



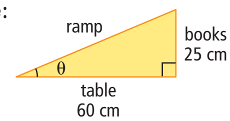
   b) 29.0 m
9. 39.8 m
10. 8485.3 m
11. a)



   b) 217 yd
   c) approximately 84 yd shorter
12. 17°
13. a) Because two sides are known, the Pythagorean relationship could be used to determine the height.
   b) The lengths of the adjacent side and the hypotenuse are known, so the cosine ratio could be used to determine the angle.
14. a)



   Comparing triangles representing the two situations shows that Barbara will cover a greater horizontal distance.
   b) 105.3 m
15. $x \approx 6.01$ m; $y \approx 10.23$ m; $\theta \approx 35.6°$
16. approximately 173.2 cm
17. **Step 1:** Example:



   $\theta \approx 22.6°$
   Step 4:
   a) As the launch angle of the ramp increases, the distance the marble travels increases. This change in distance travelled occurs because at higher angles, the marble will stay in the air longer and travel farther horizontally.
   b) Yes. Making the angle of the ramp larger would increase the speed at which the marble leaves the ramp, so it would increase the horizontal distance travelled when the ramp curves up at the end.

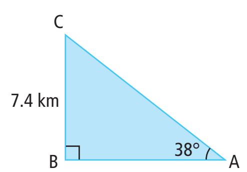
3.3 Solving Right Triangles, pages 131 to 135

1. a) $\angle = 60°$; $x \approx 8.7$; $y = 5$
   b) $\angle = 45°$; $x = 7$; $y \approx 9.9$
   c) $\angle B \approx 30.3°$; $\angle C \approx 59.7°$; $BC \approx 13.9$
   d) $\angle J = 29°$; $MD \approx 1.7$; $MJ \approx 3.4$
2. a) 14.2 cm b) 12.4 cm
3. 75°
4. a) angle of depression b) angle of elevation
   c) angle of depression d) angle of elevation
5. a) approximately 16.17°
   b) approximately 38.66°
   c) approximately 65.06°
   d) approximately 21.80°
   e) approximately 37.95°
6. 1079 m
7. The boat is not safe, because it is approximately 51.2 m from the cliff.
8. 504 ft
9. a) maximum distance: approximately 204.9 m; minimum distance: approximately 192.2 m
   b) maximum angle: approximately 87.6°; minimum angle: approximately 69.5°
10. approximately 165.1 m
11. a) approximately 6.2 mi
   b) approximately 7.9 mi

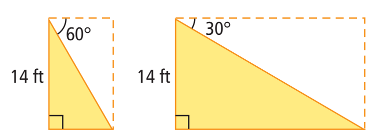
12. a) The boat on the left is closer to the helicopter, because the larger the angle of depression, the closer the object is to the base of where the object is sighted.
b) 1601 m
13. 32.3 m
14. a) 123 m
b) The truck is travelling at approximately 12.3 m/s, which is equivalent to approximately 44 km/h, so the truck driver is speeding.
15. approximately 3524 cm^3
16. a) Richard is incorrect. The angle of depression sighting down will equal the angle of elevation sighting up.
b) Because $\alpha = \theta$, knowing the angle of depression (α) gives the angle of elevation (θ). If we know the height of the window and the horizontal distance, the tangent ratio can be used to determine the value of θ. If we know the height of the window and the length of the line of sight, the sine ratio can be used to determine the value of θ. If we know the length of the line of sight and the horizontal distance, the cosine ratio can be used to determine the value of θ.

Chapter 3 Review, pages 136 to 137

1. BC = 4; AC ≈ 2.6; XZ ≈ 7.9
2. a) $x \approx 17.3$ **b)** $\theta \approx 54.0°$ **c)** $y \approx 2.3$
3. 52°
4. a) $x \approx 9.8$ **b)** $x \approx 11.0$ **c)** $\theta \approx 56.4°$
5. 41.0°
6. 12.6 ft
7. a)



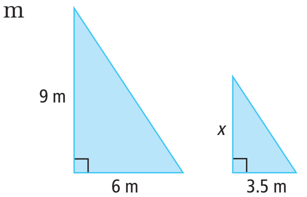
b) ∠C = 52°; AB ≈ 9.5 km; AC ≈ 12.0 km
8. approximately 208.4 m
9. The swimmer is moving away from the lifeguard, because as the angle of depression decreases, the distance from the base of the position of sighting increases.



The swimmer has travelled approximately 16.2 ft.

Chapter 3 Practice Test, pages 138 to 139

1. C
2. B
3. A
4. D
5. 5.3 m



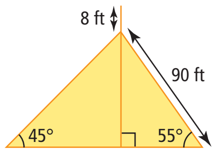
6. a) 0.3057 **b)** 0.9272 **c)** 0.9205
7. a) 15° **b)** 80° **c)** 42°
8. The second ratio should be cos 18° not sin 18°. The cosine ratio is used in conjunction with the angle's adjacent side and hypotenuse.
9. a) 9.5°
b) No. The puck will be approximately 51 in. high when it reaches the net, which is only 48 in. high.

Unit 1 Review, pages 140 to 143

1. Example: millimetre: thickness of one sheet of Bristo board; centimetre: width of small finger; metre: length of a large step; inch: width of adult male thumb at base of nail; foot: distance from elbow to wrist; yard: length of a large step
2. a) 2.5 cm, because two fingers would total 1 in., which is equal to approximately 2.54 cm.
b) 38.1 cm, because 5 hand widths total 15 in., and (15)(2.54) = 38.1.
c) 6.4 m, because 14 steps total 7 yd, which is equal to 252 in. There are approximately 39.37 in. in 1 m. 252 ÷ 39.37 = 6.4
3. a) 350 cm **b)** 42 in.
c) 8.723 km **d)** 1.2954 m
e) approximately 26.4 in.
f) approximately 8.7 mi
4. 19 ft 2 in.
5. 5 ft $1\frac{1}{2}$ in.
6. a) 4.5 m^2 **b)** 49 561 600 yd^2
7. a) 1728 cm^2 **b)** 1200 cm^2
8. a) 5 cm
b) approximately 5.89 cm^3
9. approximately 11 879 $in.^3$
10. a) $x \approx 17.7$ cm; $\theta \approx 43°$
b) $x \approx 26.1$ cm; $\theta \approx 32°$

11. 29.7 m

12. a)



b) 82 ft

c) The wire with the unknown length is likely longer than the other wire, because the angle of depression for that wire is less than the angle of depression for the wire that is 90 ft long. The length of the wire is 104.5 ft.

d) 125.5 ft

13. approximately 21.7 m

14. 897.8 m

Unit 1 Test, pages 144 to 145

1. A

2. B

3. C

4. B

5. 8

6. 44

7. 8.0

8. a) 15 cm **b)** 720 cm^3

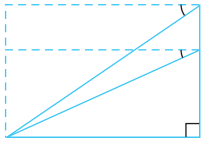
c) The box must have sides that are at least 12 cm by 12 cm by 15 cm in order for the purse to fit, so the volume needs to be at least 2160 cm^3. The larger size box (2200 cm^3) would fit the purse, providing that the sides are 12 cm by 12 cm by 15 cm.

9. a) Example: $\sin 40° = \dfrac{5}{\text{length of AC}}$

b) 7.8 cm **c)** 4 cm

10. a) approximately 14.0°

b) The angle of depression will decrease because he will not be looking downward as steeply toward the net.



Chapter 4

4.1 Square Roots and Cube Roots, pages 158 to 161

1. a) 49 **b)** −2500 **c)** 9

d) $\frac{16}{5}$ **e)** $\frac{3}{4}$ **f)** $\frac{9}{16}$

2. a) 8 **b)** −64 **c)** −125

d) 2 **e)** $\frac{1}{72}$ **f)** $\frac{8}{27}$

3. a) 7 **b)** 13 **c)** 10

d) 2 **e)** 2 **f)** $3x$

4. a) 1 **b)** 6 **c)** 20

d) 2 **e)** $\frac{3}{5}$ **f)** $4a$

5. a) both; (1)(1) = 1, (1)(1)(1) = 1

b) perfect cube; (10)(10)(10) = 1000

c) perfect square; (9)(9) = 81

d) perfect square; (13)(13) = 169

e) perfect cube; (6)(6)(6) = 216

f) perfect square; (32)(32) = 1024

6. a) perfect square **b)** perfect cube

c) perfect square **d)** perfect square

e) both **f)** both

7. Find all prime factors of the radicand. Group the prime factors into two equal groups. Calculate the value of one group.

a) 10 **b)** 2 **c)** 9 **d)** 3 **e)** 12 **f)** 24

8. a) 14 **b)** 16 **c)** 21 **d)** 15 **e)** 31 **f)** 17

9. 20 m

10. 7 ft

11. a) 40 **b)** 38 ft

c) Example: My answer is 2 ft less than my estimate.

12. 1 m by 1 m or 2 m by 2 m or 3 m by 3 m

13. a) 120 squares **b)** Designs may vary.

c) Example: The length of the diagonal of each square is the square root of the sum of two sides squared.

14. 36 in.

15. a) 17 in.

b) Example: It is a 17 by 17 by 17 cube, so 17 represents the cube root of the volume.

16. 1000 mm^3

17. a) 22.2 km **b)** 6.3 h

18. 26 cm

19. a) 2880 cm^2

b) 13 824 cm^3; 24 cm by 24 cm by 24 cm

20. a)

Number	0	1	2	3	4	5	6	7	8
Number Squared	0	1	4	9	16	25	36	49	64

b) vertical axis: number squared; horizontal axis: number

c) 0.2; 2

d) Find 5 on the horizontal axis, slide up to the curve, and left to the vertical axis.

e) Find 49 on the vertical axis, slide right to the curve, and down to the horizontal axis.

f) $4.2^2 = 17.64$ **g)** 39

21. a) Example: What is the square root of $\frac{4}{9}$?
b) Example: What is the cube root of $\frac{8}{27}$?

4.2 Integral Exponents, pages 169 to 173

1. a) A positive exponent would be used for calculating population in future years, n being the number of years past 2005. A negative exponent would be used to calculate the population in years before 2005.
b) A positive exponent would be used for calculating the amount of radioactive substance a number of periods, n, after the sample was measured. A negative exponent would be used to calculate the amount of radioactive substance a number of periods before the sample was measured.
c) A positive exponent would be used for calculating the number of bacteria a number of periods, n, after the initial number in the culture was counted. A negative exponent would be used to calculate the number of bacteria in the culture a number of periods before the count was done.

2. a) $\frac{1}{b^3}$ b) $\frac{x}{y^4}$ c) $\frac{2}{x^2}$ d) $\frac{2x^2}{y}$ e) $\frac{-4}{x^5}$ f) $\frac{-2}{x^3y^4}$

3. Yes. A negative exponent in the numerator is a positive exponent in the denominator.

4. a) $\frac{1}{4^2}$ b) $\frac{1}{3^2}$ c) $\frac{1}{12^4}$ d) $\frac{1}{8^3}$ e) $\frac{1}{5^8}$ f) $\frac{3^6}{2^{15}}$ g) $\frac{4^2}{5^2}$ h) 3.2^6 i) $4(2^3)$

5. a) $\frac{t^6}{s^2}$ b) $\frac{1}{h^{10}}$ c) $8t^4$ d) $\frac{8}{x^{12}}$ e) $\frac{1}{n^{24}}$ f) x^6y^{24}

6. a) $2^6 = 64$ b) $\left(\frac{3}{2}\right)^9 = 38.4434$ c) $5^{-4} = 0.0016$ d) $(6^0)^{-3} = 1$ e) $8^8 = 16\ 777\ 216$ f) $\left(\frac{3}{4}\right)^6 = 0.1780$

7. a) 1250; 78 b) 80 000

8. $593 979.4

9. 1000

10. a) i) 8000 ii) 256 000 iii) 500
b) the beginning of the time period

11. 1.298×10^{19} miles

12. 250 000 mm

13. a) 405 b) 328

14. 3.2×10^{24}

15. 4.1 volts

16. 2 weeks ago

17. a) the doubling approach b) $40.96

18. 20 days

19. a) 14 164 b) 17 284

20. a) $\frac{2}{3}$ b) -4 c) -3 d) -2

21. 4

22. a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$
b) $32 000, $16 000, $8000, $4000, $2000, $1000
c) $1000 d) 9 agencies

23. a) 1200 b) 768 c) 491.5

24. a) 100 b) starting time
c) times before the starting time

25. Example: A power with a negative exponent moves from the numerator to the denominator, while a power with a positive exponent remains in the numerator. For example, $2^{-2} = \frac{1}{2^2}$, $2^2 = \frac{2^2}{1}$.

26. Example: In the half-life of a chemical substance, a negative exponent means the value decreases by a certain proportion over time.

27. a) 3^5 is larger. The greater the exponent, the greater the result will be if the base is the same.
b) The greater the base, the greater the result will be when the exponent is the same.
c) 6^{222}, 2^{666}, 5^{333}, 3^{555}, 4^{444}

4.3 Rational Exponents, pages 180 to 183

1. a) $x^{\frac{16}{3}}$ b) b^2 c) a^3 d) $k^{7.8}$ e) 2 f) $\frac{-2a^2}{3}$ g) $-8x^2$ h) $27x^3$ i) $5x$

2. a) $x^{\frac{7}{3}}$ b) $\frac{1}{3^3}$ c) $\frac{1}{m^{\frac{10}{3}}}$ d) $\frac{1}{3p^{\frac{5}{2}}}$ e) $\frac{1}{x^9y^6}$ f) $\frac{243x^5}{32y^{10}}$

3. a) 2 b) $\frac{5}{4}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$ e) $\frac{1}{2}$ f) 3

4. a) 4 b) 2 c) -81 d) 3 e) $\frac{216}{125}$ f) $\frac{1}{6}$

5. a) 0.0370 b) 6208.3751 c) 0.6905 d) 0.25 e) 77.5305 f) 0.1768

6. a) 402 b) 383 c) 207 d) 179

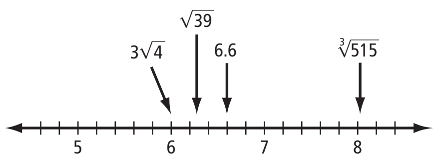
7. a) $t^{1.7}$ b) $4x$

8. a) 3-year term deposit at 1.5%
b) \$5.28
9. a) the starting population, in millions
b) 1 255 286 **c)** 1 138 950
10. a) 12; 24; 48; $6(2)^m$ **b)** 543
c) No, tank size and increased pollution of the tank would limit population growth.
11. a) \$5111.33 **b)** \$15 642.66
12. a) 7.6 times larger **b)** 8 times
13. a) $\frac{2}{5}$ **b)** $\frac{3}{4}$
14. a) 4 °C **b)** 1.3 °C
15. 5.80×10^{10} m
16. 7 folds
17. a) 14 mg/mL
b) Answers may range from 1.5 h to 1.75 h. Example: 1.66 h
18. Example: Rational exponents can be used to model the decrease in value of a car that you own. For example, consider a \$10 000 car that decreases in value at a rate of 15% per year. You can model this situation using the equation $V = 10\ 000(0.85)^n$, where V represents the value of the car and n is the number of years.
19. Example: A common error is multiplying the exponents instead of adding them when multiplying powers with the same base. You could rewrite the rational exponents in the margin and add them separately to help avoid multiplying them.

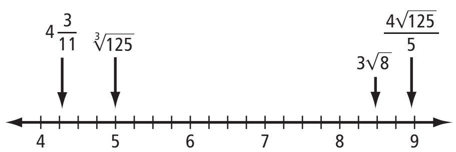
4.4 Irrational Numbers, pages 192 to 195

1. a) $(\sqrt{4})^3$ **b)** $\sqrt[5]{32}$ **c)** $\sqrt{64}$
d) $\sqrt[4]{\frac{1}{100}}$ **e)** $\sqrt[3]{\frac{y^4}{x^3}}$ **f)** $\sqrt{m^{3n}}$
2. a) $(12p)^{\frac{3}{2}}$ **b)** $5^{\frac{3}{5}}$ **c)** $x^{\frac{3}{4}}$
d) $\left(\frac{s^3}{t^5}\right)^{\frac{1}{3}}$ or $\frac{s}{t^{\frac{5}{3}}}$ **e)** $y^{\frac{5}{6}}$ **f)** $8^{\frac{1}{n}}$
3. a) 0.6 **b)** 3
c) 16.4924 **d)** 16.1662
e) 1.4071 **f)** 2.2678
4. a) $\sqrt{99}$ **b)** $\sqrt{98}$ **c)** $\sqrt{45}$
d) $\sqrt{28}$ **e)** $\sqrt{27}$ **f)** $\sqrt{600}$
5. a) $\sqrt[3]{56}$ **b)** $\sqrt[3]{81}$ **c)** $\sqrt[3]{5000}$
d) $\sqrt[3]{128}$ **e)** $\sqrt[4]{162}$ **f)** $\sqrt[4]{80}$
6. a) $2\sqrt{3}$ **b)** $5\sqrt{2}$ **c)** $4\sqrt{3}$
d) $6\sqrt{2}$ **e)** $3\sqrt{5}$ **f)** $10\sqrt{5}$
7. a) $2\sqrt[3]{3}$ **b)** $3\sqrt[3]{2}$ **c)** $3\sqrt[3]{9}$
d) $2\sqrt[3]{5}$ **e)** $2\sqrt[4]{2}$ **f)** $3\sqrt[4]{3}$

8. a) $\sqrt{0.25}$, $\frac{5}{8}$, $0.\overline{6}$, $\sqrt[3]{0.84}$; $\sqrt[3]{0.84}$ is an irrational number.
b) $\sqrt[4]{625}$, $\sqrt{225}$, $15\frac{4}{5}$, $3\sqrt{28}$; $3\sqrt{28}$ is an irrational number.
9. a)



$\sqrt{39}$ and $\sqrt[3]{515}$ are irrational numbers.
b)



$\frac{4\sqrt{125}}{5}$ and $3\sqrt{8}$ are irrational numbers.
10. 3.375 cm
11. 1.35 m
12. 40 mph
13. a) 37.08 cm **b)** 2224.8 cm^2
14. a) 85 min **b)** 35 800 km
15. 3.24 ft
16. $0.862 \approx \sqrt[3]{8P}$
$0.862 \approx 2\sqrt[3]{P}$
$0.431 \approx \sqrt[3]{P}$
$P \approx (0.431)^3$
$P \approx 0.08$
The store should offer an 8% discount.
17. 6.3 A
18. 2
19. a) $2^{\frac{1}{5}}$ **b)** $256^{\frac{1}{8}}$
20. Sometimes true. Not true when $x < 0$.
21. a) 8
b) Rational numbers: 1 by 1, 2 by 2, 3 by 3, 4 by 4. Irrational numbers: $\sqrt{2}$ by $\sqrt{2}$, $2\sqrt{2}$ by $2\sqrt{2}$, $\sqrt{5}$ by $\sqrt{5}$, $\sqrt{10}$ by $\sqrt{10}$
c) 1, 2, 4, 5, 8, 9, 10
22. The denominator is the root and the numerator is the power: $\sqrt[m]{x^n} = x^{\frac{n}{m}}$.

Chapter 4 Review, pages 196 to 198

1. a) perfect square **b)** both
c) perfect cube **d)** perfect square
e) both **f)** perfect square
2. $144 = (2)(2)(2)(2)(3)(3)$; square root: $(2)(2)(3) = 12$
3. a) 11 **b)** 6 **c)** 20
4. 11 cm by 11 cm by 11 cm
5. \$540

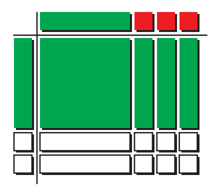
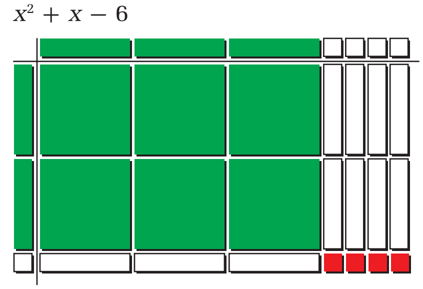
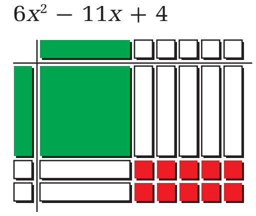
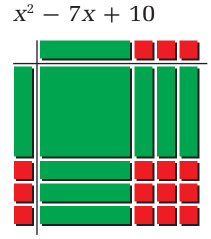
6. a) $\frac{1}{x^8}$ b) s^6
c) $(-2.6)^6$ d) $(4k)^5$
7. a) 81 b) 0.0370
c) 0.4823 d) $\frac{x^2}{1\ 048\ 576}$
8. a) 1.03 m b) 5 bounces
9. a) 15.625 g b) 8000 g
10. a) 1470; 1543; 1620; $1400(1.05)^n$
b) 3369 c) 1209
11. a) $\left(\frac{1}{x}\right)^{\frac{1}{3}}$ b) 4
c) $\frac{1}{8g^6}$ d) $8t^7$
12. a) $(3)(3) = 9$
b) Example: A negative exponent in the numerator is a positive exponent in the denominator.
13. a) 7.3004 b) 0.1111
c) 0.001 d) 45.2548
14. $27^{\frac{2}{3}} = 9$, not 18; $9x^{\frac{8}{3}}$
15. \$7785.51
16. 0.166 km
17. \$465 658.88
18. a) $\sqrt[5]{x^3}$ b) $\sqrt[3]{(27t^2)^2}$ c) $\sqrt{\frac{g^3}{18}}$
19. a) $(xp)^{\frac{5}{2}}$ b) $2^{\frac{5}{3}}$ c) $3\left(x^{\frac{4}{5}}\right)$
20. a) $\sqrt{108}$ b) $\sqrt{40}$
c) $\sqrt[3]{320}$ d) $\sqrt[3]{-16}$
21. a) $6\sqrt{5}$ b) $8\sqrt{3}$
c) $4\sqrt[3]{2}$ d) $2\sqrt[4]{3}$
22. a) Irrational numbers: $\frac{\pi}{3}$, $\sqrt{0.9}$, $\sqrt[5]{96}$;
Order: $\sqrt[5]{96}$, $\frac{\pi}{3}$, $\sqrt{0.9}$, $0.\overline{24}$
b) Irrational number: $18^{\frac{1}{2}}$;
Order: $6.\overline{2}$, $2\sqrt[3]{27}$, $\sqrt{36}$, $18^{\frac{1}{2}}$
24. a) 9.8 cm b) 452.4 cm^2

Chapter 4 Practice Test, pages 199 to 201

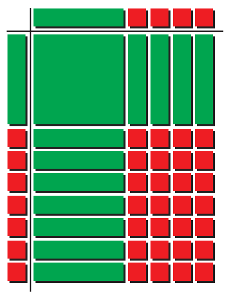
1. D
2. A
3. D
4. C
5. A
6. A
7. B
8. a) $(2)(2)(2) = 8$ while $(3)(3)(3) = 27$, so the cube root of 10.648 is closer to 2.
b) 2.2 m by 2.2 m by 2.2 m
9. Example: An irrational number cannot be made into a ratio or fraction, so it cannot be rational.
10. a) 13 cm by 13 cm b) 4.3 cm
11. Subtract the exponents in step 2; $\frac{1}{2}$
12. 31.25 g
13. 224.8 km
14. a) 1580 b) 1758
15. a) Convert the numbers to decimals and order on the number line.
b) Change all mixed radicals to entire radicals and compare.
c) $6\sqrt{3}$, $4\sqrt{12}$, $3\sqrt{27}$, $3\sqrt{48}$

Chapter 5

5.1 Multiplying Polynomials, pages 209 to 213

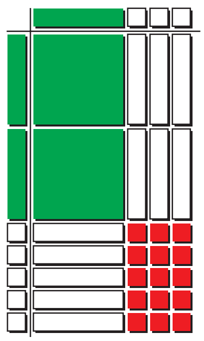
1. a)

$x^2 + x - 6$
b)

$6x^2 - 11x + 4$
c)

$x^2 - 7x + 10$
d)

$x^2 + 6x + 9$

e)



$x^2 + 11x + 28$

f)



$2x^2 - 11x + 15$

2. a) $2x^2 + 3x - 2$ **b)** $2x - 1$ by $x + 2$

3. a) $x^2 + 3x - 10$ **b)** $x^2 - 6x + 9$
c) $c^2 - d^2$ **d)** $4x^2 + 5xy + y^2$
e) $y^2 + 6y + 9$ **f)** $24j^2 - 6k^2$

4. a) $3x^3 - 5x^2 + 8x$ **b)** $7ab^2 + ab - a$
c) $6x^3 - 22x^2 + 36$ **d)** $10x^3 + 3x^2 - 14x + 5$
e) $12s^4 - 5s^3 + 22s^2 + 6s$
f) $2y^4 + 11y^3 + 21y^2 + 11y - 5$

5. a) B **b)** H **c)** F **d)** D
e) J **f)** E **g)** A **h)** G

6. a) $6n^2 - 9n + 8$ **b)** $-7f^2 + 4f - 29$
c) $9b^2 - 8bd + 7d^2$ **d)** $40x^2 - 90x - 50$
e) $14a^2 - 35ac - 28c^2$
f) $4y^4 - 14y^3 - 53y^2 - 41y - 6$

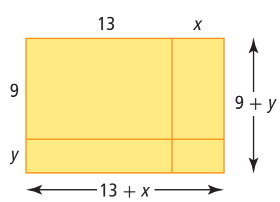
7. $A = (x + 4)(x + 4)$; $A = x^2 + 8x + 16$

10. $A = (x - 7)(x - 4)$; $A = x^2 - 11x + 28$

11. $A = \pi(3x + 2)^2$; $A = 9\pi x^2 + 12\pi x + 4\pi$

12. a) No. Step 3 is incorrect.
b) Example: $p = 1$, $-5 \neq -15$

13. a)



b) $A = (y + 9)(x + 13)$ **c)** 154 m^2

14. a) $x + 2$ by $x - 1$ **b)** $A = (x + 2)(x - 1)$
c) The new rug has the greater area by 1 ft^2.

15. a) $A = (3x + 8)(2x + 4) = 6x^2 + 28x + 32$
b) 1232 cm^2

16. a) In the check, the left side does not equal the right side.
b) In step 1, André multiplied -4 and 5 to get $+20$. This is actually equal to -20.

17. a) As the price of a burger increases, the average number of burgers sold decreases.
b) $p = \dfrac{550 - b}{100}$ **c)** $R = \dfrac{550n - bn}{100}$

18. a) The product of the first and last numbers is 2 less than the product of the middle numbers.
b) $n + 1$, $n + 2$, $n + 3$
c) Example: The first and last product is $n^2 + 3n$; the middle product is $n^2 + 3n + 2$. I noticed that the product of the middle values is 2 more than the product of the first and last values.

19. a) $3t + 4$ **b)** 1530

5.2 Common Factors, pages 220 to 223

1. a) 20: 1, 2, 4, 5, 10, 20; 30: 1, 2, 3, 5, 6, 10, 15, 30; GCF: 10
b) 28: 1, 2, 4, 7, 14, 28; 40: 1, 2, 4, 5, 8, 10, 20, 40; GCF: 4
c) 30: 1, 2, 3, 5, 6, 10, 15, 30; 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48; GCF: 6
d) 36: 1, 2, 3, 4, 6, 9, 12, 18, 36; 27: 1, 3, 9, 27; GCF: 9

2. a) 12 **b)** 48 **c)** 27
d) 2 **e)** 25

3. a) 48 **b)** 60 **c)** 90
d) 150 **e)** 132

4. a) $3ab$ **b)** $27m^2n$ **c)** $8x^2y^2$
d) $4a^2c$ **e)** p^3q^3

5. a) $5(x + 3)$ **b)** $y(3y - 5)$
c) $w^2(x + y - z)$ **d)** $6ab(a^2 - 3b)$
e) $3x(3x^2 - 4x + 2)$

6. a) $3ab$ **b)** $s^2 - 5$ **c)** $d - 7$
d) $8x - 1$ **e)** $4xy$

7. a) $(y - 2)(3y + 4)$ **b)** $(a - 4)(5a - 2)$
c) $(c - 4)(2x + 7)$ **d)** $(x - 3)(3x - 8)$
e) $(2y + 1)(y^3 - 5)$

8. 36 cm

9. Example: When you list the factors of a number, you list all the numbers that divide evenly into the number. When you list the multiples of a number, you list the products of the number and all natural numbers.

10. a) $3x + 9$; 3 by $x + 3$; $3(x + 3)$
b) $2x^2 + 3x$; x by $2x + 3$; $x(2x + 3)$

11. Example:
a) $6x^2 + 18x$ **b)** $8a^2b - 4ab$
c) $4m^4n^2 + 6m^3n^3 - 10m^3n^2$

12. a) Incorrect: $3x \div 3x \neq 0$;
Correct: $3x(5x - 1)$
b) Incorrect: $(x - 2) \div (x - 2) \neq 0$;
Correct: $(x - 2)(5x - 1)$
c) Incorrect: GCF $\neq 9ab$;
Correct: $9a^2b^2(b - 3 + 9ab)$
d) Incorrect: factoring incomplete;
Correct: $2(x + 4)(2f + 1)$
e) Incorrect: expression not simplified;
Correct: $2(p^2 - 7p - 5)$

13. 6

14. $4r^2(4 - \pi)$

15. 6 in. by 6 in.

16. Example: $15x$ by $x + 2$

17. 3484, 5226

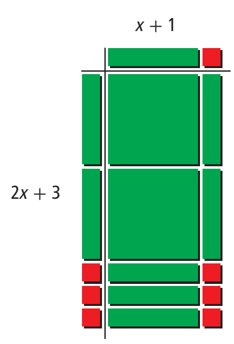
18. a) $(2x + 5)^2 + (2x + 2)^2 + (2x - 1)^2$
b) $12x^2 + 24x + 30$ **c)** $6(2x^2 + 4x + 5)$

19. a) $SA = b(b + 2s)$ **b)** 65 cm^2
c) Example: The surface areas are the same, but the equations used to calculate them are different.
d) Example: It is less complicated to find the surface area using the factored form.

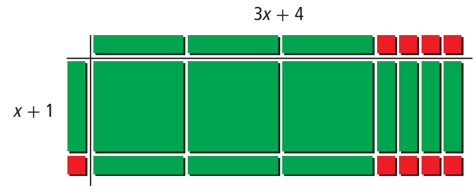
5.3 Factoring Trinomials, pages 234 to 237

1. a) $x^2 + 4x + 3$; $(x + 1)(x + 3)$
b) $x^2 + 2x + 1$; $(x + 1)(x + 1)$
c) $x^2 + x - 2$; $(x + 2)(x - 1)$
d) $x^2 + 5x + 4$; $(x + 4)(x + 1)$

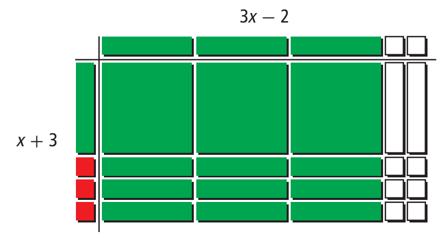
2. a) $(2x + 3)(x + 1)$



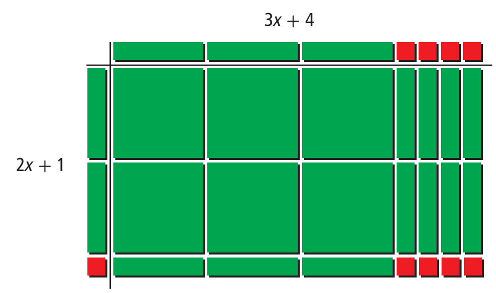
b) $(3x + 4)(x + 1)$



c) $(3x - 2)(x + 3)$



d) $(3x + 4)(2x + 1)$



3. a) 5 and 9 **b)** -2 and -3
c) 5 and -2 **d)** -10 and 2

4. a) $(x + 2)(x + 5)$ **b)** $(j + 3)(j + 9)$
c) $(k + 4)(k + 1)$ **d)** not factorable
e) $(d + 6)(d + 4)$ **f)** not factorable

5. a) $(m - 5)(m - 2)$ **b)** $(s + 5)(s - 2)$
c) $(f - 6)(f - 1)$ **d)** $(g - 7)(g + 2)$
e) $(b - 4)(b + 1)$ **f)** $2(r - 3s)(r - 4s)$

6. a) $(2x + 5)(x + 1)$ **b)** $(3y + 8)(2y + 1)$
c) $(3m + 4)(m + 2)$ **d)** not factorable
e) $(4q + 3)(3q + 2)$ **f)** $(3x + y)(x + 2y)$

7. a) $(4x - 3)(x - 2)$ **b)** not factorable
c) $(x - 2)(x - 3)$ **d)** $(2m - 3)(m + 3)$
e) $3(2x + y)(x - y)$ **f)** $(4y - 1)(3y + 1)$
g) $(6c - 5d)(c + 2d)$ **h)** $(k + 3)(4k + 3)$
i) $(a + 3b)(a + 8b)$ **j)** $(6m + n)(m + 2n)$

8. a) $x + 10$ and $x + 8$; 25 cm by 23 cm
b) $3x + 8$ and $2x - 1$; 53 cm by 29 cm

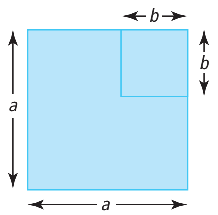
9. Example:
a) 7, 8 **b)** 4, 5
c) 2, 7 **d)** 3, 9

10. Example:
a) 8, 9 **b)** 9, 20 **c)** −2, −3 **d)** 8, 15
11. Example:
a) ±8, ±17 **b)** ±20, ±28 **c)** ±13, ±23
12. Example:
a) $k = 2$ **b)** $k = 2$ **c)** $k = 2$
13. Example:
a) $5x^2 + x + 16$
b) No two numbers multiply to 80 and add to 1.
15. $h = -(t - 5)(5t + 2)$; 34 m
16. $(40 - 2x)$; $(18 + x)$
17. any three of the following values: −16, −11, −8, 8, 11, 16
18. $12x + 20y$. Factor the expression and then multiply the length of a single side (factor) by 4.
19. First factor out 3. Then, factor the new expression $10x^2 - 13xy - 3y^2$; $3(5x + y)(2x - 3y)$
20. a) rectangle **b)** $2x - 1$ by $4x + 7$
21. Example: For factorable trinomials, the operations of factoring the trinomial and multiplying the resulting binomials are opposite operations. For example, the product of $(x + 5)(x - 3)$ results in the trinomial $x^2 + 2x - 15$, and the result of factoring the trinomial $x^2 + 2x - 15$ is $(x + 5)(x - 3)$.

5.4 Factoring Special Trinomials, pages 246 to 251

1. a) $(x + 2)(x - 2)$ **b)** $(2x + 3)(2x - 3)$
c) $(x + 4)(x + 4)$ **d)** $(x - 3)(x - 3)$
2. a) $x^2 - 64$ **b)** $4x^2 - 25$
c) $9a^2 - 4b^2$ **d)** $3t^2 - 75$
3. a) $x^2 + 6x + 9$ **b)** $25a^2 - 30ab + 9b^2$
c) $4h^2 + 12h + 9$ **d)** $5x^2 - 20xy + 20y^2$
4. a) $m^2 - y^2 = (m - y)(m + y)$
b) $16r^6 - 81 = (4r^3 - 9)(4r^3 + 9)$
c) $x^2 - 12x + 36 = (x - 6)^2$
d) $4x^2 + 20x + 25 = (2x + 5)^2$
e) $25x^2 + 70x + 49 = (5x + 7)(5x + 7)$
5. a) $(x + 4)(x - 4)$ **b)** $(b + 11)(b - 11)$
c) not factorable **d)** $(3a + 4b)(3a - 4b)$
e) $(6c + 7d)(6c - 7d)$ **f)** not factorable
g) not factorable **h)** $(10 + 3t)(10 - 3t)$
6. a) $(x + 6)(x + 6)$ **b)** $(x + 5)(x + 5)$
c) not factorable **d)** $(m - 13)(m - 13)$
e) $(4k - 1)(4k - 1)$ **f)** $(7 - m)(7 - m)$
g) not factorable **h)** $(6a + 7)(6a + 7)$
7. a) $5(t^2 - 20)$ **b)** $10xy(x + 3)(x - 3)$
c) $4(x^2 - 12x + 9)$ **d)** $2x(3x + 2)(3x + 2)$
e) $(x^2 + 4)(x + 2)(x - 2)$ **f)** $(x + 3)^2(x - 3)^2$

8. a) ±10; $(x + 5)^2$; $(x - 5)^2$
b) ±20; $(a + 10)^2$; $(a - 10)^2$
c) ±70; $(5b + 7)^2$; $(5b - 7)^2$
d) ±132; $(6t + 11)^2$; $(6t - 11)^2$
9. a) $-16b$ is not a perfect square term.
b) There are no pairs of integers that have a product of −12 and a sum of −7.
c) The trinomial is not of the form $(ax)^2 - 2abx + b^2$.
d) $49t^2 + 100$ is not a difference of squares.
11. a) 280 **b)** 460 **c)** 600 **d)** −600
13. $(x + y)(x - y)$
14. a) $\pi(r + 4)^2 - \pi r^2$ **b)** $8\pi(r + 2)$
c) 201.1 cm²
15. a) $[3(2x - 3)]^2 - (2x - 3)^2$
$= [3(2x - 3) - (2x - 3)][3(2x - 3) + (2x - 3)]$, or $[4x - 6][8x - 12]$
b) $32x^2 - 96x + 72$
c) Example: $x = 1$; $8 = 8$
16. Example: The top striped rectangle has an area of $x(x - y)$. The bottom striped rectangle has an area of $y(x - y)$. Adding these areas gives the difference between the areas of the larger and smaller squares. The difference of squares is $x^2 - y^2 = (x - y(x + y)$.
17. $28 - 8x$
18. $6x + 10$
19. a) Never true. $(-b)^2 \neq -b^2$
b) Sometimes true. It is true if $a = 0$ or $b = 0$.
c) Sometimes true. When $b = 0$,
$a^2 - 0^2 = a^2 - 2a(0) + 0^2$
$a^2 = a^2$
d) Always true. $(a + b)^2 = a^2 + 2ab + b^2$.
20. Rahim is correct; $4(4x^2 + y^2)$ cannot be factored further.
21. $x + 3y$ by $x - 3y$ by $xy - 7$
22. $16x^2 - 52x + 36$
23. a) $x^2 - y^2 = x + y$
Factor as a difference of squares to get $x - y = 1$.
b) any pair of consecutive integers from 11 to 20, for example 11 and 12, 12 and 13, and so on
24. a) $b = 2\sqrt{c}$ **b)** $b = 2\sqrt{ac}$
25.



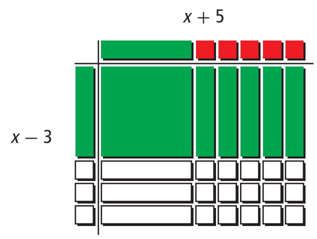
$(a - b)(a + b) = a^2 - ab + ab - b^2 = a^2 - b^2$

26. $x^2 + 2bx + b^2$ has factors $(x + b)^2$ and $x^2 - 2bx + b^2$ has factors $(x - b)^2$.

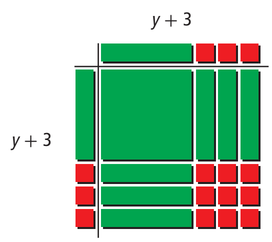
27. $30^2 - 1 = 899$; $60^2 - 1 = 3599$
   a) Example: $a^2 - b^2 = (a + b)(a - b)$ represents a difference of squares and also the product of two numbers that differ by 2. In this case, the average of a and b represents half the difference between the numbers. Since the two numbers differ by 2, adding 1 to the average gives the larger number and subtracting 1 gives the smaller number.
   b) Square the average of the two numbers and subtract 9.
   c) (average − 3)(average + 3)

Chapter 5 Review, pages 252 to 253

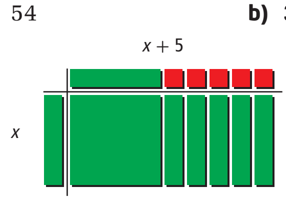
1. **a)**



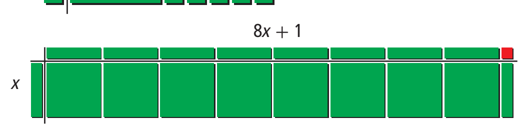
   b)



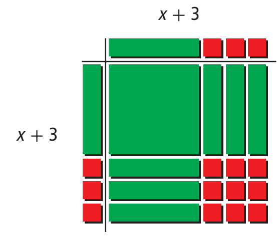
2. **a)** $x^2 + 10x + 21$ **b)** $b^2 - 81$
   c) $y^2 - 121$ **d)** $15a^2 + 58ab + 48b^2$
   e) $-20x^2 - 100xb - 125b^2$
   f) $36b^2 - a^2$
3. **a)** $a^3 + 3a^2 - 16a - 6$ **b)** $19b^3 + 2b^2 - 16b$
4. $x(x - 3) + 2(9)$; $x^2 - 3x + 18$
5. $10x^2 + 100x + 250$
6. **a)** 16 **b)** 27 **c)** 6
   d) $2x$ **e)** 10 **f)** x
7. **a)** 54 **b)** 375
8. **a)**



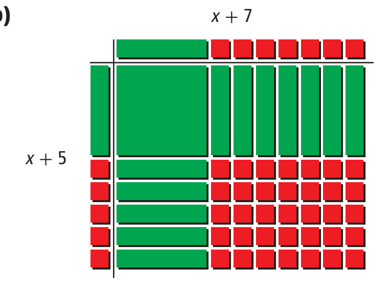
   b)



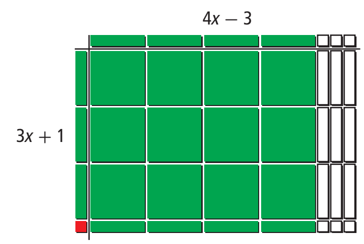
9. $xy(2x + 5)$
10. **a)**



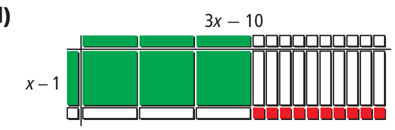
   b)



   c)



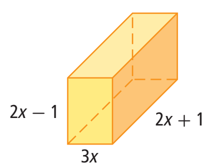
   d)



11. **a)** $(x - 6)(x + 2)$ **b)** $(x - 3)(x - 4)$
   c) $3(5x + 4)(2x - 1)$ **d)** $-2(x + 6)(3x - 1)$
   e) $-2(x - 3)(x - 5)$ **f)** $x(x + 7)(x - 4)$
12. $(x - 9)$ and $(x - 10)$; 2 cm by 1 cm
13. **a)** $(x + 10)(x - 10)$ **b)** $(c + 5)(c - 5)$
   c) $(3x + 4)(3x - 4)$ **d)** $2(8 + 3x)(8 - 3x)$
   e) $(1 + 15y)(1 - 15y)$ **f)** $-3(x + 3y)(x - 3y)$
14. **a)** $(y + 8)^2$ **b)** $(x - 10)^2$
   c) $9(5 - y)^2$ **d)** $(11c + 14d)^2$
15. **a)** x by $2x + 3$ by $2x + 3$
   b) a rectangular prism with a square base with sides $2x + 3$ and height x
   c) 270 cm^2
16. $x^2 + y^2 + 4x + 4y + 8$

Chapter 5 Practice Test, pages 254 to 255

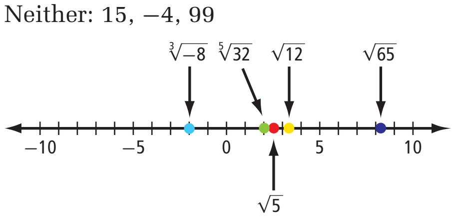
1. A
2. B
3. A
4. C
5. GCF: 4; LCM: 15 960
6. a) $x^2 - 12x + 27$ **b)** $4x^2 + 4x - 3$
c) $-x^2 + 24x - 66$ **d)** $11c^2 + 4cd + d^2$
e) $-10x^2 - 2x + 9$
f) $c^2 + 9d^2 + 12cd - 6c - 3$
7. a) $6x^4 + 17x^3 + 5x^2$ **b)** 252 cm^3
8. a) $(x + 5)^2$ **b)** $(5r - 2s)^2$
c) $5(x + 1)(x - 1)$ **d)** $(1 + 7m)(1 - 7m)$
e) $(m + 3)(5m + 2)$ **f)** $(m - 7n)(m - 2n)$
9. a) $y(3y - 1)(y - 2)$ **b)** $4(m^2 + 4)$
c) $(2y + 1)(3y - 1)$ **d)** $(x - 4)(m - 2)$
e) $(x + y)(2 + y)$ **f)** $t(3 - 2t)(3 + 2t)$
10. a) $(2x + 5)(5x - 8)$ **b)** 69 mm by 152 mm
11. $A = \pi(2x + 3)^2$ m; $A = \pi(4x^2 + 12x + 9)$ m
12. No. The expression $4y^2 - 6y - 9$ cannot be factored over the integers. The correct answer should be $2(4y^2 - 6y - 9)$.
13. a) $3x(2x + 1)(2x - 1)$
b)



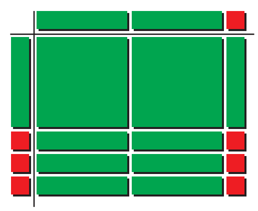
c) 11 cm by 13 cm by 18 cm

Unit 2 Review, pages 256 to 259

1. B
2. D
3. C
4. A
5. Perfect squares: 16, $\sqrt{16} = 4$; 169, $\sqrt{169} = 13$.
Perfect cubes: -8, $\sqrt[3]{-8} = -2$; 27, $\sqrt[3]{27} = 3$;
125, $\sqrt[3]{125} = 5$; 1000, $\sqrt[3]{1000} = 10$.
Neither: 15, -4, 99
6.

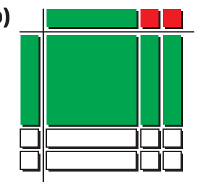


7. 81 in.^{2}
8. 9 cm^2
9. a) $2\sqrt{3}$ **b)** $9\sqrt{2}$ **c)** $2\sqrt[3]{2}$
10. a) $\sqrt{20}$ **b)** $\sqrt{75}$ **c)** $\sqrt[3]{40}$
11. a) $\sqrt[5]{7^4}$ **b)** $\sqrt[3]{\dfrac{27}{8}}$ **c)** $\sqrt[4]{6x^2}$
12. a) $\dfrac{m^{\frac{3}{2}}}{n}$ **b)** $6^{\frac{3}{4}}$ **c)** $2s^{\frac{4}{3}}$
13. a) $\left(\dfrac{1}{3}\right)^{\frac{11}{2}}$ **b)** $\dfrac{1}{y^3}$ **c)** $\dfrac{1}{3}$
14. a) $\dfrac{125}{8}$ **b)** 150 **c)** 100
15. a)



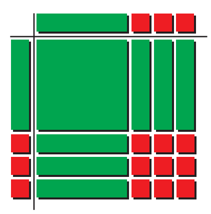
$2x^2 + 7x + 3$

b)



$x^2 - 4$

c)



$x^2 + 6x + 9$

16. a) $a^2 + 3a - 28$ **b)** $10x^2 + 19x + 6$
c) $-x^2 + 25$ **d)** $9y^2 + 24y + 16$
e) $4a^2 - 13ab + 3b^2$ **f)** $2v^3 - 6v^2 - 5v + 9$
17. Example:
a) $k = 7$ **b)** $k = 2$
18. a) $161 \neq 12$
b) No. $4x(11x) = 44x^2$ not $44x$; $4x(-7) = -28x$ not $-24x$; $-1(-7) = 7$ not 6.
Correct: $8x^3 + 42x^2 - 39x + 7$
19. a) not factorable **b)** $2(5x - 3y)^2$
c) not factorable
20. k is an integer that is divisible by 2.
21. a) $7x$ **b)** $5x^2$ **c)** $3ab(a - 1)$
22. a) not factorable **b)** $(v + 3)(2v - 3)$
c) $-2(x + 5)(x - 2)$ **d)** $(2y + 5)(2y - 5)$
e) $(x - 20)(x - 1)$ **f)** $-(3x - 2)(5x + 3)$
23. a) Julio divided the first and last terms by 2, but subtracted 2 from the middle term instead of dividing by 2.
b) $2(x + 3)(x + 3)$
24. a) $(4a + 6)(4a - 6)$ **b)** $16a^2 - 36$
c) 36 units^2
25. $r = 7n + 8$

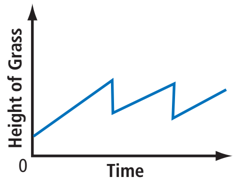
Unit 2 Test, pages 260 to 261

1. D
2. C
3. A
4. D
5. A
6. 12
7. 5
8. 19
9. 4
10. $\frac{1}{20^{\frac{1}{6}}}$
11. **a)** $2x^2 + 9xy - 5y^2$ **b)** $6a^3 - 5a^2 - 20a + 21$ **c)** $3x^3 - 7x^2 + 7x + 1$
12. **a)** $(x - 9)(x - 1)$ **b)** $(a - 2)(4a + 3)$ **c)** $(4x + y)(4x - y)$
13. **a)** $x^2 - 1$ **b)** $14x^2 + 17x - 3$

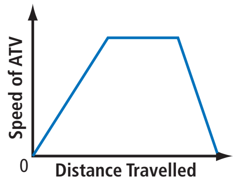
Chapter 6

6.1 Graphs of Relations, pages 274 to 278

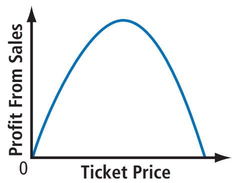
1. AB: There is a constant slow decrease of Quantity B. The segment is falling and shallow.
   BC: Quantity B is not changing, the segment is horizontal.
   CD: Quantity B is decreasing, but not at a constant rate. The curve is initially steep and falling, then becomes less steep as it gradually approaches horizontal.
   DE: Quantity B is increasing, but not at a constant rate. The curve is initially horizontal, then gradually begins to increase until it rises steeply.
   EF: Quantity B is decreasing quickly at a constant rate. The segment is falling and steep.
   FG: There is a constant increase of Quantity B. The segment is rising at approximately 45°.
2. **a)** **i)** A **ii)** C **iii)** D
   b) Example: Graph B: the number of people in a building as they enter the building, watch a concert, and exit the building
3. **a)**



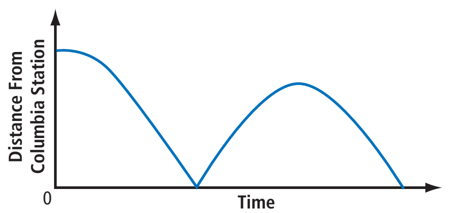
b)



c)

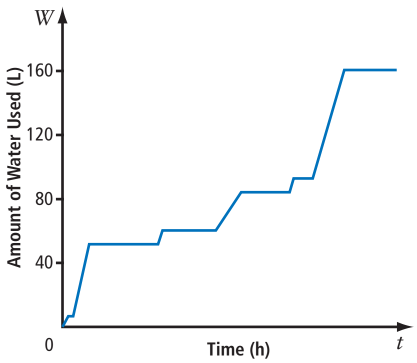


4. Examples:
   a) A jogger jogs home, and then leaves his house walking. The vertical axis is the distance from home, and the horizontal axis is time.
   b) A car slows down to a stop at a stop sign, and then accelerates to a constant speed. The vertical axis is speed, and the horizontal axis is time.
   c) A ball is thrown up and left to bounce on the floor. The vertical axis is height, and the horizontal axis is time.
5. Example:



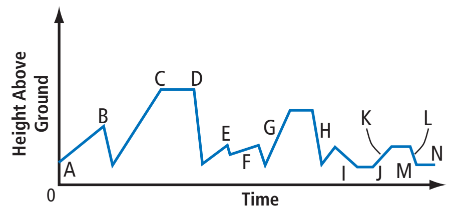
6. Examples: The blue line represents vinyl albums. It is the oldest format, so many units were sold many years ago, but few have been sold for a number of recent years. The red line represents cassette tapes. This format is newer than vinyl albums and was very popular for a while, but in the last few years it has no longer been a viable option. The green line represents compact discs. This format quickly became popular, but has been declining rapidly in popularity. The yellow line represents digital downloads. It is the most recently developed format, and has rapidly become the most popular format.
7. Example: Uriash initially travelled on flat ground at a constant speed. He went up and down a number of slopes travelling away from home. Then, he travelled at a constant speed on flat terrain. Next, he travelled back toward home going up and down slopes. Finally, he returned home on flat terrain at a constant speed.

8. Example: I flushed the toilet and washed my hands. I took a shower. I flushed the toilet and washed my hands. I ran the dishwasher. I flushed the toilet and took a bath.

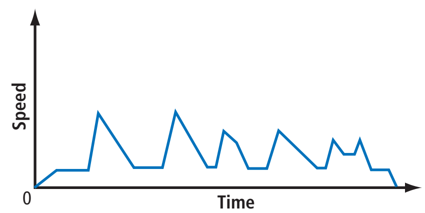


9. Examples:

a)

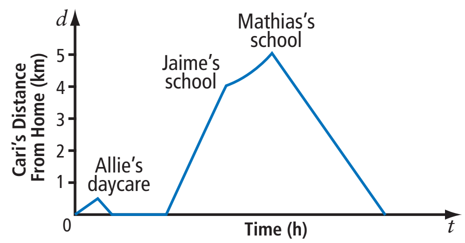


b)

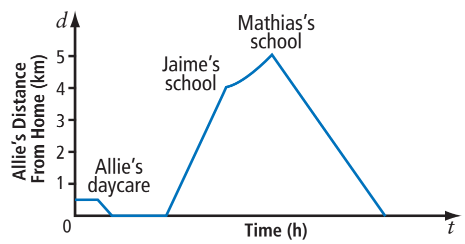


10. Examples:

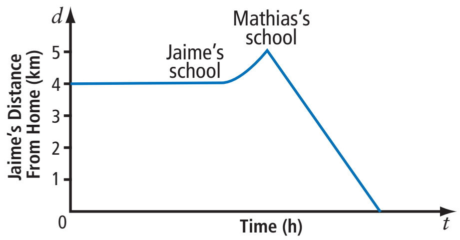
a)



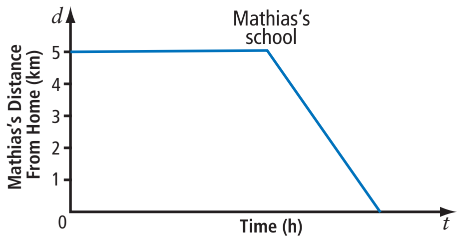
b)



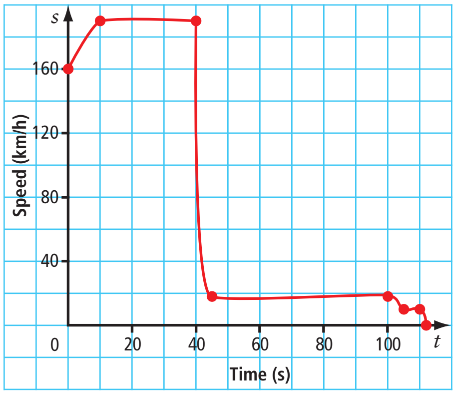
c)



d)



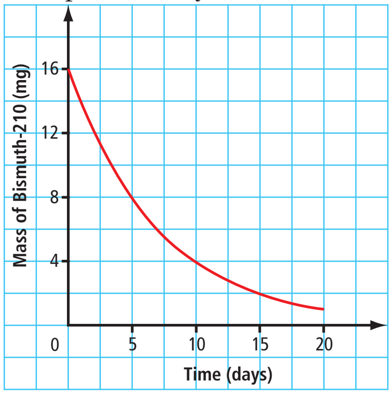
11. Example:



12. Example: Stage 1: The birth and mortality rates are similar, so the total population remains constant. Stage 2: The mortality rate declines rapidly while the birth rate remains nearly constant, so the total population rises at a constant rate. Stage 3: This stage is similar to stage 2. The birth rate is greater than the mortality rate, although the mortality rate begins to approach the birth rate, so the total population increase begins to taper off. Stage 4: The birth and mortality rates become similar, so the total population increase continues to taper off. Stage 5: The mortality rate is greater than the birth rate, so the total population begins to decline.

13. a) Example: 10 000 years

b)



14. The cost is constant up to a certain quantity of time. Example: The cost is \$1 for the first hour, \$2 for between one and two hours, and \$3 for between two and three hours.

15. Examples:

a) Total time passed cannot be taken away.

b) Distance already travelled cannot be taken away.

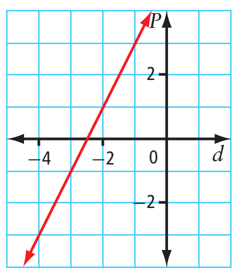
6.2 Linear Relations, pages 287 to 291

1. a) (−2, 5), (−1, 6), (0, 7), (1, 8), (2, 9), (3, 10), (4, 22)

b)

x	y
0	0
1	1 and −1
2	2 and −2
3	3 and −3

c)



d) Each child ate one orange.

2. a) linear; Each integral change in r results in a constant change of 2π in C.

b) non-linear; The degree is 2.

c) linear; An integral change in x results in a constant change in y.

d) non-linear; The values of x increase by greater and greater amounts but the values of y are increasing by the same constant.

e) linear; Each increase of 5 in the value of x results in an increase of 10 in the value of y.

f) linear; Each integral change in the value of x results in a constant change in the value of y.

g) linear; As the values of x increase, the values of y stay the same.

h) non-linear; The relation does not form a straight line.

3. a) dependent variable: A, independent variable: r

b) dependent variable: V, independent variable: t

c) dependent variable: A, independent variable: n

d) dependent variable: profit, independent variable: year

e) dependent variable: e, independent variable: c

4. Graph B. The relation does not have a degree of 1, so it cannot be linear. Only Graph B is non-linear.

5. a) linear; For each additional bushel of wheat sold, the amount of money received goes up by \$6.

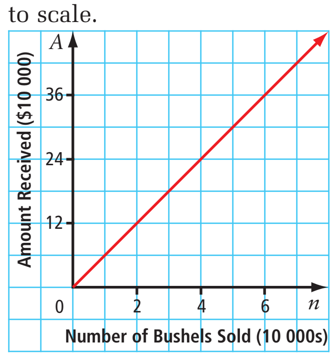
b) The amount of money received, A, is the dependent variable, and the number of bushels sold, n, is the independent variable.

c)

Number of Bushels Sold, n	Amount Received, A (\$)
0	0
10 000	60 000
20 000	120 000
30 000	180 000
40 000	240 000
50 000	300 000

d) discrete; The farmer would not get paid for parts of bushels. However, because a single bushel is such a small unit compared to 10 000 bushel units, the relation would appear to be continuous on a graph drawn to scale.

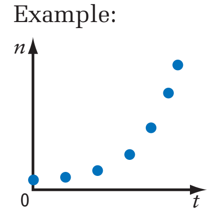
e)



6. a) Example:

x	y
0	0
1	−2
2	2
3	−4
4	4

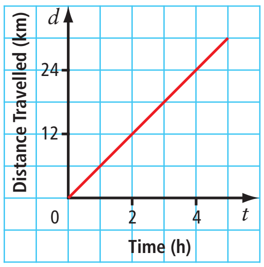
b) Example:



c) Example: $n = 4m + 1$

7. a) linear; The degree is 1.
b) dependent variable: D, independent variable: d
c) 1.5 m
d) Yes. The apparent depth is 2.1 m. The relation is continuous, because fractions of metres are feasible.

8. a) The dependent variable, d, represents the distance travelled, in kilometres. The independent variable, t, represents time, in hours.
b) (0, 0), (1, 6), (2, 12), (3, 18), (4, 24), and (5, 30)
c) continuous; The whale can swim for a fraction of an hour or cover a distance that is a fraction of a kilometre.
d)



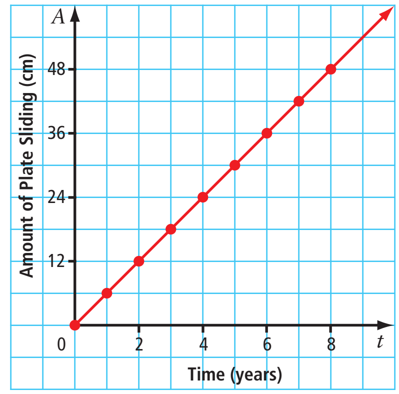
e) linear; The change in the distance travelled is constant with a constant change in the time.

9. a)

t	A
0	9.0
1	8.2
2	7.4
3	6.6
4	5.8
5	5.0
6	4.2
7	3.4
8	2.6
9	1.8
10	1.0
11	0.2
12	0.0

b) 12 years

10. a) non-linear; As the magnitude changes by one, the difference in size changes by ten times, so the difference in size of the earthquake is not constant.
b) (0, 0), (1, 6), (2, 12), (3, 18), (4, 24), (5, 30), (6, 36), (7, 42), and (8, 48)
c)



Example: Yes, the points should be connected, because fractions of years and fractions of centimetres are logical.

11. a) linear; As the values of x increase by two, the values of y increase by k.
b) linear; As the values of x increase by one, the values of y increase by $3n$.

12. a) Graph A is linear and Graph B is non-linear. In Graph A, the difference between values on the vertical axis is constant. In Graph B, the difference between values on the vertical axis is increasing.
b) For simple interest, the interest calculation is a certain percent of the original amount, so the amount added each year is the same, as shown in Graph A. For compound interest, the interest is added to the amount each year, so the interest earned each year gets larger, as shown in Graph B.

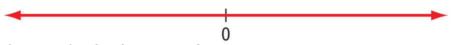
13. Example: I prefer an equation, because it is easy to see whether or not the relation is linear.

14. Example: research on the Internet or interviewing people

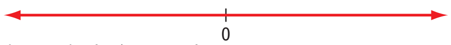
6.3 Domain and Range, pages 301 to 304

1. a) the real numbers between −8 and 30 inclusive, $[-8, 30]$, $\{n \mid -8 \le n \le 30, n \in \mathrm{R}\}$
b) the real numbers less than or equal to 0, $(\infty, 0]$, $\{n \mid n \le 0, n \in \mathrm{R}\}$
c) all real numbers greater than or equal to −2, $[-2, \infty)$, $\{n \mid n \ge -2, n \in \mathrm{R}\}$
d) the real numbers greater than 50 and less than or equal to 100, $(50, 100]$, $\{n \mid 50 < n \le 100, n \in \mathrm{R}\}$

2. a) domain: all real numbers



(∞, ∞), $\{x \mid x \in \mathrm{R}\}$
range: all real numbers



(∞, ∞), $\{y \mid y \in \mathrm{R}\}$

b) domain: all real numbers from 2 to 8, inclusive

$[2, 8]$, $\{x \mid 2 \leq x \leq 8, x \in \mathrm{R}\}$
range: all real numbers from 1 to 7, inclusive

$[1, 7]$, $\{y \mid 1 \leq y \leq 7, y \in \mathrm{R}\}$

c) domain: all real numbers greater than or equal to -4

$[-4, \infty)$, $\{x \mid x \geq -4, x \in \mathrm{R}\}$
range: all real numbers greater than or equal to 0

$[0, \infty)$, $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d) domain: all real numbers from -2 to 2, inclusive

$[-2, 2]$, $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$
range: all real numbers from -5 to -1, inclusive

$[-5, -1]$, $\{y \mid -5 \leq y \leq -1, y \in \mathrm{R}\}$

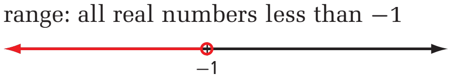
e) domain: all real numbers

(∞, ∞), $\{x \mid x \in \mathrm{R}\}$
range: all real numbers less than and including 7

$(\infty, 7]$, $\{y \mid y \leq 7, y \in \mathrm{R}\}$

f) domain: all real numbers less than 1

$(\infty, 1)$, $\{x \mid x < 1, x \in \mathrm{R}\}$
range: all real numbers less than -1



$(\infty, -1)$, $\{y \mid y < -1, y \in \mathrm{R}\}$

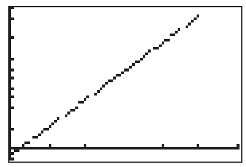
3. a) domain: $\{-4, 0, 1, 2, 3\}$, range: $\{-1, 0, 1, 4, 5, 6, 7\}$

b) domain: $\{-4, -2, 0, 2, 4, 6\}$, range: $\{5, 7, 9\}$

c) domain: $\{50, 100, 150, 200\}$, range: $\{10, 20, 30, 40\}$

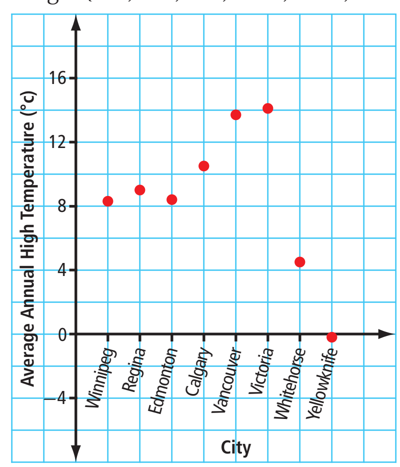
4. a) $[-3.5, 66.5]$

b) Example: Window values: Xmin = 0, Xmax = 30, Ymin = -5, Ymax = 70



5. a) domain: {Winnipeg, Regina, Edmonton, Calgary, Vancouver, Victoria, Whitehorse, Yellowknife}, range: $\{8.3, 9.1, 8.5, 10.5, 13.7, 14.1, 4.5, -0.2\}$

b)

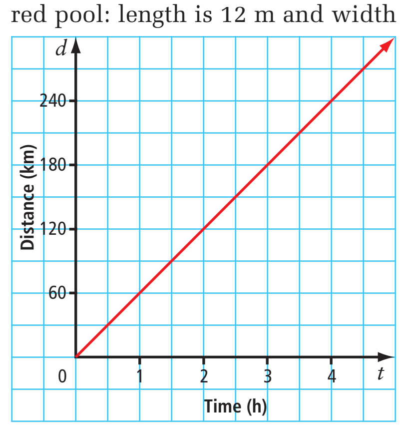


6. a) domain: $[2.5, 11.5]$, range: $[0, 6]$

b) domain: $\{x \mid 1 \leq x \leq 13, x \in \mathrm{R}\}$, range: $\{y \mid 0 \leq y \leq 6, y \in \mathrm{R}\}$

c) blue pool: length is 9 m and width is 6 m; red pool: length is 12 m and width is 6 m

7. a)



b) Examples: domain: the times between 0 h and approximately 3.22 h, [0, 3.22]; range: the distances between 0 km and 193 km inclusive, [0, 193]

8. a) dependent variable axis label: Water Depth (ft), d;
independent variable axis label: Time (h), t

b) Point A represents the high-tide water depth of 15.9 ft at 12:00 a.m. (0 h) at the beginning of the period, at 12:00 p.m. (12 h) at the middle of the period, and at 12:00 a.m. (24 h) at the end of the period. Point B represents the low-tide water depth of 4.5 ft at 6:00 a.m. (6 h) and at 6:00 p.m. (18 h). Point C is the origin, at the beginning of the time period (0 h). Point D is the end of the period (24 h).

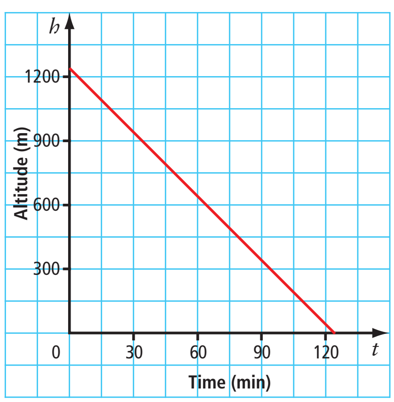
c) domain: the times between 0 h and 24 h, inclusive

[0, 24], $\{t \mid 0 \le t \le 24, t \in \mathrm{R}\}$
range: the water depths between 4.5 ft and 15.9 ft, inclusive

[4.5, 15.9], $\{d \mid 4.5 \le d \le 15.9, d \in \mathrm{R}\}$

9. a) 123.6 min

b)



c) Example: No. Once the balloon has landed, the situation of the model changes, causing a restriction on the domain and range.

d) The domain represents the times from when the balloon is at its highest until the balloon reaches the ground. The range represents the heights of the balloon from its highest point to ground level.

e) domain: the times between 0 min and 123.6 min, inclusive

[0, 123.6], $\{t \mid 0 \le t \le 123.6, t \in \mathrm{R}\}$
range: the altitudes between 0 m and 1236 m, inclusive

[0, 1236], $\{h \mid 0 \le h \le 1236, h \in \mathrm{R}\}$

10. $m = 2$ and $k = 14$

11. domain: [0, 7997.5]; range: [0, 2285]

12. Example: Given pairs of numbers that are related in some way, the domain is all of the possible first numbers of the relation, and the range is all of the possible second numbers of the relation.

13. Example: Hockey teams: The number of teams is the independent variable and the total number of players is the dependent variable. The restrictions are in place because there must be a limited number of teams, and each team has a limit on the number of players it can have.

6.4 Functions, pages 311 to 314

1. a) function; Each value of x has one value of y.
b) function; Each value of x has one value of y.
c) function; Each value of x has one value of y.
d) not a function; The values of x of 1 and 4 each have more than one value of y.
e) function; Each name has one age.
f) function; The graph passes the vertical line test.
g) not a function; There are two values of y associated with $x = -2$.

2. $A(r) = 4\pi r^2$

3. $C = 3n + 50$

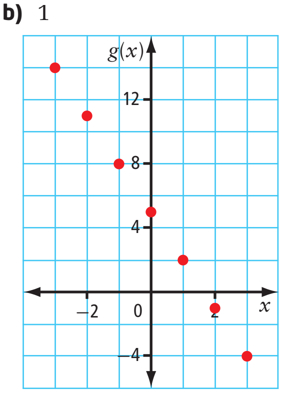
4. a) 12 **b)** −38 **c)** 5

5. a) 7 **b)** −1 **c)** −12

6. a) 2 **b)** 1

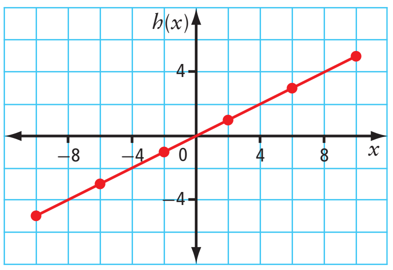
7. a)

x	$g(x)$
−3	14
−2	11
−1	8
0	5
1	2
2	−1
3	−4



b) Example:

x	$h(x)$
−10	−5
−6	−3
−2	−1
2	1
6	3
10	5



8. a) The variable w represents the number of weeks.

b) $M(w)$ is the function that shows the amount that Mike saves each week. $A(w)$ is the function that shows the amount that Ali spends each week.

c) $M(4) = 280$ and $A(4) = 120$; After four weeks, Mike has \$280 and Ali has \$120.

d) $w = 10$; After 10 weeks, Ali will have spent all of her money.

9. a) 177.523 cm

b) 174.636 cm; The function $h(36.87)$ represents the height of a female whose humerus measures 36.87 cm.

c) Example: [100, 230]; It is unlikely that either a man or woman would be shorter than 100 cm or taller than 230 cm.

d) Example: For $h(100)$, the radius bone for men measures approximately 4.1 cm. For $h(230)$, it measures approximately 43.5 cm. So, the domain is [4.1, 43.5]. For women, the radius bone measures approximately 5.7 cm and 45.1 cm. So, the domain is [5.7, 45.1]

e) Example: A male with a radius measuring 29 cm should have a height of 182.1 cm. If his height is 180 cm, the prediction is relatively accurate.

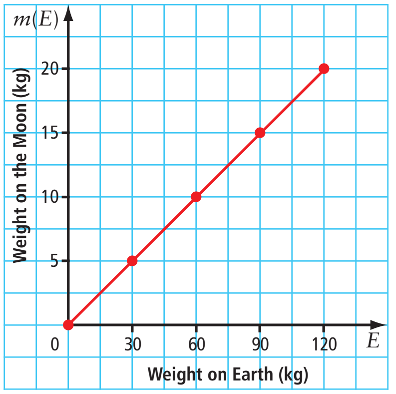
10. a) You would be lighter on the moon, because you divide your Earth weight by six to calculate your weight on the moon.

b) approximately 13.3 kg

c) Example: A person who weighs 90 kg on Earth would weigh 15 kg on the moon.

d) Example: [0, 120]

Weight on Earth (kg)	Weight on the Moon (kg)
0	0
30	5
60	10
90	15
120	20



11. a) 1236 km/h

b) 1071 km/h

c) $M(V) = \dfrac{V}{1236}$ and $M(V) = \dfrac{V}{1071}$

d) approximately 1.68

12. a) $f(x) = 3x + 2$

b) Example: $f(x) = 5x - 2$; $f(1) = 3$, $f(-1) = -7$, $f(2) = 8$, and $f(-2) = -12$

13. a) Yes. Each percent of carbon-14 value is paired with only one age value.

b) Example: approximately 90 years; This is the age of the bison bone.

c) Example: These bison bones would have about 50% of their carbon-14 remaining.

14. a) Yes. No value of x has more than one value of y.

b) $f(-4) = 0.5$; $f(1) = 0.5$; $f(3) = 4$; $f(5) = 6$

15. a) $h(4x) = 8x - 5$

b) $h(2x + 3) = 4x + 1$

c) $h\left(\frac{x}{2} - 1\right) = x - 7$

16. Example: Relations are pairs of numbers that are related to each other in some way. Functions are also pairs of numbers that are related, but they cannot be related in such a way that any value of x has more than one value of y (the first number cannot be paired with more than one second number).

17. Example: The expression $f(2)$ means that the value of x is 2. The expression $f(x) = 2$ means that the function value, or value of y, is 2.

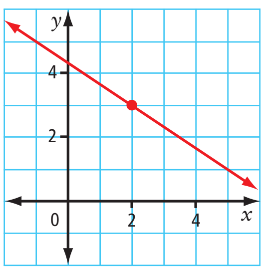
18. Examples:
   a) Jean-Marie is applying the distributive property. He thinks that f is being multiplied by the variable and the constant in the brackets.
   b) The expression $f(x + 2)$ means that for any function $f(x)$, substitute $(x + 2)$ for x.

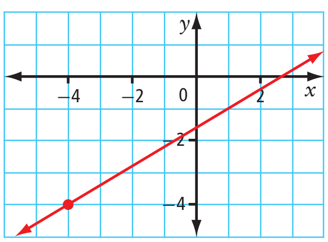
6.5 Slope, pages 325 to 329

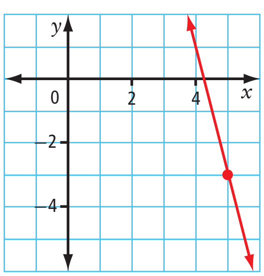
1. a) negative b) positive c) positive
   d) zero e) negative

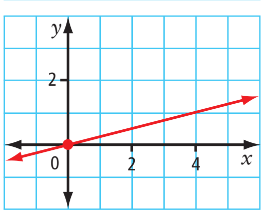
2. a) $\frac{2}{3}$ b) -1

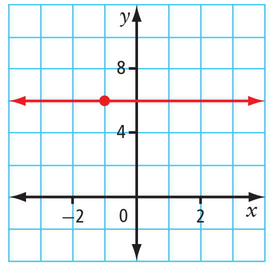
3. a) $\frac{4}{7}$ b) 0 c) $-\frac{2}{3}$
   d) 3 e) undefined f) $\frac{1}{2}$

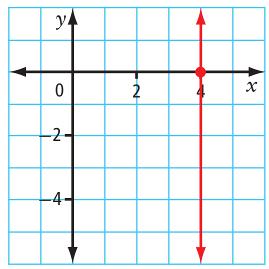
4. a)


b)


c)


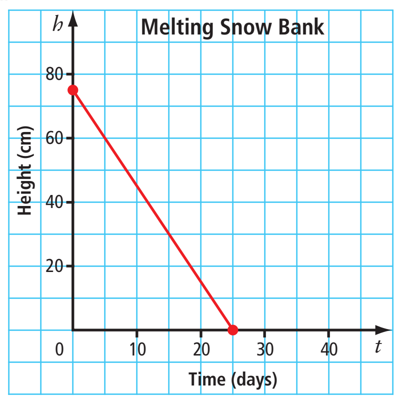
d)


e)


f)


5. -6.5 °C/km

6. approximately -8.3 m/s

7. a)


   b) the height of the snow bank after the number of days defined by that point
   c) The segment ends at (25, 0). After 25 days, the snow bank has completely melted.
   d) The slope represents the rate at which the snow bank is losing height, which is 3 cm per day.

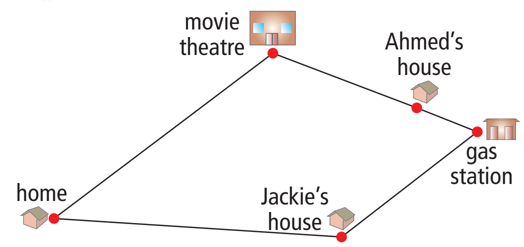
8. a) 216 in. or 18 ft
   b) approximately $216\frac{3}{4}$ in. or 18 ft $\frac{3}{4}$ in.
   c) approximately 24 ft 1 in.

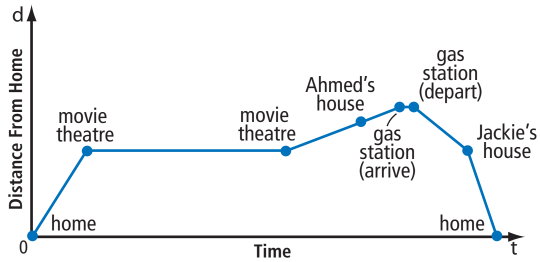
9. a) $\frac{8}{100}$ or 8 : 100
   b) Example: For every 100 units forward, the height of the road drops 8 units.

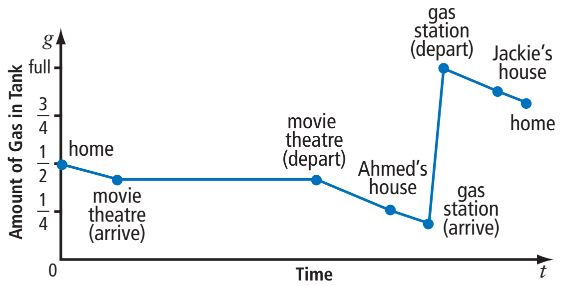
10. -3 ft/yr

11. **a)** The wood bison population diminished at a rate of approximately 1804 wood bison per year.
b) The wood bison population increased at a rate of approximately 47 wood bison per year.
12. **a)** 933 000 trees/yr
b) This rate of change represents the average increase per year in the number of trees in Alberta infested with the pine beetle from 2004 to 2007.
c) Example: If the same rate of increase continues, then in 2012 the number of trees infested with pine beetles will be 7 465 000, or almost 7.5 million infested trees.
13. **a)** 341 m/s
b) the speed of sound
c) approximately 10 221 m, or approximately 10 km
14. approximately 0.72
15. **a)** before heating: 125 cm^3, after heating: approximately 125.75 cm^3
b) 75.1501
16. $\frac{2x}{3}$
17. Example: The ratio of the rise versus the run remains the same (or constant) between any two points on the line.
18. **a)** Example: The tangent ratio is equal to the ratio of the length of the side opposite the angle (or rise), to the length of the side adjacent to the angle (or run). So, the measure of the angle is $\tan^{-1}\left(\frac{1}{16}\right)$.
b) approximately 3.6°
19. **Step 1:** 2900 m, 2720 m, and 2640 m
Step 2: The highest peak, 2900 m, closest to the bottom left corner, is Big Sister.
Step 3: Example: Calculate the slope between the lowest point on the mountain and the point that represents the peak.
Step 4: Example: AB: 5, BC: 0.1, CD: 1.1, DE: 2.6, FG: 0.6; Sections CD and FG pose an avalanche risk.
Step 5: Example: The slope is either too flat for the snow to easily fall, or too steep to hold any snow.

Chapter 6 Review, pages 330 to 334

1. Examples:
a)




b)


2. Example: The green line represents televisions. As people age, they tend to go out less and use more home entertainment devices such as television. The red line represents cell phones. Older generations were not raised with access to cell phones. The blue line represents computers. The younger groups are more comfortable with and dependent on this device because they were raised with it.
3. Example: Container 1: Graph A; The container would fill up at a constant, gradual rate.
Container 2: Graph D; The container would fill up quickly at a constant rate.
Container 3: Graph B; The lower portion of the container would fill up more slowly than the upper part.
Container 4: Graph C; The container would fill up more quickly at the bottom and slower at the top.

4. a) non-linear; The distance from the sun would not increase or decrease at a constant rate, and there would be times when the distances would repeat.

b) linear; The values of x are increasing by five each time, and the corresponding values of y are changing at a constant rate.

c) non-linear; The degree is 2.

d) non-linear; The values of x are increasing at a constant rate, but the values of y are not.

e) non-linear; The graph is not a single straight line.

5. B and D

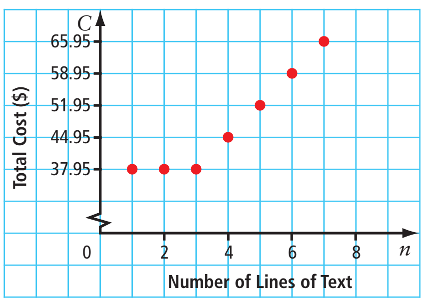
6. a) No. There is no change in the cost for the first three lines of text.

b) dependent variable: total cost; independent variable: number of lines of text

c) (3, 37.95), (4, 44.95), (5, 51.95), (6, 58.95), (7, 65.95)

d) discrete

e)



7. a) domain: $\{-9, -5, 0, 2\}$, range: $\{5, 8\}$

b) domain: $\{-1, 0, 1, 2, 3\}$, range: $\{-3, -1, 1, 3, 5\}$

8. a) domain:

$(-5, 5)$, $\{x \mid -5 < x < 5, x \in \mathrm{R}\}$

range:

$(1, 5)$, $\{y \mid 1 < y < 5, y \in \mathrm{R}\}$

b) domain:

$(\infty, 4]$, $\{x \mid x \leq 4, x \in \mathrm{R}\}$

range:

(∞, ∞), $\{y \mid y \in \mathrm{R}\}$

9. $C = \pi d$

10. $V(r) = r^3$

11. a) function; Each value of x has only one value of y.

b) not a function; It fails the vertical line test.

c) not a function; Black hair colour has one male and one female.

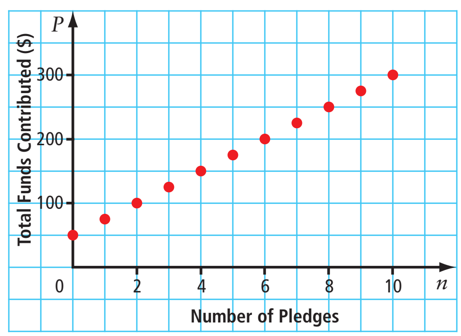
d) not a function; The value of x of 8.6 has more than one value of y.

12. a) approximately 24.4 kg

b) 37.8 kg

13. a)

Number of Pledges	Total Funds Contributed ($)
0	50
1	75
2	100
3	125
4	150
5	175
6	200
7	225
8	250
9	275
10	300



b) \$250; If Amber has eight pledges, she will have \$250 to donate.

c) more than 25 pledges

d) Example: For each number of pledges, there is only one possible total for the amount raised.

14. a) $\frac{3}{5}$

b) undefined

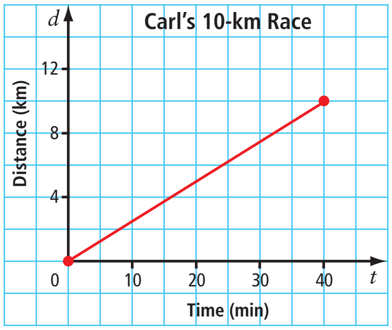
c) 0

d) -4

15. (450, 0) and (−450, 0)

16. a) $\frac{1}{4}$

b) -2

17. a) 

b) Carl's distance covered and the time he has taken to cover that distance

c) (40, 10); This point represents the end of the 10-km race, 40 min after Carl started.

d) his speed, 0.25 km/min

18. a) approximately 1155 people per year

b) This number represents the average rate of increase per year of people aged 12 or older who have asthma in Manitoba.

c) approximately 87 292 people

Chapter 6 Practice Test, pages 335 to 337

1. C

2. D

3. B

4. C

5. D

6. a) 1

b) Examples: (2, 1), (3, 2), and (4, 3)

7. a) approximately 5.4 m/s

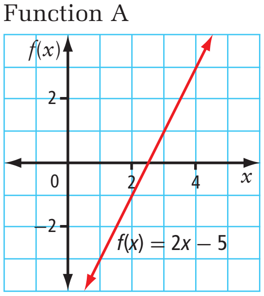
b) This rate represents his actual speed, which combines his speed from rowing and the current speed.

8. a) the boat's speed, in metres per second, after accelerating for 6 s

b) 20 m/s

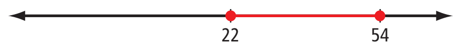
c) 7.2 s

9. Function A



10. a) the time, in hours, beginning at 24:00, and lasting for 3 days

b) Example:

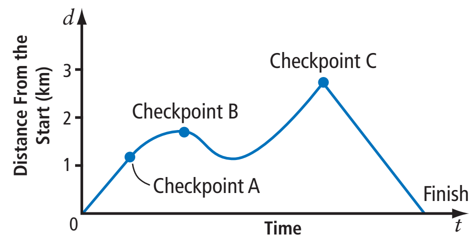


c) Example: [30, 39]

d) Example: $\{t \mid 34 \leq t \leq 37, t \in R\}$

e) Example: The temperatures are more stable at the deeper depths.

11.



Chapter 7

7.1 Slope-Intercept Form, pages 349 to 356

1. a) $m = -5$, y-intercept: (0, 4)

b) $m = \frac{3}{4}$, y-intercept: (0, 1)

c) $m = 1$, y-intercept: (0, −7)

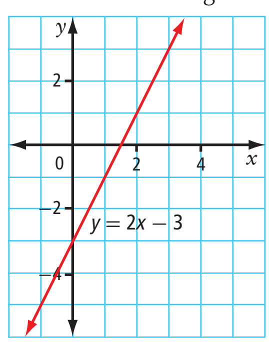
d) $m = -4$, y-intercept: (0, 0)

e) $m = 0$, y-intercept: (0, −3)

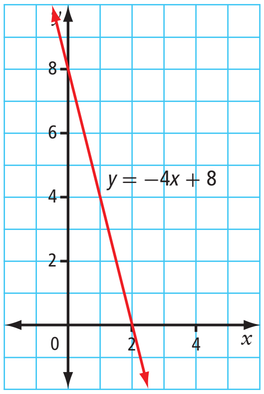
f) $m = 0.5$, y-intercept: (0, −1.25)

2. a) $m = -3$, y-intercept: (0, 2)

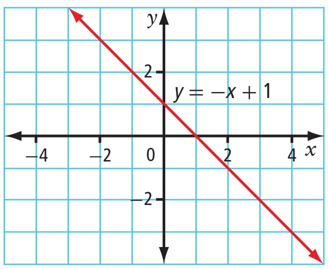
b) Example: Plot the point (0, 2). From the point (0, 2), go 3 units down and 1 unit to the right. Draw a line through the two points given.

3. a) 

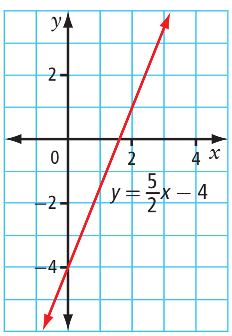
b)



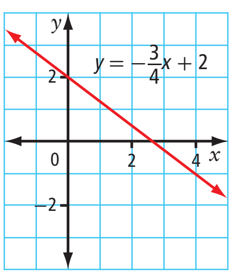
c)



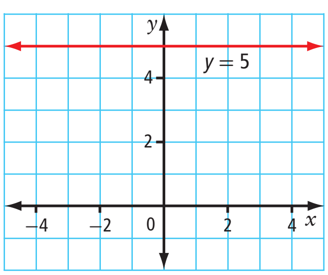
d)



e)



f)



4. a) step 3

b) $y = \frac{3}{2}x - 4$; The slope of the line is $\frac{3}{2}$ and the y-intercept is -4.

5. a) $y = -2x + 6$; $m = -2$, y-intercept: $(0, 6)$

b) $y = -3x - 9$; $m = -3$, y-intercept: $(0, -9)$

c) $y = -\frac{5}{6}x + \frac{4}{3}$; $m = -\frac{5}{6}$, y-intercept: $\left(0, \frac{4}{3}\right)$

d) $y = 6x - 4$; $m = 6$, y-intercept: $(0, -4)$

e) $y = 7x + 9$; $m = 7$, y-intercept: $(0, 9)$

f) $y = 2x - \frac{3}{4}$; $m = 2$, y-intercept: $\left(0, -\frac{3}{4}\right)$

6. a) $y = -3x + 2$ **b)** $y = \frac{5}{6}x - 4$

c) $y = -0.75x - 5$ **d)** $y = x - 7$

e) $y = -x$ **f)** $y = \frac{1}{3}$

7. a) A and C; The slopes are positive.

b) B and D; The slopes are negative.

c) C, B, D, A

d) D

8. a) $m = 4$, y-intercept: $(0, 4)$; $y = 4x + 4$

b) $m = -\frac{1}{2}$, y-intercept: $(0, -1)$; $y = -\frac{1}{2}x - 1$

c) $m = -2$, y-intercept: $(0, 6)$; $y = -2x + 6$

d) $m = \frac{2}{3}$, y-intercept: $(0, 4)$; $y = \frac{2}{3}x + 4$

e) $m = 0$, y-intercept: $(0, -2.5)$; $y = -2.5$

f) $m = \frac{2}{5}$, y-intercept: $(0, 4)$; $y = \frac{2}{5}x + 4$

9. a) -3 **b)** 18 **c)** -11 **d)** -5

10. a) 1 **b)** -5 **c)** $-\frac{3}{2}$ **d)** $-\frac{1}{6}$

11. Examples: $y = \frac{13}{5}x + 64$ and $y = -\frac{13}{5}x + 64$

12. a) $m = -2$, y-intercept: $(0, 3)$; Graph C

b) $m = 2$, y-intercept: $(0, -3)$; Graph D

c) $m = \frac{1}{2}$, y-intercept: $(0, -3)$; Graph B

d) $m = -\frac{1}{2}$, y-intercept: $(0, 3)$; Graph A

13. a) $C = 300 + 6.25n$

b) $T = 3.60 + 1.48x$

c) $D = 1024 + 54t$

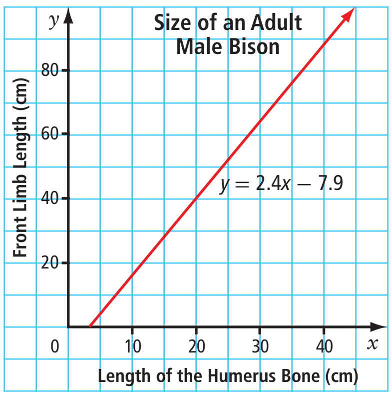
d) $L = 2500 - 120t$

14. $y_1 = \frac{10}{3}x + 10$, $y_2 = -\frac{10}{3}x + 10$, $y_3 = -\frac{10}{3}x - 10$, $y_4 = \frac{10}{3}x - 10$, $y_5 = -x$, $y_6 = x$, $y_7 = -\frac{3}{10}x + 3$, $y_8 = \frac{3}{10}x + 3$, $y_9 = -\frac{3}{10}x - 3$, $y_{10} = \frac{3}{10}x - 3$

15. a) spring 1: $L = \frac{3}{2}x + 8$, spring 2: $L = -\frac{3}{2}x + 24$

b) Example: The mass is sitting on top of the spring instead of being suspended from it. The length of spring 2 compresses by 1.5 cm per gram.

16. a)

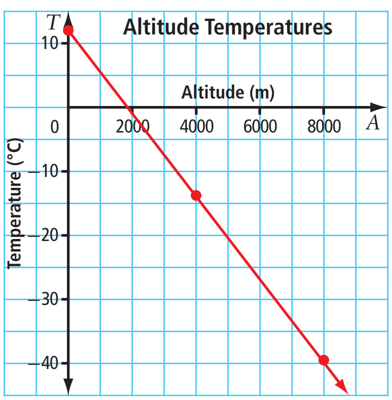


b) 89 cm, 70 cm

c) 26%

17. $y = -3x + 7$

18. a)



b) $m = -\frac{4}{625}$; The temperature drops 4 °C for every 625 m increase in altitude.

c) y-intercept: (0, 12); The temperature at ground level is 12 °C.

d) $T = -\frac{4}{625}A + 12$

e) −26.1 °C

f) above 5000 m

19. $y = -\frac{1}{4}x + 4$

20. −20

21. −9

22. $y = x + 3$

23. 99 points

24. a) Example: To determine the slope, apply the slope formula using two of the points from the table. To determine the y-intercept, look for the corresponding y-coordinate when the x-coordinate is 0.

b) Example: To determine the slope, find two points on the graph and apply the slope formula. To determine the y-intercept, read the value of y when $x = 0$ from the graph.

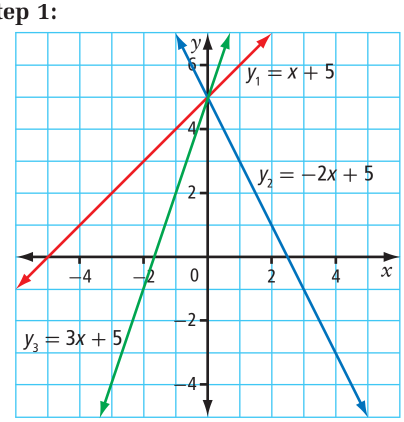
c) Example: The slope is the value of m, and the y-intercept is the value of b.

25. a) Example: Write the equation of the line in the form $y = mx + b$, where m is the slope and b is the y-intercept.

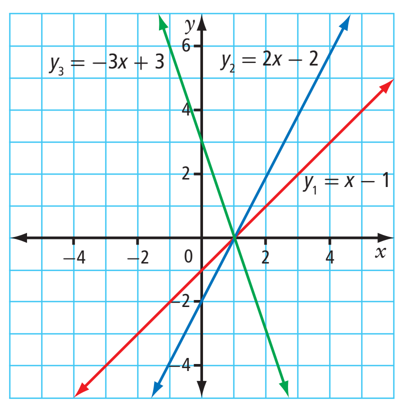
b) Example: Start with the y-intercept and then use the slope to determine a second point. Draw the line passing through the two points.

26. Step 1:

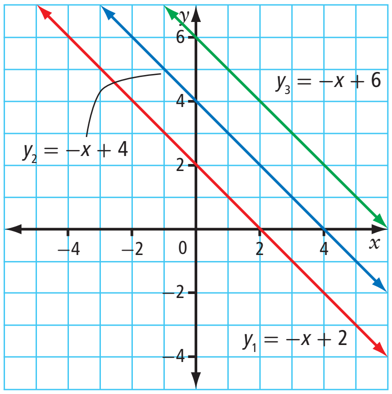
a)



b)



c)



Step 2: Examples:

a) same y-intercept but different slopes

b) same x-intercept but different slopes

c) same slope but different y-intercepts; parallel lines

Step 3: Examples:

a) $y = -x + 5$

b) $y = -x + 1$

c) $y = -x - 2$

Step 4: Example: $y_1 = 2x$, $y_2 = -x$, $y_3 = 3x$; All lines pass through the origin, (0, 0).

7.2 General Form, pages 365 to 369

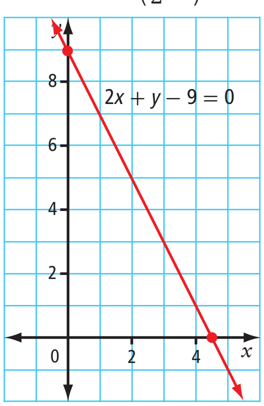
1. Jasmine forgot to multiply 4 by 2. $3x + 2y - 8 = 0$

2. a) $7x - y - 5 = 0$ **b)** $x + y - 8 = 0$

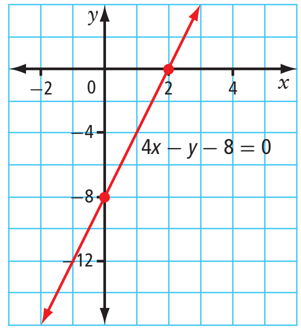
c) $3x - 2y + 8 = 0$ **d)** $3x + 5y + 10 = 0$

e) $5x - 20y - 6 = 0$ **f)** $20x + 8y - 1 = 0$

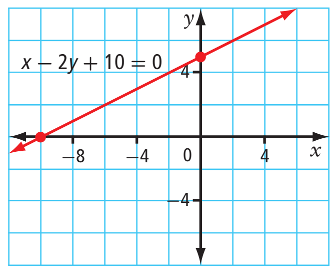
3. a) x-intercept: $\left(\frac{9}{2}, 0\right)$, y-intercept: (0, 9)



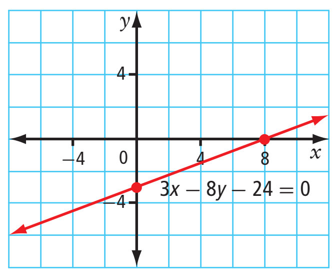
b) x-intercept: (2, 0), y-intercept: (0, −8)



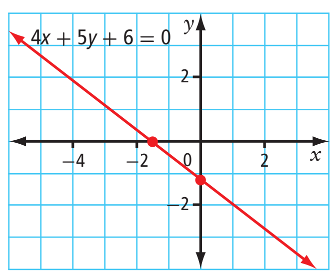
c) x-intercept: (−10, 0), y-intercept: (0, 5)



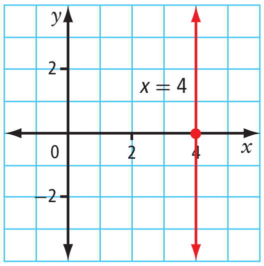
d) x-intercept: (8, 0), y-intercept: (0, −3)



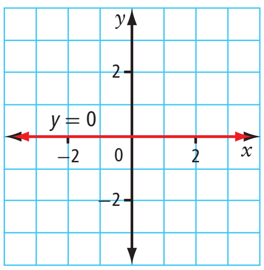
e) x-intercept: $\left(-\frac{3}{2}, 0\right)$, y-intercept: $\left(0, -\frac{6}{5}\right)$



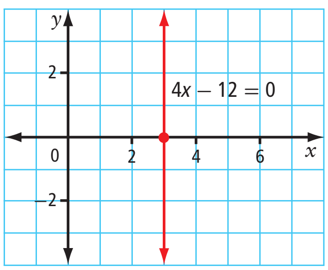
f) x-intercept: (4, 0), no y-intercept



g) $\{x \mid x \in \mathrm{R}\}$, y-intercept: (0, 0)



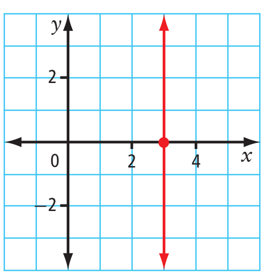
h) x-intercept: (3, 0), no y-intercept



4. a) domain: $\{x \in \mathrm{R}\}$, range: $\{2\}$, no x-intercept, y-intercept: (0, 2), $m = 0$; $y - 2 = 0$

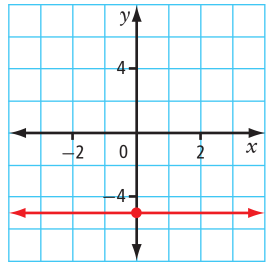
b) domain: $\{-3\}$, range: $\{y \in \mathrm{R}\}$, no y-intercept, x-intercept: $(-3, 0)$, slope is undefined; $x + 3 = 0$

5. a)



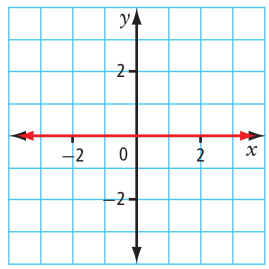
$x - 3 = 0$

b)



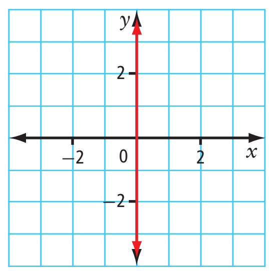
$y + 5 = 0$

c)



$y = 0$

d)



$x = 0$

6. a) 2 **b)** 4 **c)** 8 **d)** 5
e) 3 **f)** 6 **g)** 7 **h)** 1

7. a) $x - 3 = 0$ **b)** $y - 6 = 0$
c) $y = 0$ **d)** $x = 0$

8. $x - 3 = 0$

9. Let g represent the number of 125-mL servings of green peas. Let b represent the number of 125-mL servings of baked beans. Let B represent the number of 125-mL servings of bran buds.
$4g + 16B - 21 = 0$ or $7b + 16B - 21 = 0$;
green peas: 157 mL; baked beans: 90 mL
Note that answers were rounded up to ensure the desired amount of each vegetable was consumed. If the answers had been rounded down, the desired amounts would not have been achieved.

10. a) y-intercept: (0, 1200); The distance between Saskatoon and Vancouver is 1200 km. x-intercept: (2.0, 0); It takes 2 h to fly from Saskatoon to Vancouver.

b) domain: $\{x \mid 0 \le x \le 2, x \in \mathrm{R}\}$, range: $\{y \mid 0 \le y \le 1200, y \in \mathrm{R}\}$

c) $m = -600$; The speed of the plane is 600 km/h.

d) $600x + y - 1200 = 0$

e) 1 h 40 min

f) 750 km

11. a) $8x + 11y = 440$

b) x-intercept: (55, 0); Luc needs to swim backstroke for 55 min in order to burn 440 cal. y-intercept: (0, 40); Luc needs to swim butterfly for 40 min in order to burn 440 cal.

c) 33 min

12. a) $1200x + 1800y = 10\ 000$

b) approximately 0.50 m^3

c) approximately 6%

13. a) $8a + 12d - 1120 = 0$

b) Example: Find the a-intercept and the d-intercept and draw the line passing through these two points.

c) 80 advance tickets, 40 tickets at the door

14. a) $\frac{4}{3}$ **b)** 3 **c)** -10

15. a) (6, 6) **b)** (12, 4) **c)** (3, 7)

16. -4

17. a) 25 square units

b) 49 square units

18. a) Example: I prefer the slope-intercept form, because it is easy to find two points on the graph.

b) Example: Use the general form to find the x-intercept.

19. a) Example: Substitute $y = 0$ in the equation and solve for x.

b) Example: Substitute $x = 0$ into the equation and solve for y, or rewrite the equation in slope-intercept form.

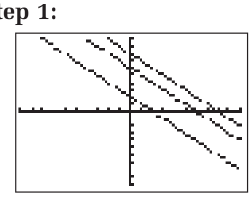
20. a) vertical

b) horizontal

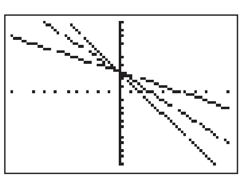
c) oblique

21. Step 1:

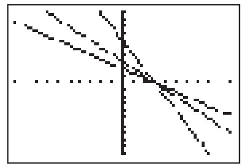
a)



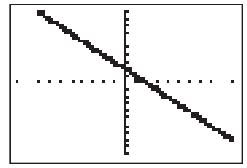
b)



c)



d)



Step 2:

a) parameter C; If only parameter C is changed, the graphs of the equations will have the same slope but different x-intercepts and y-intercepts.

b) parameter A; If only parameter A is changed, the graphs of the equations will have the same y-intercept but different slopes and x-intercepts.

c) parameter B; If only parameter B is changed, the graphs of the equations will have the same x-intercept but different slopes and y-intercepts.

d) All parameters have changed so that the second and third equations are multiples of the first equation. When parameters A, B, and C of one equation are multiples of those of another equation, the graphs represent the same line.

Step 3: Example: Parameters A and B influence the slope of the line, while parameter C influences the x-intercept and y-intercept.

7.3 Slope-Point Form, pages 377 to 382

1. a) $y = x - 8$, $x - y - 8 = 0$

b) $y = 2x + 2$, $2x - y + 2 = 0$

c) $y = 4x + 10$, $4x - y + 10 = 0$

d) $y = -5x - 17$, $5x + y + 17 = 0$

e) $y = -\frac{1}{2}x - 1$, $x + 2y + 2 = 0$

f) $y = -\frac{2}{3}x - 5$, $2x + 3y + 15 = 0$

2. a) $y - 2 = 2(x - 3)$

b) $y + 3 = -\frac{3}{2}(x - 1)$

c) $y + 2 = \frac{1}{2}(x + 4)$

d) $y - 2 = -\frac{1}{3}(x + 4)$

3. a) $y + 2 = 6(x - 5)$, $y = 6x - 32$, $6x - y - 32 = 0$

b) $y + 5 = -2(x + 3)$, $y = -2x - 11$, $2x + y + 11 = 0$

c) $y - 3 = \frac{1}{2}(x + 8)$, $y = \frac{1}{2}x + 7$, $x - 2y + 14 = 0$

d) $y + 6 = -\frac{2}{3}(x - 12)$, $y = -\frac{2}{3}x + 2$, $2x + 3y - 6 = 0$

4. a) $m = \frac{2}{3}$, $(6, 1)$

b) Example: From the point (6, 1), go 2 units up and 3 units right to find the second point. Draw the line passing through the two points.

5. Examples:

a) $y - 1 = 4(x - 1)$

b) $y + 2 = -3(x + 1)$

c) $y - 2 = -\frac{1}{2}(x + 2)$

d) $y + 1 = \frac{2}{3}(x - 0)$

6. a) $y - 1 = 4(x - 5)$, $y = 4x - 19$, $4x - y - 19 = 0$

b) $y + 8 = -3(x - 5)$, $y = -3x + 7$, $3x + y - 7 = 0$

c) $y - 5 = -\frac{1}{2}(x - 4)$, $y = -\frac{1}{2}x + 7$, $x + 2y - 14 = 0$

d) $y + 3 = \frac{3}{4}(x - 8)$, $y = \frac{3}{4}x - 9$, $3x - 4y - 36 = 0$

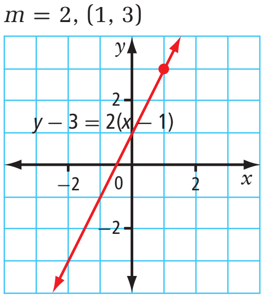
e) $y + 1 = \frac{3}{2}(x - 5)$, $y = \frac{3}{2}x - \frac{17}{2}$, $3x - 2y - 17 = 0$

f) $y - 6 = \frac{3}{2}(x - 3)$, $y = \frac{3}{2}x + \frac{3}{2}$, $3x - 2y + 3 = 0$

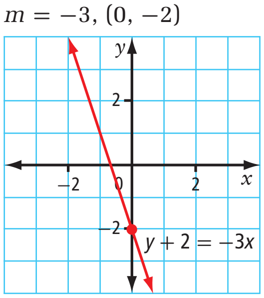
7. a) Example: Compare the graphs of the equations using slope-point form, or rewrite the equations in slope-intercept form and compare the equations.

b) 1 and 4, 2 and 3; In slope-intercept form, the equations in each pair are the same.

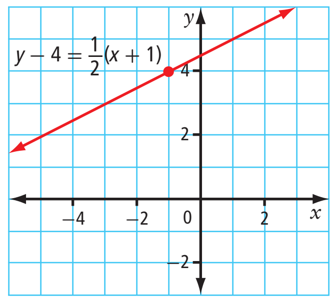
8. a) $m = 2$, $(1, 3)$



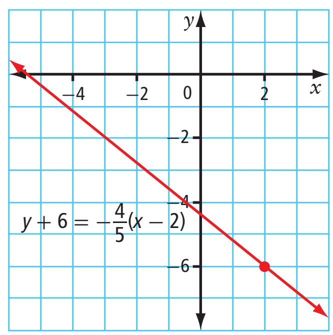
b) $m = -3$, $(0, -2)$



c) $m = \frac{1}{2}$, $(-1, 4)$



d) $m = -\frac{4}{5}$, $(2, -6)$



9. a) Example: Find the slope and use one of the points in slope-point form or in slope-intercept form of the equation. Plot the two points and draw the line that passes through them. Read the y-intercept from the graph.

b) 10

10. a) $y = 2x + 1$

b) Example: $y - 7 = 2(x - 3)$

c) same graph

11. a) $y + 5 = 0(x - 4)$, $y = -5$

b) $y - 4 = -3(x + 2)$, $y = -3x - 2$

c) $y - 0 = \frac{1}{2}(x - 8)$, $y = \frac{1}{2}x - 4$

d) $y + 4 = -2(x - 3)$, $y = -2x + 2$

12. a) $y - 0 = 3(x - 4)$, $3x - y - 12 = 0$

b) $y + 1 = -4(x - 2)$, $4x + y - 7 = 0$

c) $y - 2 = -\frac{1}{2}(x - 0)$, $x + 2y - 4 = 0$

d) $y + 6 = 3(x - 0)$, $3x - y - 6 = 0$

13. a) $V - 29 = 1.2(t - 24)$

b) 129 h

c) No. The tank contained 0.2 m^3 before it started filling.

14. a) $\frac{3}{5}$

b) 0.6 m/s/°C

c) $V - 335 = \frac{3}{5}(t - 6)$

d) 352.4 m/s

e) approximately 28 °C

15. -2

16. -3

17. a) (1, 30) and (9, 33.7)

b) $m = 0.4625$

c) 462 500 people/year

d) $p - 30 = 0.4625(t - 1)$

e) 37.4 million people

18. a) Example: $p - 30 = \frac{3}{5}(n - 0)$, where p represents the number of grams of protein in the dinner and n represents the number of yellow potatoes that are eaten.

b) $m = \frac{3}{5}$; It represents 0.6 g protein per yellow potato.

c) p-intercept $= 30$; The steak contains 30 g of protein.

d) Example: domain: $\{n \mid 0 \leq n \leq 5, n \in W\}$, range: $\{p \mid 0 \leq p \leq 33, p \in R\}$

19. $y = m(x - n)$

20. $y + 4 = \frac{1}{2}(x + 4)$

21. Substitute the coordinates of the y-intercept, $(0, b)$.

22. Example: two points, slope and one point, relationships with other lines

23. Example: $y = mx + b$ for questions about slope or y-intercept; $y - y_1 = m(x - x_1)$ for questions involving two points or a point and slope

24. Step 1: Example:

Length of Humerus Bone (cm)	Person's Height (cm)
36	176
40	177

Step 3: The equation of the line passing through these two points is $y = 0.25x + 167$.

Step 4: Example: For a measurement of 37 cm, the predicted height of the teacher is 176.25 cm or approximately 176 cm.

7.4 Parallel and Perpendicular Lines, pages 390 to 395

1. a) $m = 5$; $m = -\frac{1}{5}$

b) $m = -7$; $m = \frac{1}{7}$

c) $m = -\frac{1}{3}$; $m = 3$

d) $m = \frac{6}{7}$; $m = -\frac{7}{6}$

e) $m = 0.5$; $m = -2$

f) $m = -0.75$; $m = \frac{4}{3}$

g) $m = 0$, m is undefined

h) m is undefined, $m = 0$

2. a) $m = \frac{3}{7}$, $m = -\frac{7}{3}$

b) $m = -1$, $m = 1$

c) $m = -3$, $m = \frac{1}{3}$

d) $m = -2$, $m = \frac{1}{2}$

e) $m = \frac{3}{2}$, $m = -\frac{2}{3}$

f) $m = -\frac{5}{4}$, $m = \frac{4}{5}$

g) $m = 0$, m is undefined

h) m is undefined, $m = 0$

3. a) $m = -2$ **b)** $m = \frac{1}{2}$

4. a) $n = 20$; $n = -5$

b) $n = -72$; $n = 8$

c) $n = \frac{27}{2}$; $n = -6$

d) $n = -\frac{6}{7}$; $n = \frac{21}{2}$

5. a) neither; The slopes are neither equal nor negative reciprocals.

b) perpendicular; The slopes are negative reciprocals.

c) parallel; The slopes are equal.

d) perpendicular; The slopes are negative reciprocals.

e) neither; The slopes are neither equal nor negative reciprocals.

f) perpendicular; The slopes are negative reciprocals.

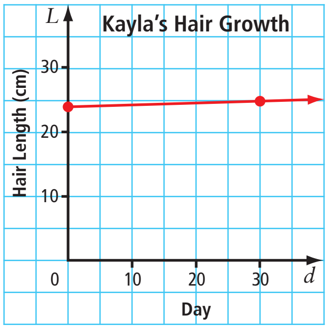
6. a) $y = 2x - 8$ **b)** $y = -3x - 1$

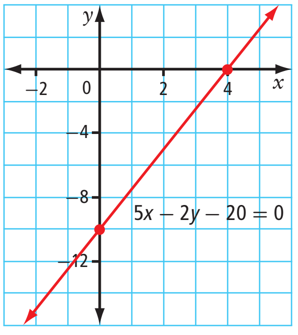
c) $y = -5x + 7$ **d)** $y = 3x - 14$

e) $y = 4$ **f)** $x = -1$

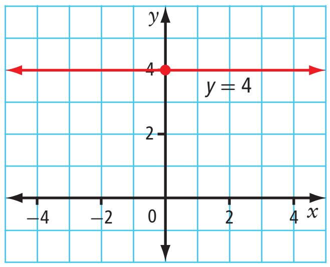
7. a) $y = -\frac{1}{3}x + 8$ b) $y = \frac{1}{4}x - 4$
c) $y = 3x - 24$ d) $y = -\frac{3}{4}x - \frac{5}{2}$
e) $y = 7$ f) $x = 4$
8. a) Example: No. You have to prove it mathematically.
b) Example: Calculate and compare the slopes of the two lines. If the slopes are equal, then the lines are parallel.
c) No. Since $m_{AB} = \frac{1}{3}$ and $m_{CD} = \frac{1}{2}$, the two line segments are not parallel.
9. Yes. Example: Since $m_{AB} = m_{CD} = -\frac{2}{7}$, side AB is parallel to side CD. Since $m_{AD} = m_{BC} = -3$, side AD is parallel to side BC. Therefore, ABCD is a parallelogram, because opposite sides are parallel.
10. a) $y - 5 = 0$ b) $x - 7 = 0$
11. a) $n = -8$ b) $n = 5$
12. Example: $m_{AB} = \frac{2}{7}$ and $m_{AC} = -\frac{7}{2}$. Since the slopes of sides AB and AC are negative reciprocals, AB and AC are perpendicular to each other, and △ABC is a right triangle.
13. a) $y = -5x - 6$
b) $y = 5x - 3$
c) $y = \frac{4}{5}x - 4$
14. $y = 2x - 1$
15. $n = -8$
16. $y = -3x + 16$
17. Example: No. The slopes of the two graphs are not equal.
18. Example: $y_1 = \frac{14}{23}x + \frac{42}{23}$, $y_2 = \frac{14}{23}x$, $y_3 = \frac{14}{23}x - 2$; Note: The slope of the middle line is parallel to the other two lines.
19. $n = -10$
20. $n = 3$
21. $\frac{\sqrt{5}}{5}$ units; It is the perpendicular distance between the lines.
22. (1, 0) and (4, 0)
23. $n = 6$, $n = -6$
24. $n = 0$
25. Example: Sometimes true. Vertical and horizontal lines are perpendicular to each other. However, their slopes are not negative reciprocals. For oblique lines, it is always true.
26. Example: I would prefer to use the slope-intercept or slope-point form, because it is easy to compare the slopes.

Chapter 7 Review, pages 396 to 398

1. a) $m = -5$, y-intercept: (0, 6)
b) $m = \frac{5}{6}$, y-intercept: (0, 2)
2. a) Substitute $-\frac{4}{5}$ for m and 6 for b in $y = mx + b$. $y = -\frac{4}{5}x + 6$
b) Substitute 0 for m and -8 for b in $y = mx + b$. $y = -8$
3. a)


b) $m = 0.04$; Kayla's hair grows 0.04 cm per day.
c) y-intercept: (0, 24); Kayla's starting hair length is 24 cm.
d) $L = 0.04d + 24$
e) 53.2 cm
4. Example: Express the equation in slope-intercept form. Start from the y-intercept, 4, on the y-axis. Locate another point by moving 2 units down and 5 units to the right, since the slope is $-\frac{2}{5}$. Draw a line through the points, (0, 4) and (5, 2).
5. a) (0, 6), (−2, 0); $3x - y + 6 = 0$
b) (3, 0), no y-intercept; $x - 3 = 0$
6. a) x-intercept: (4, 0), y-intercept: (0, −10)


b) no x-intercept, y-intercept: (0, 4)



7. a) Let b represent the number of bannock wheels sold. Let c represent the number of buffalo burgers sold. $2b + 3c - 600 = 0$

b) b-intercept $= 300$; If they sold only bannock wheels, they would have sold 300. c-intercept $= 200$; If they sold only buffalo burgers, they would have sold 200.

c) domain: $\{b \mid 0 \leq b \leq 300, b \in W\}$, range: $\{c \mid 0 \leq c \leq 200, c \in W\}$

d) 110 buffalo burgers

8. a) $y - 1 = -4(x + 2)$, $y = -4x - 7$, $4x + y + 7 = 0$

b) $y + 3 = \frac{1}{2}(x - 8)$, $y = \frac{1}{2}x - 7$, $x - 2y - 14 = 0$

9. Example: Find the slope using the two points: $m = -4$. Write the equation in slope-point form using the slope and one of the points: $y - 2 = -4(x - 3)$. $y = -4x + 14$ or $4x + y - 14 = 0$

10. a) $m = -0.0035$; The temperature at which water boils decreases by 0.0035 °C for every metre increase in altitude.

b) $T = -0.0035d + 100$

c) 86 °C

11. a) $m = \frac{5}{2}$ **b)** $m = -\frac{2}{5}$

12. a) $n = -12$ **b)** $n = 27$

13. a) A and D, A and E, D and E; The slopes are equal but the intercepts are different.

b) B and A, B and D, B and E; The slopes are negative reciprocals.

c) D and E; The equations of the lines are the same.

14. $y = 2$; Example: The slope of $y = -7$ is 0 and the graph is a horizontal line. Therefore, the equation of a horizontal line through $(-1, 2)$ is $y = 2$.

15. $4x - 3y + 45 = 0$; Example: Determine the slope of the given line. Then, substitute the negative reciprocal of that slope and the coordinates of the given point into the slope-point form. Express in general form.

16. Example: First, determine that the slope of $2x + 5y + 10 = 0$ is $-\frac{2}{5}$. The slope of a line perpendicular to this one is $\frac{5}{2}$. Next, determine that the x-intercept of $3x - 2y = 12$ is 4. Then, use $m = \frac{5}{2}$ and (4, 0) to write the equation of the line in slope-point form. Express it in general form. $5x - 2y - 20 = 0$

17. $y = -\frac{4}{3}x + 18$

Chapter 7 Practice Test, pages 399 to 401

1. C

2. B

3. B

4. C

5. A

6. A

7. a) The product of their slopes is -1.

b) $n = 9$

8. $x - 2 = 0$

9. a) Let a represent the number of adult tickets sold. Let s represent the number of student tickets sold. $10a + 6s = 2900$

b) 125 adult tickets

10. A, C, and D; Once expressed in slope-intercept form, they all have the same equation.

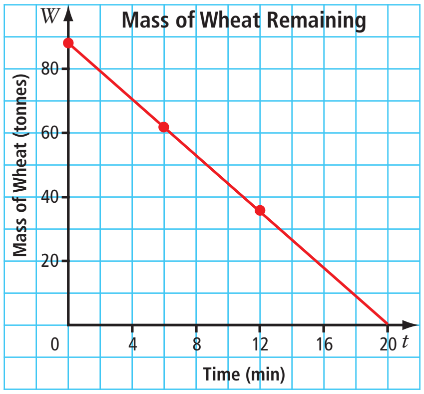
11. a) D-intercept: (0, 30); This represents the total length of the trail: 30 km.

b) Example: The equations of the two scenarios represent parallel lines, since they will have the same slope. Parallel lines never intersect. Therefore, Felicia will never catch up to Jacob.

12. $5x + y - 14 = 0$; Example: First, determine that the slope of $5x + y - 1 = 0$ is -5. A parallel line will have the same slope. Use $m = -5$ and $(3, -1)$ to write an equation in slope-point form, and then express it in general form.

13. Example: 1) Plot the point $(-3, 4)$ and use the slope of -2 to locate another point. Draw a line through the two points. 2) Express the equation in slope-intercept form. Then, use the slope of -2 and the y-intercept of -2 to draw the graph of the line. 3) Express the equation in general form. Then, find the intercepts and use these to draw the graph of the line.

14. a)



b) $W = -4.4t + 88$

c) t-intercept $= 20$; After 20 min, the car is empty. W-intercept $= 88.0$; Before unloading, there is 88.0 tonnes of grain in the railway car.

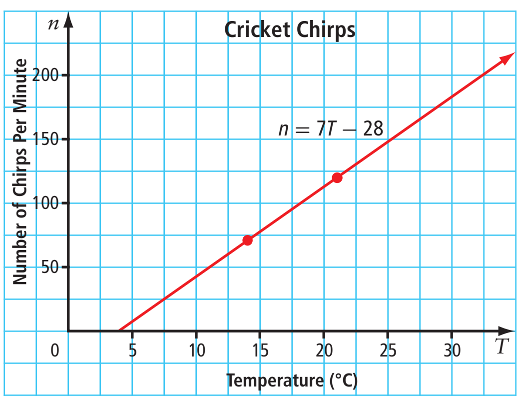
d) $m = -4.4$; The railway car is unloaded at a rate of 4.4 tonnes per minute.

e) domain: $\{t \mid 0 \leq t \leq 20, t \in \mathrm{R}\}$, range: $\{W \mid 0 \leq W \leq 88.0, W \in \mathrm{R}\}$

f) 10 min

15. a) $n = 7T - 28$

b) Example: Plot the two points (14, 70) and (21, 119) and draw a line through them.

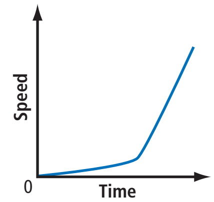


c) 28 °C

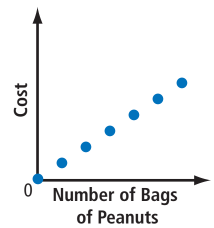
Unit 3 Review, pages 406 to 407

1. Examples:

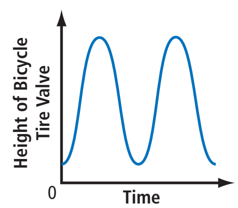
a) Time is greater than or equal to zero and stops when the rider is at the bottom of the hill. The relation is continuous.



b) The number of bags of peanuts bought must be a whole number. The relation is discrete.



c) Time is greater than or equal to zero. The relation is continuous.



2. Examples:

a) The height of a plant over time.

b) After coming home from school, Timmy visited his friend and returned home again.

3. a)

Weeks Worked	Earnings ($)
0	0
1	875
2	1750
3	2625
4	3500
5	4375

b) This relation is a function. Each domain value occurs only once.

c) The relation is discrete, since Rachel is only paid daily.

d) Let E represent Rachel's weekly earnings, in dollars. Let n represent the number of weeks worked. $E = 875n$

4. **a)** function **b)** function
c) function **d)** not a function

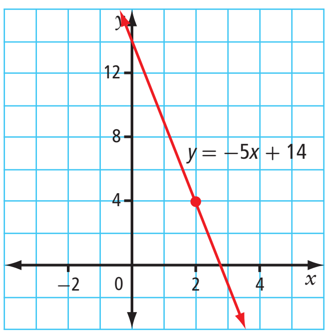
5. **a)** independent variable: time, dependent variable: litres of blood
b) $m = 5$
c) A human heart can pump 5 L of blood per minute.

6. **a)** function **b)** non-function

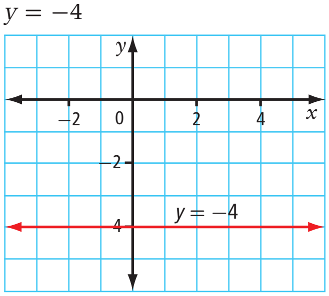
7. $\frac{5}{7}$, 2, 0

8. **a)** $m = \frac{1}{2}$, y-intercept: (0, 6)
b) $m = -\frac{3}{5}$, y-intercept: (0, 2)
c) $m = \frac{1}{3}$, y-intercept: (0, 0)
d) $m = 0$, y-intercept: (0, −1)

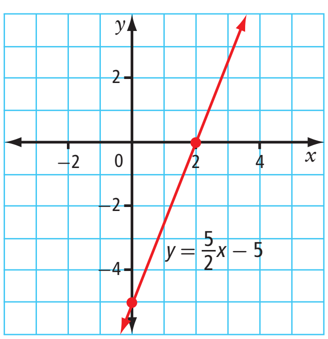
9. **a)** $y = -5x + 14$



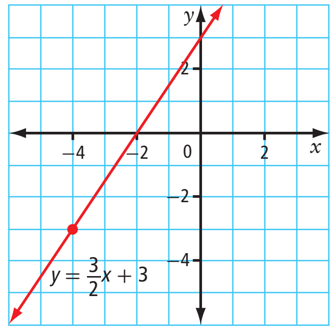
b) $y = -4$



c) $y = \frac{5}{2}x - 5$



d) $y = \frac{3}{2}x + 3$



10. **a)** (0, 6) and (4, 0); $3x + 2y - 12 = 0$
b) (0, 3) and (−2, 0); $3x - 2y + 6 = 0$

11. **a)** $x - 1 = 0$
b) $3x + 5y + 21 = 0$

Unit 3 Test, pages 408 to 409

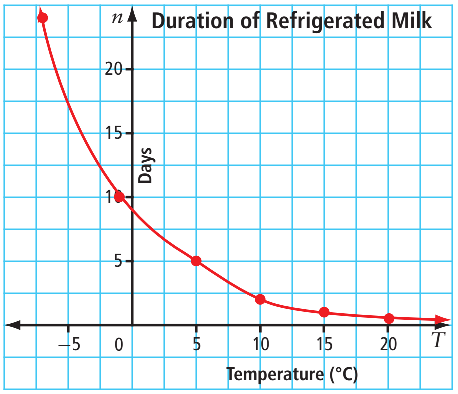
1. C
2. D
3. B
4. D
5. A
6. 3
7. 2
8. **a)** $\{n \mid 0 \le n \le 8.5, n \in \mathrm{R}\}$, where n represents the number of hours you study.
b)

Number of Hours of Study	Mark (%)
0	32
1	40
2	48
3	56
4	64

c) $M = 8n + 32$, where n represents the number of hours you study and M represents the predicted mark as a percent.
d) 8.5 h
e) Example: No, a linear model is not a valid model because there are many factors that can affect performance on an exam other than hours of study.

9. $2x + y + 6 = 0$

10. a)



b) 8 days

11. a) Example: The equation contains two variables and is of degree one. For each domain value there will be only one range value.

b) discrete; The cost for the lessons is based on half hour increments.

c) Example: (0, 25), (0.5, 40), (1, 55), (1.5, 70), (2, 85)

d) 3.5 h

Chapter 8

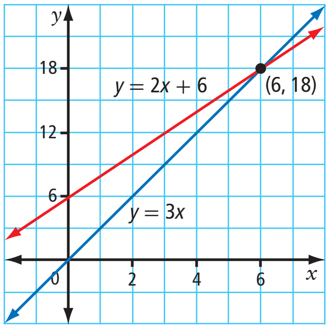
8.1 Systems of Linear Equations and Graphs, pages 426 to 431

1. Yes. Example: The table of values shows that at $x = 1$, both outputs are 3. The graph shows that the point of intersection is (1, 3).

2. No. Example: The calculator screen shows that (5.2, 3) is a solution to $7x - 2y = 30.4$ because the left side equals the right side. However, it is not a solution to $4x + y = 25.1$ because (5.2, 3) produced 23.8 and not 25.1.

3. a)

x	$y = 2x + 6$	$y = 3x$
0	6	0
1	8	3
2	10	6
3	12	9
4	14	12
5	16	15
6	18	18
7	20	21



b) Example: The solution is (6, 18). From the table of values, both values of y are 18 when $x = 6$. From the graph, the point of intersection is (6, 18).

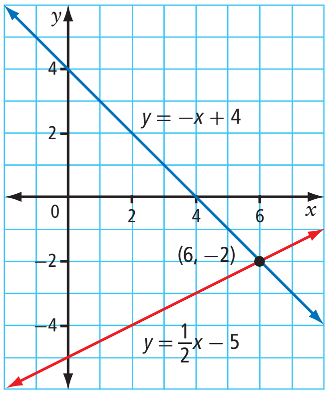
c) Since $2(6) + 6 = 18$ and $3(6) = 18$, the ordered pair (6, 18) satisfies both equations and is the solution.

4. a) Example:

x	$y = -x + 4$	$y = \frac{1}{2}x - 5$
0	4	−5
2	2	−4
4	0	−3
6	−2	−2
8	−4	−1
10	−6	0

The table of values shows that when $x = 6$, $y = -2$ in each of the equations. Therefore, (6, −2) is the solution.

b)



c) Example: The solution is the ordered pair common to both equations and the point of intersection of the two linear graphs.

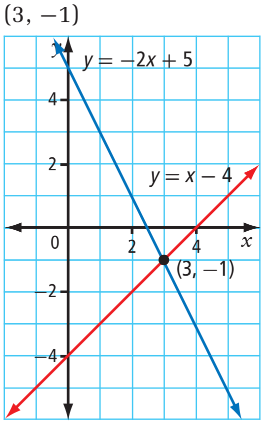
5. a) Yes. Example: The point with coordinates (4, 7) satisfies both equations. (4, 7) is the solution to the linear system.

b) No. Example: The point with coordinates (−1, 3) satisfies the equation $4x + 3y = 5$, but not the equation $x + 4y = 13$. (−1, 3) is not a solution to the linear system.

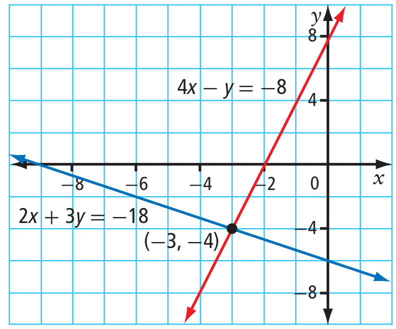
c) Yes. Example: The point with coordinates (−6, −10) satisfies both equations. (−6, −10) is the solution to the linear system.

d) No. Example: The point with coordinates (1.2, 2.4) satisfies the equation $y = 4.5x - 3$, but not the equation $12x - 3y = 7$. (1.2, 2.4) is not a solution to the linear system.

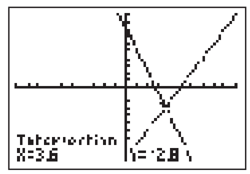
6. a) (3, −1)



b) (−3, −4)

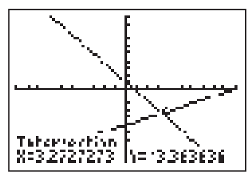


7. a)



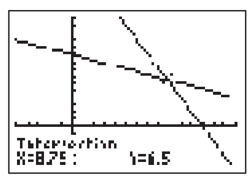
(3.6, −2.8)

b)



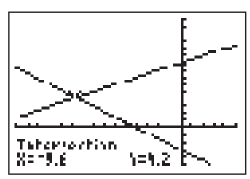
$\left(\frac{36}{11}, -\frac{37}{11}\right)$ or about (3.27, −3.36)

c)



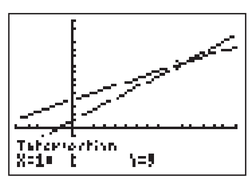
(8.75, 6.5)

d)



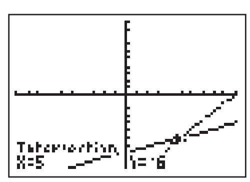
(−9.6, 4.2)

8. a)



(10, 9)

b)



(5, −6)

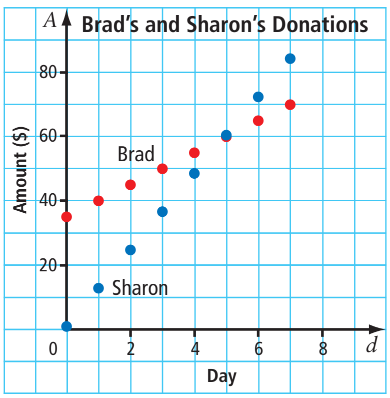
9. a) No. Example: The point (2, 5) lies on the graph of $x + 4y = 22$ but not on the graph of $3x - y = 2$. It is not a solution to the linear system.

b) Yes. Example: The point $(-3, -2)$ lies on the graph of both lines as the point of intersection. It is the solution to the linear system.

10. a)

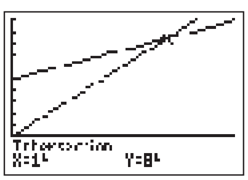
Brad	
Day	**Amount ($)**
0	35
1	40
2	45
3	50
4	55
5	60
6	65
7	70

Sharon	
Day	**Amount ($)**
0	0
1	12
2	24
3	36
4	48
5	60
6	72
7	84



b) (5, 60); It would take 5 days for both Brad and Sharon to collect $60.

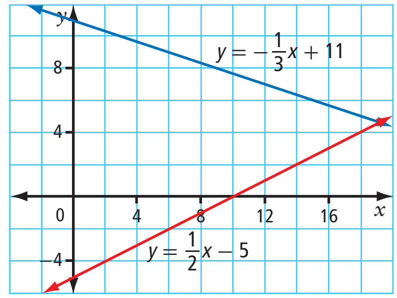
11. a)

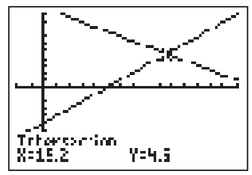


b) 14 necklaces

c) From the graph, the cost for 20 necklaces is $99 and the revenue is $120. The profit is $120 – $99, or $21. This is the vertical gap between the graphs of the two lines at 20 necklaces.

12. a) (19.2, 4.6)

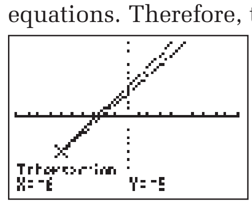


b)  (19.2, 4.6)

c) Example: Technology can give you a more accurate answer if the numbers are not integers, and is usually faster. However, sometimes you need to adjust the viewing window to find the solution. Alternatively, the x-intercept and y-intercept method may be easier to graph manually than manipulating the equations into the form $y = mx + b$ for the graphing calculator.

13. Example: Yes. While the table of values does not show the same value of $f(x)$ for a given value of x, the pattern of changes shows that (4.5, 32) would be halfway between 4 and 5 for both equations. Therefore, this is the solution.

14.



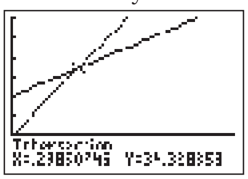
The coordinates of the point $(-6, -5)$ satisfy both equations, $3x - 2y = -8$ and $4x - 3y = -9$. Example: The two equations are almost coincident, so your pencil lines would need to be very accurate to distinguish where the point of intersection is. A graphing calculator does not have this problem.

15. a) Example: Brendan began with a 400-m head start but ran more slowly than Malcolm did. He took 9 min to complete 2800 m. Malcolm made up the difference by running 3200 m in 8 min. He passed Brendan 4.5 min into the run, around 1800 m from the start of the timing.

b) Example: Since each person is running at a constant speed during this portion of the run, the graph represents a system of linear equations.

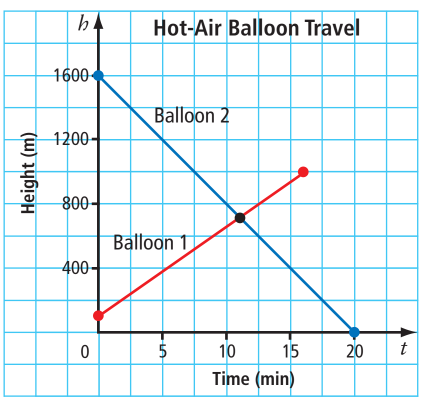
16. a) 20 km

b) At $t = 0$, the canvasback ducks have not started to fly, while the green-winged teals have already flown 20 km.



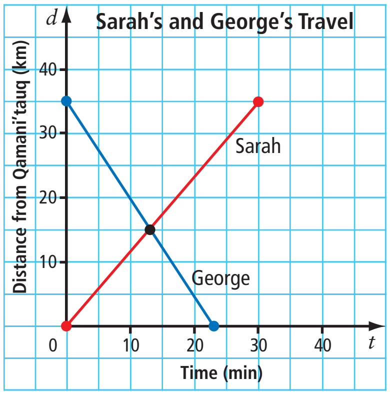
c) Example: It took the canvasback ducks about 0.3 h, or 18 min, to catch up to and pass the green-winged teals. Both types of duck were just over 34 km into the flight and 16 km from the water source. The canvasbacks arrived about 11 min before the teals despite the 20-km head start.

17.



At approximately 11 min, the two balloons pass one another, just over 719 m in the air.

18.

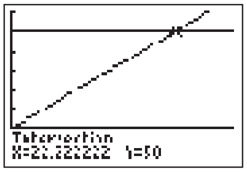


At approximately 13 min, Sarah and George pass one another, about 15 km from Qamani'tauq.

19. Example: If a person talks for over 300 min per month, that person would save money with Plan C. If a person talks for over 100 min per month but less than 300 min per month, Plan B is the least expensive. If a person talks for less than 100 min per month, Plan A is the least expensive.

20. 6.25 s

21. Example: I need to make a decision on transit passes. Do I pay \$2.25 per trip or spend \$50 per month for a pass? How many trips per month would I need to take to save money with a pass?



I would need to take over 22 trips a month to make a pass worthwhile.

22. Example: Solving a system of linear equations determines the coordinates of the point of intersection of the graphs of the equations. Verifying a solution to a system of linear equations is testing a point to see if it satisfies each equation. For example, the point of intersection of the graphs of $y = -2x + 2$ and $y = x - 7$ is the solution, $(3, -4)$. Substituting the coordinates of the point $(3, -4)$ into each equation results in a true statement.

23. a) The lines will be parallel, since the slopes are equal but the y-intercepts are different. There is no solution to this system of linear equations.

b) The lines will be perpendicular to one another, since the slopes are negative reciprocals. There is one solution to this system of linear equations.

c) The lines will intersect on the y-axis, since the slopes are different but the y-intercepts are the same. There is one solution to this system of linear equations.

d) The lines will be coincident, since the slopes and y-intercepts are the same. There are an infinite number of solutions to this system of linear equations.

8.2 Modelling and Solving Linear Systems, pages 440 to 445

1. a) Let C represent the cost, in dollars. Let s represent the number of songs downloaded. $C = 0.99s$ and $C = 0.79s + 11$

b) Let h represent the height above the ground, in metres. Let t represent time, in minutes. $h = 800 - 55t$ and $h = 80t$

c) Let R represent the material sorted, in tonnes. Let t represent time, in hours. $R = 100 + 20t$ and $R = 40t$

2. a) Let J represent Jamal's age, in years. Let M represent Maria's age, in years. $J = 3M$ and $J + 7 = 2(M + 7)$

b) Let C represent the temperature, in degrees Celsius. Let t represent time, in hours. $C = 2 - 2t$ and $C = -8 + 4t$

3. Let G represent the number of goals. Let A represent the number of assists. $G + A = 32$ and $A = 3G$

4. a) $d + q = 50$ and $0.1d + 0.25q = 6.80$

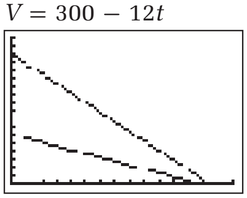
b)

Type of Coin	Value of One Coin (¢)	Number of Coins	Value of Coins (¢)
Dime	10	d	$10d$
Quarter	25	q	$25q$

$d + q = 50$ and $10d + 25q = 680$

5. a) Let V represent the volume of water remaining in each tank, in cubic metres. Let t represent time, in minutes. $V = 800 - 30t$ and $V = 300 - 12t$

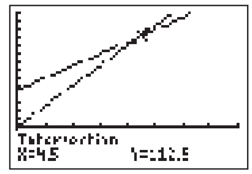
b)



c) Example: The larger tank starts with more water and drains in about 27 min. The smaller tank drains more slowly but starts with less water. It is empty in 25 min. The two graphs do not intersect until after 27 min, so they never contain the same amount of water until they are both empty.

6. a) Let L represent the length of sash woven, in centimetres. Let t represent time, in hours. $L = 45 + 15t$ and $L = 25t$

b)



c) Example: At 4 h 30 min (4.5 h), Kianna and Naomi both have 112.5 cm of sash woven. From this point on, Naomi's sash will be increasingly longer than Kianna's sash.

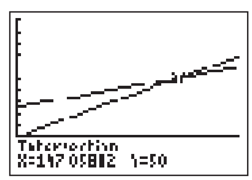
7. In 75 days, both oil wells will have produced 2625 m^3 of oil.

8. Example: If Megan is planning on driving less than 196 km in the day, she should go with the first option. If she will be driving over 196 km, then the flat rate would be more cost effective.

9. a) $C = 0.002(170n)$

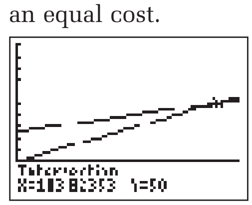
b) $C = 0.34n$ and $C = 25 + 0.17n$

c)



d) (147, 50); At about 147 showers, the cost of showering with either type of shower head is \$50.

e) Example: It will take more showers to reach an equal cost.

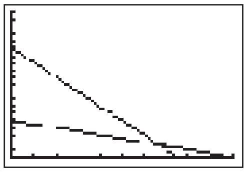


It would take almost 184 showers for the cost of showering with either type of shower head to be \$50.

10. approximately 22.7 km

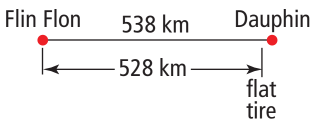
11. Yes. Example: The solution to the system of linear equations $d = 0.4 + 7.5t$ and $d = 10t$ is (0.16, 1.6). In 9.6 min (0.16 h), they will both be 1.6 km up the hill.

12. a) Let n represent the population of a species remaining. Let t represent time, in years. $n = 100 - 5t$ and $n = 300 - 20t$



b) Yes. In 13 years 4 months, there will be fewer than 34 of each species. This is the point where the graphs of the two linear equations intersect.

13. Let f represent the time driven at 90 km/h. Let s represent the time driven at 75 km/h. $f + s = 6$ and $90f + 75s = 538$; 528 km



14. They finish painting the fence in about 5.7 h, or 5 h 42 min. Chris paints almost $51\frac{1}{2}$ ft and Robert a little more than $68\frac{1}{2}$ ft.

15. In about 67 days, both trees will be approximately 153.3 cm tall.

16. Yes; about 22.9 kg

17. a) Let h represent the height above the ground, in metres. Let t represent time, in seconds. $h = 500 - 4t$ and $h = 200 + 5t$

b) In about 33.3 s, they will be approximately 366.7 m above the ground.

18. Let A represent the number of grapes that Andrea has. Let H represent the number of grapes that Hunter has. $A = 3H$ and $A = 2H + 6$; Hunter: 6 grapes, Andrea: 18 grapes

19. 51 years old

20. Let s represent the average speed of the swimmer, in metres per minute. Let c represent the average speed of the current, in metres per minute. $200 = 3(s - c)$ and $150 = 0.75(s + c)$; swimming speed: about 133.3 m/min, current speed: about 66.7 m/min

21. Let s represent the mass of sterling silver, in grams. Let p represent the mass of pure silver, in grams. $0.925s + 1p = 0.94(100)$ and $s + p = 100$

22. Let C represent the cost, in dollars. Let n represent the number of visits.

Option A: $C = 22 + 6n$, Option B: $C = 16.50\left(\frac{n}{2}\right)$;

Example: Eunji needs to decide how many times she will visit the amusement park. If she makes at least ten visits to the amusement park, the season's pass is a better deal.

23. a) Example:

Person	Time (h)	Speed (km/h)	Distance Travelled (km)
Gavin	t	110	$110t$
James	$t - 0.75$	240	$240(t - 0.75)$

b) Example: Graph $d = 110t$ and $d = 240t - 180$ and find the point of intersection. If the d-coordinate of this point is less than 300, then James will catch up to Gavin.

24. a) Example: Determine the slope of the line using points (0, 0) and (5, 70).

b) $C = 14t$ and $C = 50 + 4t$

c) Example: The point of intersection is (5, 70), so Bikes-to-Go is a better deal for more than 5 h. Spokz is a better deal if you're renting for less than 5 h.

8.3 Number of Solutions for Systems of Linear Equations, pages 454 to 459

1. a) A and B, A and C, A and D, B and D, C and D

b) B and C

2. a) infinite number of solutions; Example: Since the equations have the same slope and the same y-intercept, the graph will result in coincident lines. Therefore, this system has an infinite number of solutions.

b) one solution; Example: The equations have different slopes. So, the graph will result in two lines that intersect at one point. Therefore, this system has one solution.

c) no solution; Example: Since the equations have the same slope and different y-intercepts, the graph will result in parallel lines. Therefore, this system has no solution.

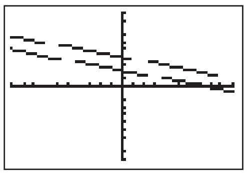
3. a) no solution; Example: Since the equations will have the same slope and different y-intercepts, the graph will result in parallel lines.

b) one solution; Example: The equations will have different slopes. So, the graph will result in two lines that intersect at one point.

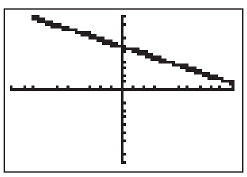
c) one solution; Example: The equations will have different slopes. So, the graph will result in two lines that intersect at one point.

4. **a)** parallel lines
   b) lines with different slopes that intersect at one point
   c) only one line shown on the graph

5. **a)** Example: Since the x-coefficient and y-coefficient are the same, the equations have the same slope. However, the constant values are different. Therefore, the graph will be a pair of parallel lines. The linear system has no solution.



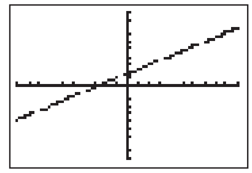
   b) Example: Double the first equation creates the same equation as the second one. So, the graph of the lines will be coincident. The linear system has an infinite number of solutions.



6. Examples:
   a) $2x - y + 1 = 0$
   b) $3x - y + 5 = 0$
   c) $4x - 2y + 10 = 0$

7. Let E represent an employee's earnings, in dollars. Let s represent the number of subscriptions sold.
   a) $E = 360 + 8.25s$ and $E = 360 + 8.25s$; infinite number of solutions; Brian and Charlie will always have the same earnings.
   b) $E = 472 + 7s$ and $E = 360 + 8.25s$; one solution; Brian will catch and pass Alyssa in earnings.
   c) $E = 360 + 8.25s$ and $E = 413 + 8.25s$; no solution; Dena will always have earned \$53 more than Charlie.

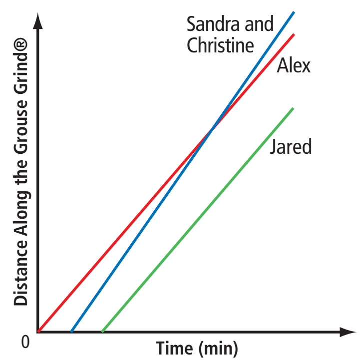
8. **a)** Compare the slopes of the two equations. Since they are different, there should be one solution.
   b)



   Example: Anton is correct because the equations have different slopes so there is one solution. Jeff was fooled because the graphs looked coincident on the screen.

9. **a)** Example: Stephanie doesn't see a point of intersection on the screen.
   b) She is incorrect. The lines have different slopes and will intersect somewhere to the left of the current screen.

10. **a)** Example:



   b) Alex and Jared: no solution; Sandra and Christine: an infinite number of solutions; Sandra or Christine and Alex: one solution; Sandra or Christine and Jared: no solution; Sandra and Christine will catch and pass Alex, but they will always be ahead of Jared and will steadily increase their lead. Jared is always the same distance behind Alex.

11. False. Example: Any point on one line is also on the other line. Not all points in the coordinate plane are on the lines.

12. **a)** No. The lines could be parallel with no solution or coincident with an infinite number of solutions.
   b) No. The lines could be coincident or have one solution (at the y-intercept).
   c) Yes. The lines will be coincident with an infinite number of solutions.

13. a) Let P represent the napkins manufactured, in kilograms. Let w represent the number of weeks. $P = 5000 + 350w$ and $P = 28\ 000 + 350w$

b) Example: Since the graphs of the lines are parallel, there is no solution to the system. Northern Paper will always have produced 23 000 kg more napkins than PaperWest.

14. a) $C = 24$

b) Any value except 24 will yield no solution ($C \neq 24$).

15. a) $y = 20x + 6$ and $y = 20x$

b) There are no solutions, because both taxis have the same fuel economy but stated in different ways. The second taxi will never catch the other taxi based on fuel used.

16. a) With the domain restricted, there is no solution. The lines do not intersect within these limits.

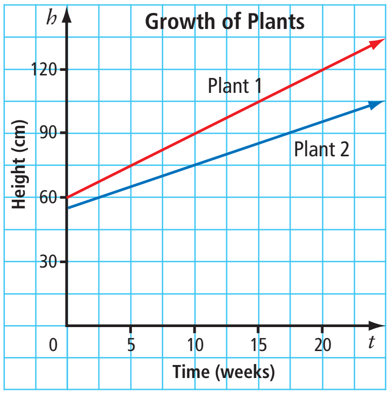
b) one solution

c) Example: Only parallel and coincident situations will be predictable. Graphs of lines with different slopes might not intersect in the window defined by the restricted domain and range.

17. a) $W = 6$ or $W = -6$

b) Another number will yield one solution. Example: $5x + 3y = 10$ and $12x + 5y = 24$ have two different slopes, $-\frac{5}{3}$ and $-\frac{12}{5}$, so the system will have one solution.

18. Wendy. Example: The shorter plant is growing slower so it will never catch up. The two equations have different slopes, but the intersection would be before the time starts.



19. Example: No. The graph of the *straight* lines would have to curve. On a globe, a line of latitude will meet a line of longitude at two points, one on either side of Earth.

20. Example: Yes, as long as there are no restrictions on the domain. With restrictions, you cannot be sure (like the plants in #18).

21. Examples:

a) In a race, a head start is given to one runner. If the two runners are running at the same speed, there will be no solution. $d = 5t$ and $d = 5t - 15$

b) In a race, a head start is given to one runner, and the second runner is faster. There will be one solution. $d = 5t$ and $d = 7t - 14$

c) In a race, the runners start at the same time and run at the same speed. They will always be beside one another (infinite number of solutions). $d = 5t$ and $d = 5t$

22. Step 1: Example: A: $2x + y = 4$ and $6x + 3y = 12$, B: $2x + y = 4$ and $2x + y = 10$, C: $2x + y = 4$ and $2x + y + 5 = 9$, D: $2x + y = 4$ and $4x + 2y = 4$

Step 2: Example: A: an infinite number of solutions, B: no solution, C: an infinite number of solutions, D: no solution

Step 3: One equation as a multiple of another produces coincident equations. If two equations have identical x-coefficients and y-coefficients but different constants, then they are parallel lines and there will be no solution.

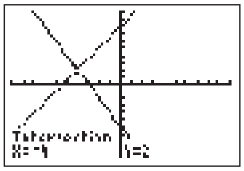
Step 4: Example: Any equation of the form $Ax + By = C$ has a slope of $-\frac{A}{B}$. Any other equation whose value of $-\frac{A}{B}$ is not the same will have one solution with the first equation, since the slopes will be different.

Chapter 8 Review, pages 460 to 462

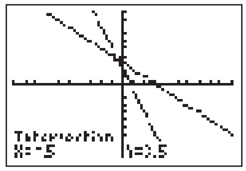
1. a) Example: Substitute $x = -7$ and $y = 4$ into each equation. Evaluate each side to determine whether the values satisfy both equations. The point $(-7, 4)$ is the solution to the linear system.

b) Example: Substitute $x = 3$ and $y = -5$ into each equation. Evaluate each side to determine whether the values satisfy both equations. The point $(3, -5)$ is not the solution to the linear system.

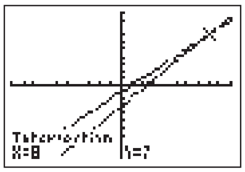
2. a) $(-4, 2)$



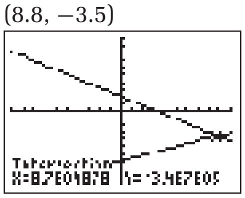
b) $(-0.5, 3.5)$



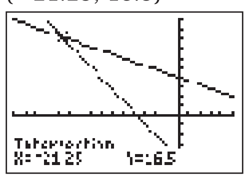
c) $(8, 7)$



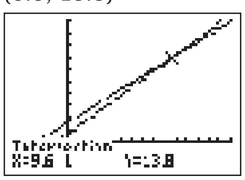
3. a) approximately $(8.8, -3.5)$



b) $(-21.25, 16.5)$

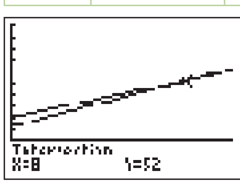


c) $(9.6, 13.8)$



4. a) Example:

t	$d = 4t + 20$	$d = 5t + 12$
0	20	12
1	24	17
2	28	22
3	32	27
4	36	32
5	40	37
6	44	42
7	48	47
8	52	52



b) Example: In the table of values, the solution occurs when the values of d are equal for the same value of t. On the graph, the solution is the point of intersection.

c) $(8, 52)$; At 8 s, both boats are at a distance of 52 m.

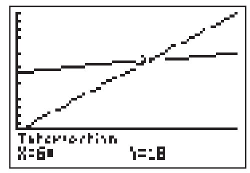
5. a) Let C represent the cost of a gym membership. Let m represent the number of months as a member. $C = 85 + 30m$ and $C = 35m$

b) Let G represent the amount of grain, in cubic metres. Let t represent time, in minutes. $G = 5 + 2.5t$ and $G = 5t$

c) Let D represent the length of road left to resurface, in metres. Let t represent time, in hours. $D = 3000 - 200t$ and $D = 4000 - 250t$

6. a) Let P represent the cost of the cell phone plan. Let m represent the number of minutes used. $P = 15 + 0.05m$ and $P = 0.3m$

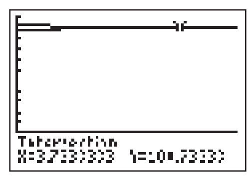
b)



c) Example: The point of intersection shows that 60 min costs $18 on both plans. If a person would use more than 60 min per month, then the better choice is Plan #1. If a person would use less than 60 min per month, then the better choice is Plan #2.

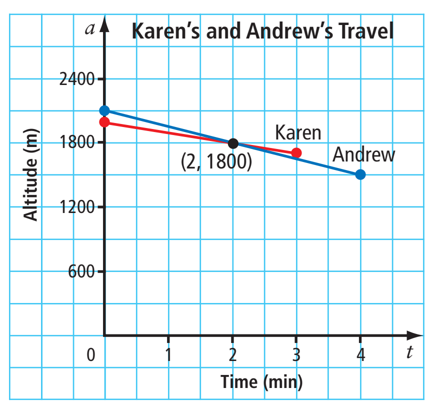
7. a) Let P represent the atmospheric pressure, in kilopascals. Let t represent time, in hours. $P = 102.6 - 0.5t$ and $P = 99.8 + 0.25t$

b)



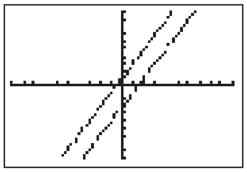
In 3 h 44 min, both cities will have the same pressure of about 100.7 kPa.

8. Andrew will pass Karen 2 min into the skiing, at 1800 m.

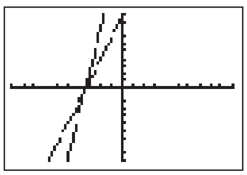


9. a) parallel lines
b) intersecting lines
c) coincident lines

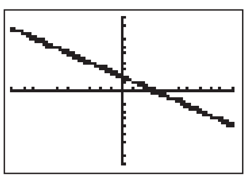
10. a) no solution; Example: Since the equations have the same slope and different y-intercepts, the graph will result in parallel lines.



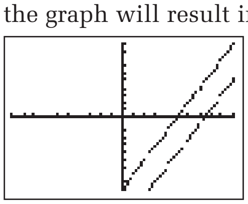
b) one solution; Example: The equations have different slopes. So, the graph will result in two lines that intersect at one point.



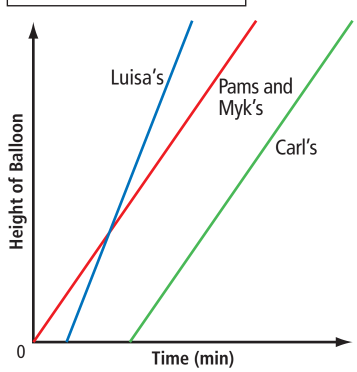
c) infinite number of solutions; Example: Since 7 times the first equation equals the second equation, the graph will result in coincident lines.



d) no solution; Example: Since the equations will have the same slope and different y-intercepts, the graph will result in parallel lines.



11. a)



b) Example: Zero solutions would be for Pam's and Carl's or Myk's and Carl's, as the graphs of their balloons are parallel. Luisa's and Carl's balloons will also never cross, because Luisa's had a head start and rises faster. One solution for Pam's and Luisa's or for Myk's and Luisa's, because their balloons have different slopes and Luisa's balloon will pass the other two. There are an infinite number of solutions for Pam's and Myk's, because their balloons are always side by side.

12. a) Let P represent the production of sports beverages, in litres. Let h represent the number of production hours. $P = 150 + 300h$ and $P = 600 + 300h$

b) There are no solutions to this system. Both companies have the same rate of production. Company B will always have 450 L more in production.

13. a) one solution

b) Example: One solution is not possible, as they are earning interest at the same rate (slope). They either started with the same amount of money (principal) and always have the same amount, or they began with different principals and will always be the same amount apart.

Chapter 8 Practice Test, pages 463 to 465

1. D
2. C
3. D
4. B
5. D

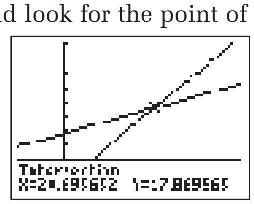
6. a) No. Example: The point (5, 2) does not lie on the graph of $x + y = 10$ or the graph of $2x - y = 3$.

b) Yes. Example: The point $(-1, 4)$ lies on the graphs of both lines as the point of intersection.

c) No. Example: The point $(8, -3)$ lies on the graph of $x + 4y + 4 = 0$ but not on the graph of $2x - 3y = 27$.

7. Example: Graph the equations on the same grid and look for the point of intersection.

8. a)



b) (20.70, 17.87)

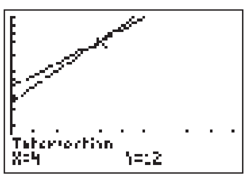
c) Example: Store the x-value and y-value after finding the point of intersection on your graphing calculator. Check that $5x - 4y$ is 32 and $3x - 7y$ is -63. Use the full decimal strings and not the rounded off values for best results.

9. a) Example: Since the x-coefficients and y-coefficients are the same, the equations have the same slope. However, the constant values are different. Therefore, the graph will be a pair of parallel lines. The linear system has no solution.

b) Example: Since the equations have the same slope and the same y-intercept, the graph will result in coincident lines. Therefore, this system has an infinite number of solutions.

10. $B = 10$

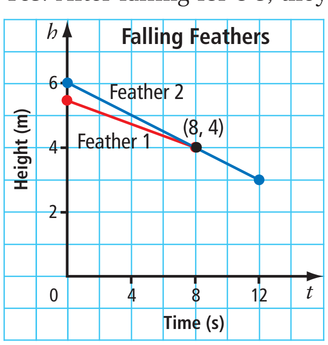
11. a) Let L represent the height of the grass, in centimetres. Let w represent the number of weeks. $L = 6 + 1.5w$ and $L = 4 + 2w$

b) 

One grass starts higher but grows more slowly. The shorter grass will grow quickly and pass the first grass in 4 weeks, when they are both 12 cm high.

12. Example: Paige had to walk 5 km to get to school, while Quinn had to walk only 3.5 km. Even though Paige had further to walk to get to school, she walked faster (0.125 km/min) than Quinn did (0.05 km/min) and arrived earlier. Paige passed Quinn 2.5 km from school and 20 min into the walk.

13. Yes. After falling for 8 s, they are both at 4 m.



14. Let d represent distance, in miles. Let t represent time, in hours.

a) $d = 11.8 + 7.2t$ and $d = 11.8 + 7.2t$; Donna and Marcus are always beside each other and thus have an infinite number of times that they are at the same point in the river.

b) $d = 12.6 + 7.0t$ and $d = 11.8 + 7.2t$; There is one solution. Donna catches Taj at 4 h and 40.6 km into the race.

c) $d = 11.8 + 7.2t$ and $d = 11.2 + 7.2t$; There is no solution. Marcus stays ahead of Rose a consistent 0.6 km throughout the race.

Chapter 9

9.1 Solving Systems of Linear Equations by Substitution, pages 474 to 479

1. a) $x = 3$ and $y = 11$

b) $x = -6$ and $y = 18$

c) $x = 12$ and $y = 5$

2. a) $x = 2$ and $y = -2$

b) $j = 2$ and $m = 16$

c) $k = 1.5$ and $n = -1$

3. a) $x = 7$ and $y = -2.9$; Example: Isolating y makes for easier calculations.

b) $x = 21\frac{3}{7}$ and $y = -37\frac{1}{7}$; Example: Isolating y makes for easier calculations.

c) $x = 4$ and $y = 3$; Example: Isolating x makes for easier calculations.

4. a) $x = 12\frac{6}{7}$ and $y = -\frac{5}{7}$

b) $x = -2$ and $y = 8$

c) $x = 17\frac{3}{4}$ and $y = -3\frac{5}{6}$

5. a) $x = 6$ and $y = 2$

b) Example: Helen's method is preferred. Although it requires two steps to isolate the variable x, the solution does not involve operations involving fractions. Jaret's method involves operations on fractions with a denominator of 2.

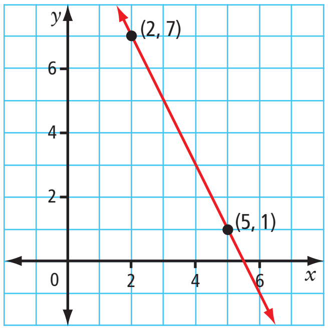
6. Let x and y be the two numbers. $x + y = 20$ and $2x = 4y + 4$; $x = 14$ and $y = 6$

7. a) approximately (0.3, 3.7)

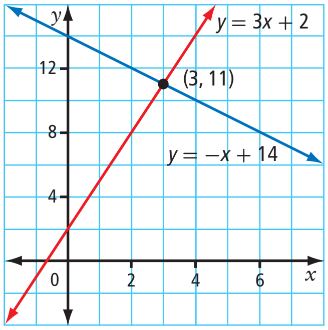
b) $\left(\frac{1}{3}, 3\frac{2}{3}\right)$

c) The algebraic approach gives exact answers.

8. **a)**, **b)**, and **c)** $x = 6.5$ and $y = 4.5$
   d) Example: The method in part b) eliminates the decimals and leaves numbers that are easy to use.
9. 32 m and 50 m
10. Vancouver: 48 cm, Whitehorse: 144 cm
11. 12 days
12. 13 h
13. 7 years
14. Rory: 16 years old,
    Rory's grandmother: 74 years old
15. Let b represent the cost of one bush and t represent the cost of one tree, both in dollars.
    $40b + 12t = 1484$ and $25b + 18t = 1421$;
    One bush costs \$23 and one tree costs \$47.
16. **a)** 12 more dimes than quarters; 23 more quarters than nickels
    b) Example: The question deals only with the quantity of each type of coin, not the value of the coins.
17. 6 students in each van and 44 students in each bus
18. whole-wheat bread: 28 L; white bread: 40 L
19. **a)** The answer must be a whole number but the ratio $\frac{7593}{24}$ is not a whole number.
    b) In step 2, he needs to multiply the expression $1.00(132 - q)$ by 100 also. Then, it becomes $100(132 - q)$.
    c) 73 quarters and 59 loonies
20. **a)** $x = -1\frac{3}{32}$ and $y = -2\frac{5}{32}$
    b) $x = \frac{2}{5}$ and $y = \frac{3}{4}$
21. $m = -2$ and $b = 11$



22. 0.5 km
23. $P = I^2R$
24. Example: Substitution results in the statement $10 = -15$, which is not true. So, the system cannot have a solution.
25. Examples:
    a) Both methods may use algebra to rearrange an equation. Both methods produce the same solution.
    b) Substitution involves working algebraically with equations, variables, and mathematical operations, while solving graphically involves drawing two graphs and finding their point of intersection.
26. Example: $y = 3x + 2$ and $x + y = 14$
    a) $x = 3$ and $y = 11$



    b) Example: The substitution method is simple. For the graphical approach, the second equation needs to be rewritten in $y = mx + b$ form.

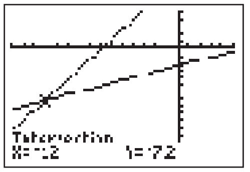
9.2 Solving Systems of Linear Equations by Elimination, pages 488 to 491

1. **a)** $x = 7$ and $y = 3$
   b) $x = 9\frac{2}{3}$ and $y = 1\frac{2}{3}$
   c) $x = 4$ and $y = 4$
2. **a)** $-3x + y = 11$ and $x - y = -5$
   b) $x - y = -7$ and $2x + y = -8$
   c) $x + 3y = 4$ and $x - y = 16$
3. $s + a = 430$ and $10s + 13a = 4804$
   a) 168 tickets
   b) 262 tickets
4. **a)** $x = 1$ and $y = 2$
   b) $x = 3$ and $y = -1$
   c) $x = -5$ and $y = -11$
5. **a)** $x = 2\frac{4}{7}$ and $y = 1\frac{1}{7}$
   b) $x = 2\frac{8}{15}$ and $y = \frac{11}{45}$
   c) $x = 4\frac{1}{2}$ and $y = 1$

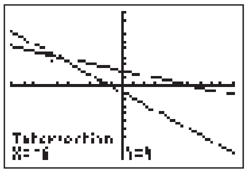
6. The system has no solution. Example: A graph of these equations would result in two parallel lines with no point of intersection.
7. 20 bicycles; 10 tricycles
8. tulip bulbs: \$14; iris bulbs: \$20
9. bagel: \$1.75; juice: \$1.25
10. 175 dogs
11. 40 trucks and 560 passenger vehicles
12. 6.75 km on the flat terrain and 1.8 km on the mountainous terrain
13. 72 min playing basketball and 18 min cycling
14. **a)** Let n represent the number of loads of laundry each sister does per week. Let V represent the number of litres of water used per week. $V = 225n$ and $V = 95n + 260$
    b) 2 loads of laundry
    c) Sharon uses 1800 L and Bev uses 1020 L. So, Sharon uses 780 L more if they both do eight loads of laundry.
15. **a)** $x = -3$ and $y = 2$
    b) $x = 1\frac{7}{25}$ and $y = \frac{56}{75}$
16. \$1750
17. 53.3 L of the 3.25% MF and 6.7 L of the 1% MF
18. $k = 12$
19. $a = -9$ and $b = 9$; There is only one solution. Example: The value of y (which equals a) can be determined from substituting 9 for x in the first equation. The value of b is determined by substituting $x = 9$ and $y = -9$ into the second equation.
20. **a)** Example: $3x + 2y = 10$ and $2x - y = 4$; $x = 2\frac{4}{7}$ and $y = 1\frac{1}{7}$
    b) It is easy to isolate y in the second equation to substitute into the first equation.
    c) Example: Any equations in which none of the coefficients of x or y in either equation is equal to one. When you must divide each term in an equation, you may have to substitute fractional expressions early in the solution.
21. Example: If it is easy to isolate either x or y in either equation without producing fractions, then substitution will be a good method. Example: $x + 5y = 7$ and $4x - 3y = -20$ For any other system, elimination is a better method.

9.3 Solving Problems Using Systems of Linear Equations, pages 498 to 501

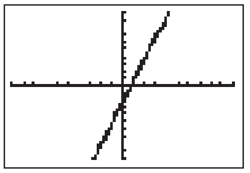
1. **a)** $x = -12$ and $y = -7.2$



   b) $x = -6$ and $y = 4$



   c) There are an infinite number of solutions.

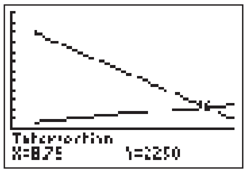


2. **a)** $x = 2$ and $y = -6.5$
   b) $m = 12.6$ and $n = 10.4$
   c) $x = \frac{1}{3}$ and $y = -\frac{2}{3}$
3. Calgary: −2.8 °C; Winnipeg: −12.7 °C
4. 6.4%
5. **a)** The equation $C = 0.3m + 16$ represents the total cost, in dollars, of the fundraiser. The equation $C = 0.75m$ represents the income, in dollars, from the sale of the muffins.
   b) 36 muffins
6. approximately 602 h
7. 85 adults and 45 students
8. If Jason drives exactly 520 km, the rental cost is the same for both companies. If he drives less than 520 km, Easy 4 U is cheaper. If he drives more than 520 km, Speed-E-Car is less expensive.

9. a) The population of fish is decreasing by 1000 each year while the number of fish eaten by osprey is increasing by 200 each year.

b) Example: Let F represent the number of fish. Let x represent the year number. $F = -1000x + 11\ 000$ and $F = 200x + 500$

c)



The solution is (8.75, 2250). This point indicates that after 8.75 years, the number of fish in the lake will equal the number of fish that are being eaten by the osprey.

d) Example: As the number of fish decreases, the osprey population will also decrease, because there will not be enough fish to keep feeding the osprey population.

10. Let x represent the depth, in metres, and let y represent the number of minutes the diver can remain at that depth. $60 = 60m + b$ and $90 = 30m + b$; $y = -x + 120$

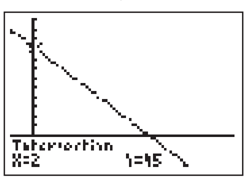
11. 81.25 min cross-country skiing and 18.75 min playing squash

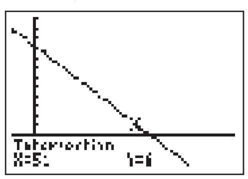
12. 36.75 square units

13. a) The constant in the second equation of the second pair is one larger than the constant of the second equation in the first pair.

b) $x = 2$ and $y = 45$; $x = 51$ and $y = 6$

c)



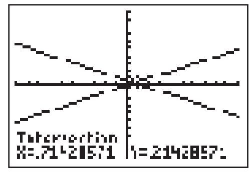


d) The large numbers make the graphing difficult for both solving for y and finding a good viewing window. The larger numbers make solving the system algebraically (elimination) tedious.

14. a) Example: $3x - y = 5$ and $2x + 7y = 57$ has the solution $x = 4$ and $y = 7$.

b) Example: If it is relatively easy to isolate y in both equations, solving graphically might be preferred. If it is relatively easy to isolate either x or y in only one of the equations, substitution would be recommended. Otherwise, elimination would be preferred.

15. a) Example: $7x - 14y = 2$ and $7x + 14y = 8$ has a solution $x = \frac{5}{7}$ and $y = \frac{3}{14}$.



b) Example: This system is difficult to solve graphically, because isolating y in both equations creates equations with fractions as coefficients and/or constants. The solution can only be approximated from the graphs. Solving by elimination is easy, because the coefficients of x are equal and the coefficients of y add to zero.

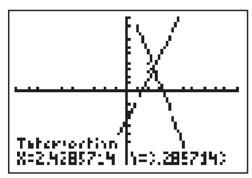
Chapter 9 Review, pages 502 to 503

1. a) $x = 3$ and $y = 8$

b) $x = 0$ and $y = -2$

c) $x = \frac{2}{9}$ and $y = -\frac{4}{3}$

2.



$x = 2\frac{3}{7}$ and $y = 3\frac{2}{7}$; Solving algebraically is preferred, because the solution is exact. Graphical solving gives only an approximate solution.

3. 32 000 km

4. \$1.40 for one song and \$4.20 for one game

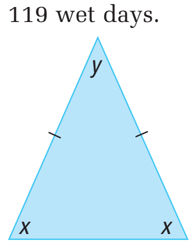
5. a) $x = 4$ and $y = -13$

b) $x = 2\frac{4}{7}$ and $y = 1\frac{1}{7}$

c) $x = \frac{3}{2}$ and $y = 0$

6. Vancouver has 166 wet days and Yellowknife has 119 wet days.

7.



The two base angles are each 46.5° and the third angle is 87°.

8. Danika ate 121.875 g of grapes and 203.125 g of oranges.

9. washing machine: $800; shower head: $25

10. a) $34 per day; $0.15 per km

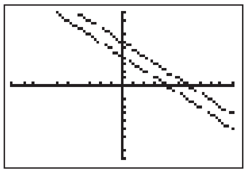
b) Example: The elimination method is easiest, because graphing and substitution are more complicated due to the coefficients of the variables.

11. a) 19 acres for developed sites and 38 acres for basic sites

b) 76 developed campsites and 57 basic campsites

12. 1 h 40 min

13. a) no solution



b) The two lines are parallel, so there is no point of intersection and thus no solution.

Chapter 9 Practice Test, pages 504 to 505

1. D

2. C

3. C

4. C

5. a) $x = 4\frac{1}{4}$ and $y = 5\frac{3}{4}$

b) $x = -1.5$ and $y = 20.5$

c) $x = 4.5$ and $y = -3.5$

6. The length is 4 m and the width is 1.4 m.

7. 187.5 g of peanuts and 112.5 g of almonds

8. 17 nickels and 32 quarters

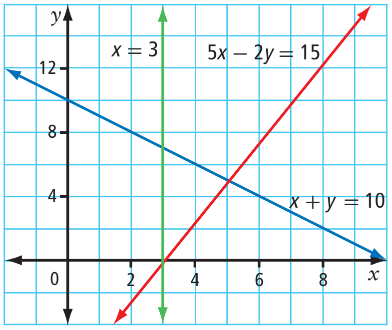
9. The green fee is $22 per game and the annual fee is $150.

10. 667 students and 29 teachers

11. a) Edmonton to Saskatoon took approximately 6.21 h. Saskatoon to Regina took approximately 3.64 h.

b) The distance from Edmonton to Saskatoon is approximately 546.89 km. Example: Multiply the number of hours to drive from Edmonton to Saskatoon (approximately 6.21 h) by the speed (88 km/h) Mallory travelled.

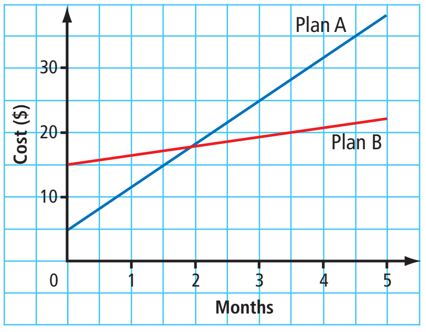
12. a)



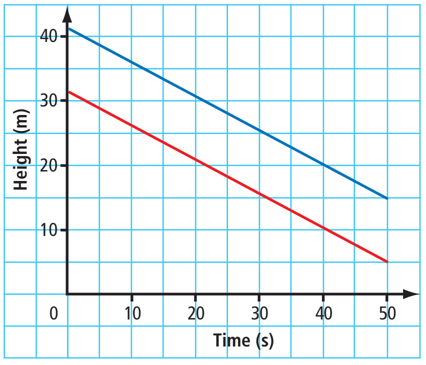
b) (3, 0), (5, 5), and (3, 7)

Unit 4 Review, pages 507 to 509

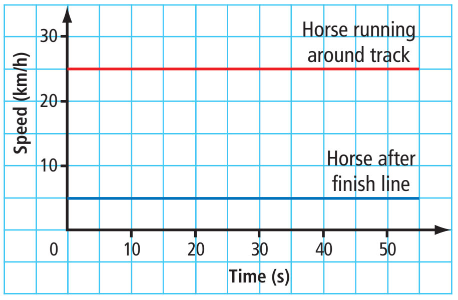
1. a)



b)



c)

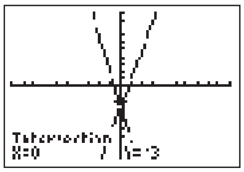


2. i) B **ii)** A **iii)** C

3. a) The point is the solution to the system of linear equations and the point of intersection of the graphs of the two lines.

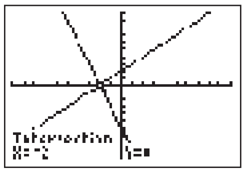
b) The point is the y-intercept of the graph of $y = -x + 3$, but the point is not on the graph of $y = \frac{3}{2}x + 2$.

4. a)



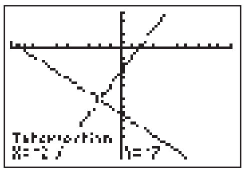
$(0, -3)$

b)



$(-2, 0)$

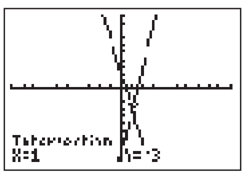
c)



$(-2, -7)$

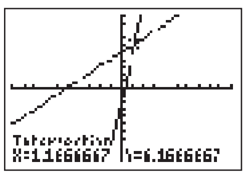
5. D

6. a)



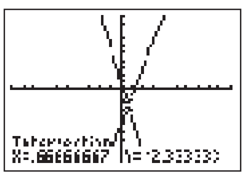
$(1, -3)$

b)



$(1.2, 6.2)$

c)



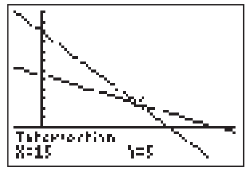
$(0.7, -2.3)$

7. This point indicates that if she rented a car for 7 days, the cost of renting from each company would be \$580.

8. a) Let d represent the number of dimes. Let q represent the number of quarters. $d + q = 20$ and $0.10d + 0.25q = 2.75$

b) $d + q = 20$ written as a function is $q(d) = 20 - d$.
Domain: $\{d \mid 0 \le d \le 20, d \in W\}$
Range: $\{q \mid 0 \le q \le 20, q \in W\}$
$0.10d + 0.25q = 2.75$ written as a function is $q(d) = -0.4d + 11$ or $q(d) = -\frac{2}{5}d + 11$.
Domain: $\{0, 5, 10, 15, 20, 25\}$
Range: $\{1, 3, 5, 7, 9, 11\}$

c)



15 dimes and 5 quarters

9. a) one solution

b) no solution

c) infinite number of solutions

10. Let a represent the number of assists. Let g represent the number of goals. $a + 2g = 23$ and $2g = a + 1$

11. a) Example: Use the substitution method, because the first equation is already solved for y.

b) Example: Use the elimination method, because substitution would involve rational expressions.

c) Example: Use the elimination method, because multiplying the first equation by 2 and adding the two equations would eliminate y.

d) Example: Use the substitution method, because it is easy to isolate x in the first equation.

12. a) $x = -1$ and $y = 5$

b) $x = 2$ and $y = 1$

c) $x = 2$ and $y = -1$

13. a) $x = 5$ and $y = 3$. Example: Use the elimination method, because adding the two equations eliminates y.

b) $x = 0$ and $y = -2$. Example: Use the elimination method, because multiplying the second equation by -3 and adding the two equations eliminates y.

c) $x = -2$ and $y = 1$. Example: Use the elimination method, because substitution would involve equations with fractions as coefficients and/or constants.

14. a) Let p represent their paddling speed, in kilometres per hour. Let c represent the speed of the current, in kilometres per hour.
$\frac{1}{3}(p + c) = 3$ and $\frac{3}{5}(p - c) = 3$

b) Their paddling speed was 7 km/h and the speed of the current was 2 km/h.

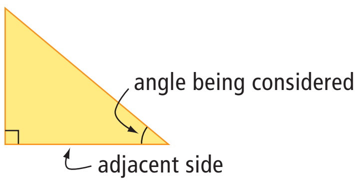
Unit 4 Test, pages 510 to 511

1. A
2. C
3. A
4. D
5. 10
6. 38
7. $x = 4$ and $y = -6$. Example: I used the elimination method, because after multiplying the first equation by 6 and the second equation by 12 to make integral coefficients, adding the two equations will eliminate y.
8. a) Let r represent the cost of one red bead.
   Let g represent the cost of one green bead.
   $25r + 15g = 2.75$ and $7r + 13g = 1.65$
   b) red bead: \$0.05, green bead: \$0.10
   c) \$1.00
9. 56 km/h

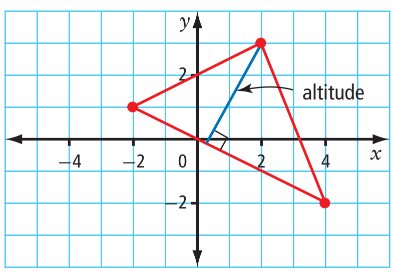
Glossary

A

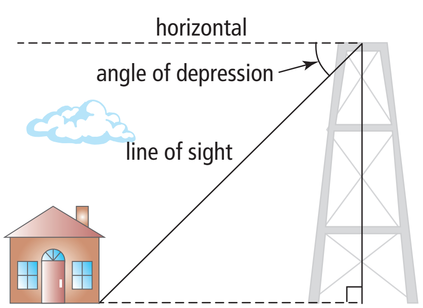
adjacent side The side that forms one of the arms of the acute angle being considered in a right triangle, but is not the hypotenuse.



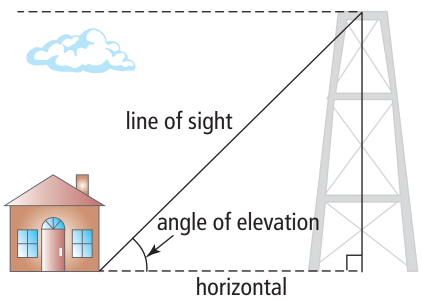
altitude (of a triangle) The perpendicular distance from a vertex to the opposite side.



angle of depression The angle formed by the horizontal and a line of sight below the horizontal.



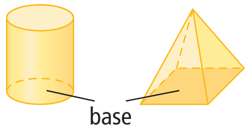
angle of elevation The angle formed by the horizontal and a line of sight above the horizontal.



apex The highest point of a pyramid, perpendicular and opposite to the base.

B

base (of a three-dimensional object) The bottom face of a three-dimensional object that is oriented in a traditional way.



binomial A polynomial with two terms.

For example, $x^2 + 3$, $m^2n + 4n$, and $2x - 5y$ are binomials.

C

circumference The boundary or perimeter of a circle. This is a linear measurement. It is often represented by the variable *C*.

coincident lines Lines that occupy the same position. In a graph of two coincident lines, any point of either line lies on the other line.

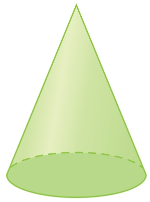
common factor A term that is a factor of two or more terms.

For example, 2 is a common factor of 4, 12, and 18, and x is a common factor of x^2, xy, and xy^2.

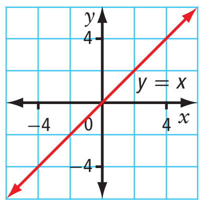
common multiple A number that is a multiple of two or more numbers.

For example, common multiples of 3 and 5 are 15, 30, 45, 60, ….

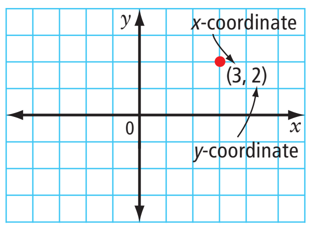
cone A three-dimensional object with a circular base and a curved lateral surface that extends from the base to the vertex.



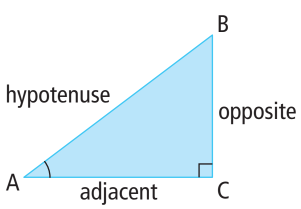
continuous data Data values on a graph that are connected.



coordinates The values in an ordered pair. The *x*-coordinate is the distance from the vertical or *y*-axis. The *y*-coordinate is the distance from the horizontal or *x*-axis.



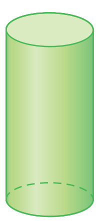
cosine ratio For an acute angle in a right triangle, the ratio of the length of the adjacent side to the length of the hypotenuse. $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$



cube root One of three equal factors of a number.

For example, $\sqrt[3]{512} = \sqrt[3]{(8)(8)(8)}$
$= 8$

cylinder A three-dimensional object with two parallel and congruent circular bases.



D

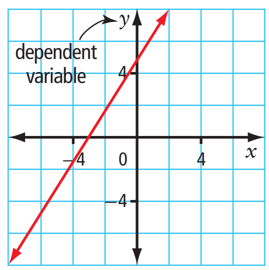
degree (of a polynomial) The degree of the highest-degree term in a polynomial.

For example, the polynomial $7a^2 - 3a$ has a degree of two.

degree (of a term) The sum of the exponents on the variables in a single term.

For example, the degree of $3x^3z^2$ is 5.
A variable with no exponent has a degree of one.

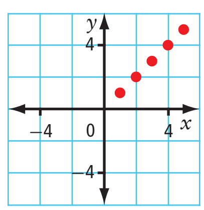
dependent variable The variable whose values depend on those of the independent variable.



difference of squares An expression of the form $a^2 - b^2$ that involves the subtraction of two squares.

For example, $x^2 - 4$ and $y^2 - 25$ are differences of squares.

discrete data Data values on a graph that are not connected.



distributive property The rule that states $a(b + c) = ab + ac$.

For example, $40(20 + 6) = (40)(20) + (40)(6)$.

domain The set of all possible values for the independent variable in a relation.

E

elimination method An algebraic method of solving a system of equations. Add or subtract the equations to eliminate one variable and solve for the other variable.

entire radical The product of 1 and a radical.

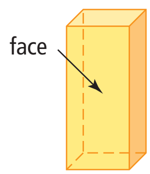
For example, $\sqrt{32}$ and $\sqrt[3]{2^5}$ are entire radicals.

exponent The number of times you multiply the base in a power by itself.

For example, in 2^3, the exponent is 3, so the base is multiplied by itself three times: $2 \times 2 \times 2 = 8$.

F

face A flat or curved surface of a three-dimensional object.



factor Any number or variable that when multiplied with one or more other numbers or variables forms a product.

For example, the factors of 12 are 1, 2, 3, 4, 6, and 12, and the factors of a^2b are a, a, and b.

function A relation in which each value of the independent variable is associated with exactly one value of the dependent variable. For every value in the domain there is a unique value in the range.

function notation A notation used when a relation is a function. It is written $f(x)$ and read as "f of x" or "f at x."

G

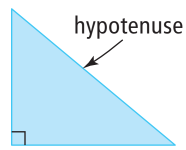
general form The equation of a line in the form $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number. This means that A will always be positive.

greatest common factor (GCF) The largest factor shared by two or more terms.

For example, the GCF of 12 and 28 is 4, and the GCF of x^2yz and x^2y^3 is x^2y.

H

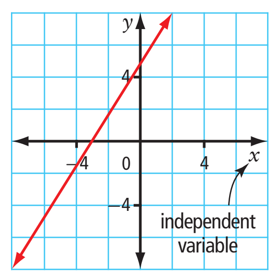
hypotenuse The side opposite the right angle in a right triangle.



I

imperial system A system of measurement based on British units.

independent variable The variable for which values are selected.



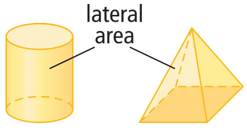
index Indicates what root to take.

irrational number A number that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$. It cannot be expressed as a terminating or repeating decimal.

For example, $\pi = 3.1415\ldots$ and $\sqrt{5} = 2.236\ldots$ are irrational numbers.

L

lateral area The surface that joins the two bases of a three-dimensional object or that joins the base to the highest point.



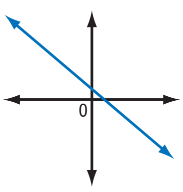
least common multiple (LCM) The smallest multiple shared by two or more terms.

For example, the LCM of 6 and 8 is 24.

like terms Terms that have the same variable(s) raised to the same exponent(s).

For example, $3x$ and $-2x$ are like terms.

linear relation A relation that forms a straight line when the data are plotted on a graph.



M

metric system A system of measurement in which all units are based on powers of ten. The metre is the basic unit of length.

mixed radical The product of a rational number and a radical.

For example, $3\sqrt{2}$ and $\frac{1}{2}\sqrt[3]{6}$ are mixed radicals.

monomial A polynomial with one term.

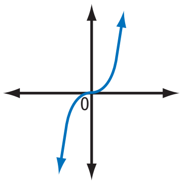
For example, 5, $2x$, $3s^2$, $-8cd$, and $\frac{n^4}{3}$ are monomials.

multiple (of a number) The product of a given number and an integral value.

For example, the multiples of 5 are 5, 10, 15, 20, ….

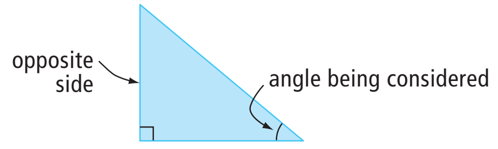
N

non-linear relation A relation that does not form a straight line when the data are plotted on a graph.



O

opposite side The side across from the acute angle being considered in a right triangle. It is the side that does not form one of the arms of the angle being considered.



P

parallel lines Lines in the same plane that do not intersect. They have the same slope but different intercepts.

parameter A variable that has a constant value in a particular equation.

perfect cube A number that can be expressed as the product of three equal factors.

For example, $64 = (4)(4)(4)$ or 4^3.

perfect square A number that can be expressed as the product of two equal factors.

For example, $16 = (4)(4)$ or 4^2.

perfect square trinomial The result of squaring a binomial.

For example, $(x + 5)^2 = x^2 + 10x + 25$ is a perfect square trinomial.

perpendicular lines Two lines that intersect at right angles (90°). These two lines have slopes that are negative reciprocals of each other.

point of intersection A point at which two lines touch or cross.

polynomial An algebraic expression formed by adding or subtracting terms.

For example, $x + 5$, $2d - 2.4$, and $3s^2 + 5s - 6$ are polynomials.

power An expression made up of a base and an exponent.

For example, in the power 6^3, 6 is the base and 3 is the exponent.

primary trigonometric ratios The three ratios—sine, cosine, and tangent—defined in a right triangle.

prime factor A factor that is a prime number; that is, a number divisible only by 1 and itself.

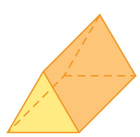
For example, the prime factors of 10 are 2 and 5.

prime factorization The process of writing a number as a product of its prime factors.

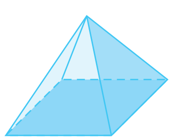
For example, the prime factorization of 24 is $2 \times 2 \times 2 \times 3$.

prism A three-dimensional object with two parallel and congruent polygon bases and rectangular sides.


rectangular prism

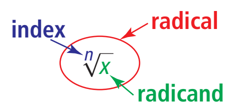

triangular prism

pyramid A three-dimensional object with one base and the same number of triangular faces as there are sides on the base.



R

radical Consists of a root symbol, an index, and a radicand. It can be rational (for example, $\sqrt{4}$) or irrational (for example, $\sqrt{2}$).



radicand The quantity under the radical sign.

range The set of all possible values for the dependent variable as the independent variable takes on all possible values of the domain.

rational exponent An exponent that can be expressed as the quotient of two integers, where the divisor is not zero.

For example, in $16^{\frac{1}{4}}$, $\frac{1}{4}$ is a rational exponent.

rational number A number that can be expressed as the quotient of two integers, where the divisor is not zero.

For example, 0.5, $\frac{3}{4}$, and -2 are rational numbers.

referent An item that an individual uses as a measurement unit for estimating.

For example, the height of a doorknob above the floor is about 1 m, or the thickness of a dime is about 1 mm.

relation An association between two quantities. It can be presented in words, as an equation, as a table of values, as ordered pairs, or as a graph.

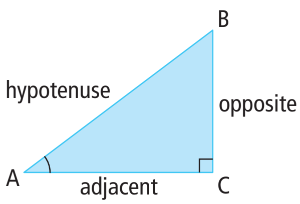
S

set notation A formal mathematical way to give the values of the domain and range.

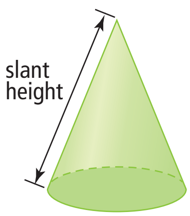
For example, the domain is $\{x \mid x \leq 10, x \in \mathrm{R}\}$ and the range is $\{y \mid y > 20, y \in \mathrm{R}\}$.

SI (Système International d'Unités) A system of measurement in which all units are based on powers of ten. The metre is the basic unit of length.

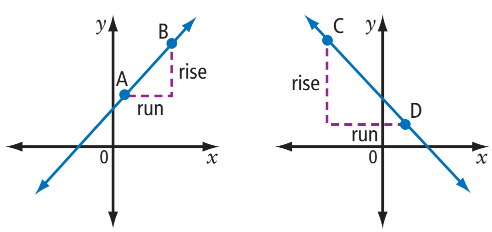
sine ratio For an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the hypotenuse. $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$



slant height The shortest lateral distance from the edge of the base of a cone or pyramid to its highest point.



slope The ratio of the vertical change, or rise, to the horizontal change, or run, of a line or line segment. Slope is not expressed with units.



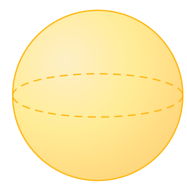
Slope is positive   Slope is negative

slope-intercept form The equation of a line in the form $y = mx + b$, where m is the slope of the line and b is the y-intercept.

slope-point form The equation of a non-vertical line in the form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) are the coordinates of a point on the line.

solution (to a system of linear equations) A point of intersection of the lines on a graph. It is an ordered pair that satisfies both equations, or a pair of values occurring in the tables of values of both equations.

sphere A round, ball-shaped object. It is a set of points in space that are a given distance (radius) from a fixed point (centre).

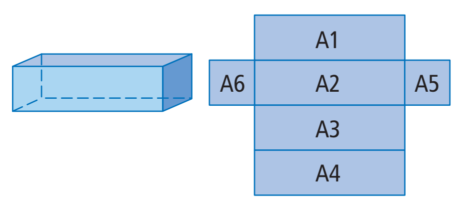


square root One of two equal factors of a number.

For example, $\sqrt{49} = \sqrt{(7)(7)}$
$= 7$

substitution method An algebraic method of solving a system of equations. Solve one equation for one variable. Then, substitute that value into the other equation and solve for the other variable.

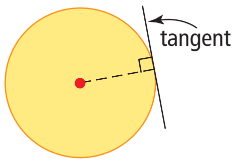
surface area The number of square units needed to cover a three-dimensional object. It is the sum of the areas of all the faces of an object.



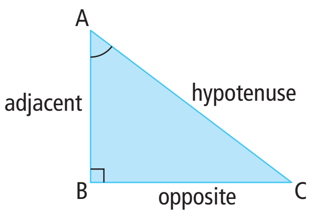
system of linear equations Two or more linear equations involving common variables.

T

tangent (of a circle) A line that touches a circle at exactly one point. The line is perpendicular to the radius at that point. The point where the line touches the circle is called the point of tangency.



tangent ratio For an acute angle in a right triangle, the ratio of the length of the opposite side to the length of the adjacent side. tan A $= \frac{\text{opposite}}{\text{adjacent}}$



term A number or a variable, or the product of numbers and variables.

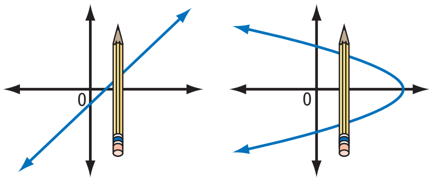
For example, the expression $5x + 3$ has two terms: $5x$ and 3.

trinomial A polynomial with three terms.

For example, $x^2 + 3x - 1$ and $2x^2 - 5xy + 10y^2$ are trinomials.

V

vertical line test A test to see if a graph represents a function. If any vertical line crosses the graph at more than one point, the relation is not a function.



volume The amount of space a three-dimensional object occupies. It is measured in cubic units.

X

x-intercept The x-coordinate of the point where a line or curve crosses the x-axis. It is the value of x when $y = 0$.

Y

y-intercept The y-coordinate of the point where a line or curve crosses the y-axis. It is the value of y when $x = 0$.

Index

S

Credits

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